Constructing de Broglie's Periodic Phenomena

A. Laidlaw
Calle Cuesta de los Cubos, 17, Velez de Benaudalla, Spain.

25 February 2016

Abstract

Les ondes de Broglie ont été initialement dérivées de la transformation Lorentz d'une onde stationnaire, $e^{-i\omega t}$, qui ne dépend pas de l'espace. Il est montré ici q'une onde stationnaire appropriée, physiquement raisonnable peut être construite à partir des ondes physiques qui se propagent à c, à la condition que chacune des lignes de champ du vecteur d'onde existe sur la surface d'une sphère au repos dans le cadre de référence comobile. Ce résultat dément l'image classique d'une particule ponctuelle émettant un champ lointain qui se propage loin de lui, et il est soutenu que, alors que cette construction des ondes de Broglie est à la fois locale et réaliste, on ne peut pas déduire les inégalités de Bell dans le contexte de de Broglie.

De Broglie waves were originally derived from the Lorentz Transformation of a standing wave, $e^{-i\omega t}$, that has no space dependence. It is shown here that a suitable, physically reasonable, standing wave can be constructed from physical waves that propagate at c, subject to the condition that any field line of the wave vector exists on the surface of a sphere at rest in the comoving frame. This result contradicts the classical picture of a point particle emitting a far field that propagates radially away from it, and it is argued that, while the present construction of de Broglie waves is both local and realistic, Bell Inequalities cannot be derived in de Broglie's context.

PACS: 03.30.+p, 03.75.-b Keywords: Solitons; Lorentz Invariance; de Broglie waves

1 Introduction

"Ce qui caractérise l'électron comme atome d'énergie, ce n'est pas la petite place qu'il occupe dans l'espace, je répète qu'il l'occupe tout entier, c'est le fait qu'il est insécable, non-subdivisible, qu'il forme une unité"[1] - Louis de Broglie [1].

While loophole free violations of the Bell Inequalities [2] point to the failure of local causality [3], it will be shown here that this implies neither "spooky"action at a distance nor the impossibility of an ontologically local, realist model, one in which ontological elements (a) do not exceed the characteristic velocity and (b) can interact only to the extent that they are colocated. This conclusion follows directly from the main hypothesis of Louis de Broglie's seminal PhD thesis, namely that a periodic phenomenon of frequency ν_0 should be associated to each massive particle of proper mass m_0 such that $h\nu_0 = m_0c^2$. As is evident from the quotation above, these periodic phenomena were conceived of as extending through all space. In fact, de Broglie went on to derive his eponymous waves from the most basic of periodic functions, $e^{-i\omega t}$, a function that has no space dependence at all.

Although the experimental evidence [4] demands that de Broglie's inherently distributed quanta be taken seriously, associating the function $e^{-i\omega t}$ to every point in space clearly involves instant correlations at a distance, raising the question whether such phenomena are acceptable in local realism. The main task of this Article is therefore to show that a local realist model for de Broglie's hypothetical periodic phenomena can be constructed from the superposition of travelling waves that propagate luminally, i.e. at c. To this end, Section 2 derives the general form for the superposition of two scalar waves of equal amplitude. Section 3 then shows that, while various proposals that would weaken de Broglie's hypothesis are not viable, de Broglie waves can be correctly implemented in wave models subject to the condition that any field line of the wave vector exists on the surface of a sphere at rest in the comoving frame. Equivalently, the wave model must comply with the little group of transformations that leaves a particle's linear momentum invariant in Special Relativity [5].

Given a reality that consists of de Broglie's inherently distributed quanta, the discussion of EPR correlations then reduces to the question how local action constrains the possible interactions between such entities. Section 4 observes that distributed interactions between de Broglie's distributed quanta can have distributed consequences, like wavefunction collapse, that may occur in different places at the same time. An important, albeit implicit, assumption in local causality [3] and the derivation of Bell Inequalities [6], is thus seen not to be valid under de Broglie's hypothesis, namely that causal relations amongst point events are necessarily mediated by retarded influences that travel between their respective locations.

[1] Translation: "What characterises the electron as an atom of energy is not the tiny region it occupies in space, I repeat it occupies the entire space, but the fact that it is indivisible, that it forms a single whole."

The consequence is that, although de Broglie's quanta can be constructed in a local realist field theory, point events in such a model aren't necessarily constrained by local causality. This example shows why local action does not imply local causality or, as Redhead put it in the 1980s [7], "ontological locality does not of necessity imply epistemological locality".

2 Superposition of 2 Luminal Waves

Consider a wave system consisting of 2 luminal waves whose general form is the real part of:

$$\psi_n = A_n \; e^{i(\mathbf{k}_n \cdot \mathbf{r} - \omega_n t)} \tag{1}$$

Where $n = 1, 2$, $\omega_n/k_n = c$ and we shall write the coordinate form of the wave vectors as $\mathbf{k}_n = \sum_{j=1}^{3} k_{nj}\hat{x}_j$. The wave amplitudes, A_n, the wave vectors, $\mathbf{k}_n$, and the wave frequencies, ω_n, might in general all be functions of the coordinates, but let us consider a simplified system with scalar waves whose amplitudes depend only on the distance to the centre of the system: $A_1 = A_2 = A(r)$. Let us also assume that the wave frequency is constant for one set of observers so that $\omega_1 = \omega_2 = \omega$ where ω is a constant, and we have $k_1 = k_2 = k$. For these observers, the superposition is then the real part of:

$$\psi = \psi_1 + \psi_2 = A[e^{i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t)} + e^{i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t)}] = 2A cos(\frac{\phi_1 - \phi_2}{2}) e^{i(\frac{\phi_1 + \phi_2}{2})} \tag{2}$$

Where $\phi_n = \mathbf{k}_n \cdot \mathbf{r} - \omega_n t$. Now, consider the same system from the point of view of an observer moving at speed v in the -ve x direction, whose coordinates are (x', y', z', t') with the frames in standard configuration. The frequencies for each wave in the primed frame can be evaluated using the relativistic Doppler shift result:

$$\omega'_n = \gamma\omega(1 + \beta cos\theta_n) = \gamma\omega(1 + \beta\frac{k_{n1}}{k}) \tag{3}$$

Where $\beta = v/c$, $\gamma = \sqrt{1/(1-\beta^2)}$ and θ_n is the angle between $\mathbf{k}_n$ and the x-axis. Using (3) with the relativistic aberration result and the fact that $c = \omega'_1/k'_1 = \omega'_2/k'_2 = \omega/k$, gives the wave vectors in the primed frame:

$$\mathbf{k}'_n = \gamma(\beta k + k_{n1})\hat{x}'_1 + k_{n2}\hat{x}'_2 + k_{n3}\hat{x}'_3 \tag{4}$$

From (3) and (4) we get $\phi'_1 - \phi'_2 = \gamma\Delta k_1(x' - \frac{\omega\beta}{k}t') + \Delta k_2 y' + \Delta k_3 z'$, where $\Delta k_j = k_{1j} - k_{2j}$ and $\phi'_1 + \phi'_2 = 2\gamma\beta k x' - 2\gamma\omega t' + [\gamma\Sigma k_1(x' - \frac{\omega\beta}{k}t') + \Sigma k_2 y' + \Sigma k_3 z']$, where $\Sigma k_j = k_{1j} + k_{2j}$. The two frames are in standard configuration so $x = \gamma(x' - vt')$, $y = y'$ and $z = z'$ whilst $\omega\beta = kv$, so these expressions can be written as:

$$\phi'_1 - \phi'_2 = (\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} \qquad ; \qquad \phi'_1 + \phi'_2 = 2\gamma\beta k x' - 2\gamma\omega t' + (\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r} \tag{5}$$

Finally, using (5) in (2) we get the superposition for the primed observer, which is the real part of:

$$\psi' = 2A \; cos[\frac{1}{2}(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r}] \;\; e^{i[\gamma\beta k x' - \gamma\omega t' + \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r}]} \tag{6}$$

The de Broglie wave propagates at speed c^2/v with a wavenumber corresponding to the linear momentum of the modelled particle: $p = \gamma m v = \gamma\hbar\omega v/c^2 = \hbar(\gamma\beta k)$, so the real part of $e^{i[\gamma\beta k x' - \gamma\omega t']}$ is a de Broglie wave propagating in the x direction. The next Section identifies the conditions under which (6) replicates the observed interference phenomena [4].

3 Two Special Cases

3.1 Case 1: Radial Propagation. The Spherical Wave Model. $\mathbf{k}_n \cdot \mathbf{r} = \pm kr$

Let us consider the particular case of a spherical wave model with balanced waves in the unprimed frame that propagate radially inwards and outwards with respect to the centre of the particle [8]. In this case, $\mathbf{k}_1 = -\mathbf{k}_2$ and $\mathbf{k}_n \cdot \mathbf{r} = \pm kr$ and (6) reduces to:

$$\psi = 2A \; cos(kr) \;\; e^{i(\gamma\beta k x' - \gamma\omega t')} \tag{7}$$

The author of [8] claims that de Broglie overlooked the significance of the new term[2], $cos(kr)$, multiplying the de Broglie wave, which he interprets as a carrier wave that is modulated by the superluminal de Broglie wave. However, the real part of (7) is plotted in Fig. 1 and it is readily seen that this does not produce the usual interference pattern at the screen in, say, a 2-slit interference experiment. For example, if we consider screen regions where the path difference is half the de Broglie wavelength, so that the (exponential) de Broglie term by itself would interfere destructively, (7) may exhibit anything between

[2] In [8], the author formed superpositions by subtraction rather than addition, so the corresponding term, see (15) of [8], became $sin(kr)$ but this is of no consequence here.

constructive and destructive interference, depending on the much shorter wavelength of the cosine term. De Broglie waves are also solutions to the Klein Gordon and Schroedinger Equations, whereas (7) is not. Consequently, rather than concluding that de Broglie overlooked this term, it is shown here that wave models with radially propagating fields are excluded[3].

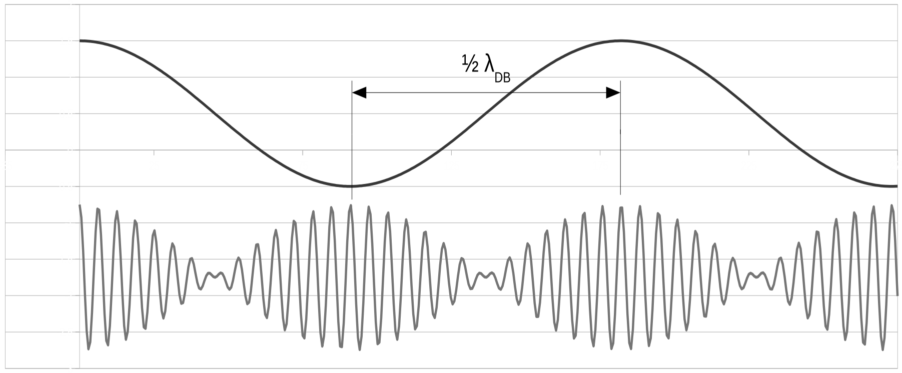

Figure 1: The de Broglie wave (top) and the real part of (7) plotted against x' for $t' = 0$, $y' = z' = 0$ and $\beta = 0.0345$, an example where the spherical wave model leads to constructive interference in screen regions where the de Broglie wave alone interferes destructively.

3.2 Case 2: Transverse Propagation. Waves that "Spin": $\mathbf{k}_n \cdot \mathbf{r} = 0$

Clearly, in order to reproduce the observed interference phenomena, we require $(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} = 0$, in which case the wave vectors in the unprimed frame must have equal radial components. However, allowing $(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r} \neq 0$ in (6) also leads to a problem. Let us write $\alpha = \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2) \cdot \hat{\mathbf{r}}$. Then the real part of the exponential term in (6) is $cos(\gamma\beta kx' - \gamma\omega t')\ cos(\alpha r) - sin(\gamma\beta kx' - \gamma\omega t')\ sin(\alpha r)$. In this case, the de Broglie wave is modified by a rapid, position dependent phase shift that similarly destroys the interference phenomenon.

Thus we require both $(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} = 0$ and $(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r} = 0$, which can only be satisfied if both $\mathbf{k}_1 \cdot \mathbf{r} = 0$ and $\mathbf{k}_2 \cdot \mathbf{r} = 0$. Substituting these in (2) and (6) gives:

$$\psi = 2Ae^{-i\omega t} \qquad \rightarrow \qquad \psi' = 2Ae^{i(\gamma\beta kx' - \gamma\omega t')} \tag{8}$$

Which is just the usual formulation of de Broglie waves. In order to correctly reproduce de Broglie waves and the experimentally observed interference patterns, the wave vectors, as seen from the unprimed frame, should be transverse to the position vector from the centre of the particle to the given point. Any field line of the wave vector then exists on the surface of some sphere at rest in the unprimed frame, which is therefore the comoving frame of the particle. Recall that this frame was identified by the condition $\omega_n = \omega$ where ω is a constant, independent of x, y, z and $\hat{\mathbf{k}}$.

This wavefield structure may be compared with the little group of transformations that preserves the linear momentum of a particle in Special Relativity [5]. For rest particles, the little group reduces to the spatial rotations group SO(3). The internal evolution of a particle as seen from the comoving frame involves no radial movements and the trajectory of any internal movement lies on the surface of some sphere at rest in this frame. Since the wave vector gives the direction of movement of a wavefield, we see that wave models that comply with the little group produce de Broglie waves.

Whilst each of the individual wavefields at any given point is arguably "spinning", or "orbiting", about the centre of the system, that does not imply that the modelled particle as a whole carries any nett angular momentum or spin. However, it seems quite reasonable to anticipate that entire wave solutions that conform to the little group may well also incorporate angular momentum and spin observables.

4 Can de Broglie's Distributed Quanta Explain Spacelike Causal Correlations?

The experimental evidence for matter waves demands that we take de Broglie's hypothesised distributed periodic phenomena seriously, and it has been shown above that his hypothesis cannot be weakened by introducing any space dependence of the phase. However, the above construction is inconsistent with the classical idea of a far field that propagates radially away from one point particle, carrying long range interactions to another. Once it is understood that each of the participating entities occupies the entire space, the usual classical idea of interaction as a thing that travels between their respective locations ceases to make sense.

How then do de Broglie's inherently distributed quanta interact with each other? As far as the Principle of local action is concerned, the indefinitely extended wavefields of two different quanta always interpenetrate and in principle they may

[3]There are several other reasons to doubt the model in [8]: The total field energy diverges. How is the supposed inward propagating wave to be sourced coherently? How can such a model address the angular momenta of particles?

interact with each other wherever they make contact, which is to say everywhere. For the present wave model, this can only take the form of local interactions between the wavefields of each quantum, and since there are no such interactions in a linear field theory, a nonlinear theory is required[4]. A suitable basic theory of nonlinear interactions between wavefields has been developed by Donev and Tashkova [9]. Under de Broglie's hypothesis, as well as occupying the entire space, the functions that are interacting have no space dependence, with the result that any local model that quantifies interactions between de Broglie's quanta will produce the same result at every space point. These distributed interactions act locally on each of the participant quanta, but they also act in parallel at every space point. Distributed interactions between de Broglie's distributed quanta necessarily have distributed consequences, that may occur in different places at the same time. These are distributed events.

This kind of interaction phenomenon is perhaps best exemplified by the nonlocal collapse of the quantum mechanical wavefunction at a measurement interaction. Since it was shown in Section 3 that de Broglie's distributed periodic phenomena are entirely plausible in local realism, it follows that distributed interactions, including measurement interactions, are equally plausible, and so are distributed changes - "nonlocal collapses" - to our description of the physical state of affairs.

Related to this, Bell Inequalities cannot be derived for de Broglie's quanta because an important presumption implicit in local causality is not valid. It is routinely presumed that all of the physical reality corresponding[5] to a point event is necessarily colocated with it. While it is reasonable to assume that the ontology that directly causes the point event is local to it, with distributed quanta, distributed interactions and distributed events it is clearly unreasonable to presume the nonexistence of any remote ontology that may also "correspond" to the point observation, in the sense of being instantly correlated to it. This remote ontology can of course directly influence subsequent, spacelike separated observations. The key point here is that causal influences between distributed events do not necessarily have to travel between the specific locations where we happen to make our observations.

The envelope function, the wave amplitude $A(r)$, makes no difference to this logic. It has been included here so that the total energy of each quantum can be finite: For example, assume, by analogy to Electromagnetics, that the field energy density $\rho_E \propto A^2$ and $A(r) \rightarrow A_{max}/r^2$ as $r \rightarrow \infty$, then the total field energy, $\int_0^\infty 4\pi r^2 \rho_E \, dr$, does not diverge as $r \rightarrow \infty$.

5 Conclusions

It has been shown that de Broglie's hypothesised widely distributed periodic phenomena can be constructed from luminally propagating waves subject to the condition that any field line of the wave vector exists on the surface of a sphere at rest in the comoving frame. Suggestions made elsewhere, that would weaken de Broglie's hypothesis by introducing space dependence of the phase of the standing wave associated to a quantum of matter, were shown to be unacceptable.

The far fields of a particle do not propagate radially away from it, as was routinely presumed in classical physics, and the usual idea of an interaction as something that travels between point particles is inappropriate for a reality that is constructed from de Broglie's inherently distributed quanta. A more appropriate interaction paradigm was suggested in which distributed interactions amongst de Broglie's periodic phenomena also provide the means to explain instantaneous causal correlations at a distance with models that are both local, in the sense of local action, and realistic. On this view, local action does not imply local causality and violation of Bell Inequalities does not imply action at a distance.

References

[1] L. de Broglie, "Recherches sur la Théorie des Quanta", *Annales de Physique* 10^e Serie Tome III (Janvier - Fevrier 1925).

[2] B. Hensen et al, "Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres" *Nature* **526** pp 682-686 (2015)

[3] T. Norsen, "JS Bell's Concept of Local Causality", *American Journal of Physics* **79**, 1261 (2011)

[4] C.J. Davisson, *Bell System Tech. J.* (USA: American Tel.) **7** (1) pp 90-105 (1928)

[5] Y.S. Kim, *Nucl.Phys.Proc.Suppl.* **102** pp 369-376, arXiv:hep-th/0104051 (2001).

[6] J.S. Bell, "On the Einstein Podolsky Rosen Paradox", *Physics* **1** 195-200 (1965).

[7] Redhead M, *"Incompleteness, Nonlocality and Realism"*, Oxford University Press (1987)

[8] D. Shanahan, *Found. Phys.* **44** pp. 349-367 arXiv:1401.4534v3 (2014).

[9] S. Donev and M. Tashkova, "From Maxwell Stresses to Nonlinear Field Equations", arXiv:physics/0604021v6 (2006).

[10] A. Einstein, B. Podolsky and N. Rosen, "Can Quantum-mechanical Description of Physical Reality be Considered Complete?", *Phys. Rev.* **47**, 777 (1935).

[4] Of course, a nonlinear theory is required in any case by the condition that any field line of the wave vector exists on the surface of a sphere.

[5] In the sense of the EPR sufficiency condition [10]. The EPR sufficiency condition may not be the only way to specify the correspondence between observables and physical reality, but, one way or another, formulating causal relations between point events requires making some such inference.