Each unidimensional system is Hamiltonian, so that each unidimensional system is quantizable

Abstract

I prove that the field of classical trajectories can be a field Hamiltonian projection of higher dimension.

I hypothesize that the same is valid for any dimension: each system is Hamiltonian, and each system is quantizable using the corrispondence principle

Unidimensional Hamiltonian System

Each unidimensional trajectory can be described by a differential equation of high order, and high degree (each derivable function can be approximate by a sum of Taylor, Fourier and Laplace series, that is a solution of a linear differential equation, but an improved approximation is a nonlinear differential equation):

$$0 = \mathcal{F}\left(y, \dot{y}, \ddot{y}, \cdots\right) = a_{10\cdots} + a_{010\cdots} y + a_{0010\cdots} \dot{y} + \cdots + a_{0101\cdots} y \ddot{y} + \cdots \tag{1}$$

$$0 = \mathcal{F}\left(y, \dot{y}, \ddot{y}, \cdots\right) = \sum_{i_0,\cdots,i_n} a_{i_0,\cdots,i_n} \frac{d^{i_0} y}{dt^{i_0}} \cdots \frac{d^{i_n} y}{dt^{i_n}} \tag{2}$$

the derive of the differential equation is linear in the higher derivative:

$$0 = \frac{d\mathcal{F}\left(y, \dot{y}, \ddot{y}, \cdots\right)}{dt} = \frac{d}{dt} \sum_{i_0,\cdots,i_n} a_{i_0,\cdots,i_n} \prod_{s=1}^{n} \left(\frac{d^s y}{dt^s}\right)^{i_s} = \sum_{k,i_0,\cdots,i_n} a_{i_0,\cdots,i_n} \prod_{s=1}^{n} i_k \left(\frac{d^s y}{dt^s}\right)^{i_s - \delta_{sk}} \frac{d^{k+1} y}{dt^{k+1}} \tag{3}$$

$$\frac{d^N y}{dt^N} = \mathcal{G}\left(y, \frac{dy}{dt}, \ldots, \frac{d^{N-1} y}{dt^{N-1}}\right) \tag{4}$$

so that each polynomial differential equation can be write linearly in the maximum derivative; so that:

$$\begin{cases} y = y_0 \\ \dot{y}_0 = y_1 \\ \dot{y}_1 = y_2 \\ \vdots \\ \dot{y}_{N-2} = y_{N-1} \\ \dot{y}_{N-1} = \mathcal{G}(y_0, \cdots, y_{N-1}) \end{cases} \tag{5}$$

this system can be the half of an Hamiltonian system $H = \sum_i p_i f_i$, that have N new momenta:

$$H = \sum_{i=0}^{N} p_i f_i = \sum_{j=0}^{N} p_i \left\{ y_{i+1} + \delta_{iN} \left[-y_{i+1} + \mathcal{G} \right] \right\} = \sum_{i=0}^{N-1} p_i y_{i+1} + p_N \mathcal{G}$$

$$\begin{cases} \dot{y}_{j \neq N} = \frac{\partial H}{\partial p_j} = f_j = y_{j+1} \\ \dot{y}_N = \frac{\partial H}{\partial p_N} = f_N = \mathcal{G} \\ \dot{p}_{j \neq N} = -\frac{\partial H}{\partial y_j} = -p_N \frac{\partial \mathcal{G}}{\partial y_j} - p_{j-1} \\ \dot{p}_N = -\frac{\partial H}{\partial y_N} = -p_{N-1} \end{cases} \tag{6}$$

the trajectories in the coordinates are ever the same, for each momentum initial condition; the volume of the phase space is an invariant in the space (coordinates,momenta) and the sum of the areas is invariant, because of there is a momenta compensation.

In this case, each quantum system is equal to the classical system:

$$\begin{array}{l} H = p_i f_i \\ i\hbar \partial_t \psi = -i\hbar \sum_i f_i \partial_i \psi \\ \boxed{0 = \partial_t \psi + \sum_i f_i \partial_i \psi} \end{array} \tag{7}$$

there are two classical solutions:

$$\begin{array}{l} \begin{cases} \dot{y}_1 = f_1 \\ \vdots \\ \dot{y}_N = f_N \end{cases} \\ \frac{dy_1}{f_1} = \cdots = \frac{dy_N}{f_N} = dt \\ \partial_t \psi + \sum_i f_i \partial_i \psi = 0 \end{array} \tag{8}$$

that is a surface solution, in an N+1 dimensional space (coordinates and times), and ψ is the solution of the differential equation.

Another solution is the Hamilton-Jacobi equation, that give the classical solution of the Hamiltonian:

$$\begin{array}{c} H = p_i f_i \\ \partial_t \psi + H(p_i = \partial_i \psi, y_i) = 0 \\ \boxed{\partial_t \psi + \sum_i f_i \partial_i \psi = 0} \end{array} \tag{9}$$

in this case the function ψ permit to calculate the momenta values like a gradient of the ψ function. Also in this case the classical solution, and the quantum solution, coincide; and the equation for the amplitude, or the probability, are equal because of the linearity of the equation.