Observation on the period of the rational number P÷d + d÷P where P is a 3-Poulet number and d its least prime factor

Abstract. In this paper I make the following observation: let P be a 3-Poulet number, d its least prime factor and q one of the other two prime factors; then the lenght of the period of the rational number P/d + d/P is for almost any P equal to q – 1 or equal to (q – 1)/n or equal to (q – 1)*n, where n positive integer.

Observation:

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Note:

The sequence of 3-Poulet numbers: 561, 645, 1105, 1729, 1905, 2465, 2821, 4371, 6601, 8481, 8911, 10585, 12801, 13741, 13981, 15841, 16705, 25761, 29341, 30121, 30889, 33153, 34945, 41665, 52633, 57421, 68101, 74665, 83665, 87249, 88561, 91001, 93961, 113201, 115921, 121465, 137149 (...). See the sequence A215672 that I submitted on OEIS.

Verifying the observation:
(true for 29 from the first 31 such Poulet numbers)

: for P = 561 = 3*11*17, the period of P/d + d/P is equal to 5347593582887700, which has the lenght 16 = 17 - 1;
: for P = 645 = 3*5*43, the period of P/d + d/P is equal to 465116279069767441860, which has the lenght 21 = (43 – 1)/2;
: for P = 1105 = 5*13*17, the period of P/d + d/P is equal to 452488687782805429864253393665158371040723981900, which has the lenght 48 = (17 – 1)*3;
: for P = 1729 = 7*13*19, the period of P/d + d/P is equal to 404858299595141700, which has the lenght 18 = 19 - 1;
: for P = 1905 = 3*5*127, the period of P/d + d/P is equal to 157480314960629921259842519685039370078740, which has the lenght 42 = (127 – 1)/3;
: for P = 2465 = 5*17*29, the period of P/d + d/P has the lenght 112 = (29 – 1)*4;
: for P = 2821 = 7*13*31, the period of P/d + d/P has the lenght 30 = 31 - 1;

: for P = 4371 = 3*31*47, the period of P/d + d/P has the lenght 690 = (47 - 1)*15;
: for P = 6601 = 7*23*41, the period of P/d + d/P has the lenght 110 = (23 - 1)*5;
: for P = 8481 = 3*11*257, the period of P/d + d/P has the lenght 256 = 257 - 1;
: for P = 8911 = 7*19*67, the period of P/d + d/P has the lenght 198 = (67 - 1)*3;
: for P = 10585 = 5*29*73, the period of P/d + d/P has the lenght 56 = (29 - 1)*2;
: for P = 12801 = 3*17*251, the period of P/d + d/P has the lenght 400 = (17 - 1)*25;
: for P = 13741 = 7*13*151, the period of P/d + d/P has the lenght 150 = 151 - 1;
: for P = 13981 = 11*31*41, the period of P/d + d/P has the lenght 15 = (31 - 1)/2;
: for P = 15841 = 7*31*73, the period of P/d + d/P has the lenght 120 = (31 - 1)*4;
: for P = 16705 = 5*13*257, the period of P/d + d/P has the lenght 768 = (257 - 1)*3;
: for P = 29341 = 13*37*61, the period of P/d + d/P has the lenght 60 = 61 - 1;
: for P = 30121 = 7*13*331, the period of P/d + d/P has the lenght 330 = 331 - 1;
: for P = 30889 = 17*23*79, the period of P/d + d/P has the lenght 286 = (23 - 1)*13;
: for P = 33153 = 3*43*257, the period of P/d + d/P has the lenght 5376 = (257 - 1)*21;
: for P = 34945 = 5*29*241, the period of P/d + d/P has the lenght 420 = (29 - 1)*15;
: for P = 41665 = 5*13*641, the period of P/d + d/P has the lenght 96 = (13 - 1)*8;
: for P = 57421 = 7*13*631, the period of P/d + d/P has the lenght 630 = 631 - 1;
: for P = 68101 = 11*41*151, the period of P/d + d/P has the lenght 75 = (151 - 1)/2;
: for P = 74665 = 5*109*137, the period of P/d + d/P has the lenght 216 = (109 - 1)*2;
: for P = 83665 = 5*29*577, the period of P/d + d/P has the lenght 4032 = (577 - 1)*7;
: for P = 87249 = 3*127*229, the period of P/d + d/P has the lenght 1596 = (229 - 1)*7;
: for P = 88561 = 11*83*97, the period of P/d + d/P has the lenght 3936 = (97 - 1)*41.

Exceptions:

: for P = 25761 = 3*31*277, the period of P/d + d/P has the lenght 345;
: for P = 52633 = 7*73*103, the period of P/d + d/P has the lenght 136.