ONE CONSTRUCTION OF AN AFFINE PLANE OVER A CORPS

Phd.Candidate. Orgest ZAKA[1], Prof.Dr.Kristaq FILIPI[2]

[1]University of Vlora "Ismail QEMALI", Email Address gertizaka@yahoo.com

[2] Polytechnic University of Tirana, Email Address f_kristaq@hotmail.com

Abstract.

In this paper, based on several meanings and statements discussed in the literature, we intend constuction a affine plane about a of whatsoever corps (K,⊕,⊙). His points conceive as ordered pairs (α,β), where α and β are elements of corps (K,⊕,⊙). Whereas straight-line in corps, the conceptualize by equations of the type $x⊙a⊕y⊙b=c$, where $a\neq 0_K$ or $b\neq 0_K$ the variables and coefficients are elements of that body. To achieve this construction we prove some theorems which show that the incidence structure $\mathcal{A}=(\mathcal{P}, \mathcal{L}, \mathcal{I})$ connected to the corps ***K*** satisfies axioms A1, A2, A3 definition of affine plane. In all proofs rely on the sense of the corps as his ring and properties derived from that definition.

Keywords: The unitary ring, integral domain, zero division, corps, incdence structurse, point connected to a corp, straight line connected to a corp, affine plane.

1. INTRODUCTION. GENERAL CONSIDERATIONS ON THEAFFINE PLANE AND THE CORPS.

In this paper initially presented some definitions and statements on which the next material.

Let us have sets $\mathcal{P}, \mathcal{L}, \mathcal{I}$, where the two first are non-empty.

Definition 1.1: The incidence structure called a ordering trio $\mathcal{A}=(\mathcal{P}, \mathcal{L}, \mathcal{I})$ where $\mathcal{P}\cap\mathcal{L}=\emptyset$ and $\mathcal{I}\subseteq\mathcal{P}\times\mathcal{L}$.

Elements of sets $\mathcal{P}$ we call points and will mark the capitalized alphabet, while those of the sets $\mathcal{L}$, we call blocks (or straight line) and will mark minuscule alphabet. As in any binary relation, the fact $(P, \ell)\in\mathcal{I}$ for $P\in\mathcal{P}$ and for $\ell\in\mathcal{L}$, it will also mark $P\,\mathcal{I}\,\ell$ and we will read, point P is incident with straight line ℓ or straight line ℓ there are incidents point P. (See [3], [4], [5], [10], [11], [12], [13], [14], [15]).

Definition 1.2. ([3], [8], [16]) Affine plane called the incidence structure $\mathcal{A}=(\mathcal{P}, \mathcal{L}, \mathcal{I})$, that satisfies the following axioms:

> **A1:** *For every two different points* ***P*** *and* ***Q*** $\in\mathcal{P}$, *there is one and only one straight line* $\ell\in\mathcal{L}$, *passing of those points.*

The straight line ℓ defined by points P and Q will mark the PQ.

> **A2:** *For a point* $P\in\mathcal{P}$, *and straight line* $\ell\in\mathcal{L}$ *such that* $(P,\ell)\notin\mathcal{I}$, *there is one and only one straight line* $\mathbf{m}\in\mathcal{L}$, *passing the point P, and such that* $\ell\cap r=\emptyset$.
>
> **A3:** *In* $\mathcal{A}$ *here are three non-incident points to a straight line.*

A1 derived from the two lines different of $\mathcal{L}$ many have a common point, in other words *two different straight lines of* $\mathcal{L}$ *or do not have in common or have only one common point.*
In affine plane $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), these statements are true.

Proposition 1.1. ([3], [5]) *In affine plane* $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), *there are four points, all three of which are not incident with a straight line (three points are called non-collinear).*

Proposition 1.2. ([3], [6],]) *In affine plane* $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), exists four different straight line.

Proposition 1.2. ([3], [8]) *In affine plane* $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), *every straight line is incident with at least two different points.*

Proposition 1.3. ([3], [9]) *In affine plane* $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), *every point is incidents at least three of straight line.*

Proposition 1.4. ([3]) *On a finite affine plane* $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), *every straight line contains the same number of points and in every point the same number of straight line passes. Furthermore, there is the natural number* $n \in \mathbb{N}$, $n \geq 2$, such that:

- **1)** *In each of straight line* $\ell \in \mathcal{L}$, *the number of incidents is points with him is n.*
- **2)** *For every point* $P \in \mathcal{P}$, *of affine plane* $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), *it has exactly n + 1 straight line incident with him.*
- **3)** *In a finite affine plane* $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), *there are exactly* n^2 *points.*
- **4)** *In a finite affine plane* $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), *there are exactly* $n^2 + n$ *straight line.*

The number *n* in Proposition 1.4, it called order of affine plane $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$), it is distinctly that the less order a finite affine plane, is *n = 2. In a such affine plane it is with four points and six straight lines, shown in Fig.1.*

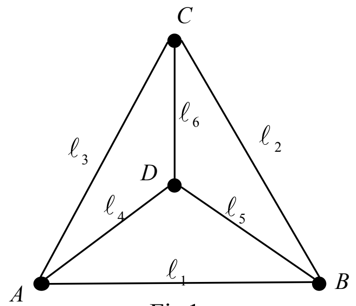

Fig.1

Definition 1.3. ([1]). *The ring called structures* $(B,\oplus,\odot)$, *that has the properties:*

- **1)** *structure* $(B,\oplus)$, *is an abelian group;*
- **2)** *The second action* $\odot$ *It is associative ;*
- **3)** *The second action* $\odot$ *is distributive of the first operation of the first* $\oplus$.

In a ring (B,$\oplus$,$\odot$) also included the action deduction – accompanying each (a, b) from B, sums

$$a \oplus (-b)$$

well

$$a \oplus (-b) = a - b$$

Proposition 1.5 ([1], [7]). *In a unitary ring* $(B,\oplus,\odot)$, *having more than one element, the unitary element* 1_B *is different from* 0_B.

Definition 1.4 ([1], [2]). Corp called rings $(K,\oplus,\odot)$ that has the properties:

1) K is at least one element different from zero.
2) $K^* = K - \{0_K\}$ it is a subset of the stable of K about multiplication;
3) $(K^*, \odot)$ *is a group.*

THEOREM 1.1. ([2]) If $(K, \oplus, \odot)$ **is the corp, then:**

1) **it is the unitary element (is the unitary ring);**
2) **there is no zero divisor (is integral domain);**
3) **They have single solutions in K equations $a \odot x = b$ and $x \odot a = b$, where $a \neq 0_K$ and b are two elements what do you want of K.**

2. TRANSFORMS OF A INCIDENCE STRUCTURES RELATING TO A CORPS IN A AFFINE PLANE.

Definition 2.1. Let it be $(K, \oplus, \odot)$ a corps. A ordered pairs *(α, β) by coordinates α, β ∈K, called point connected to the corp K.*

Sets K^2 of points associated with corps K mark $\mathcal{P}$.

Definition 2.2. Let be *a, b, c*∈*K*.. Sets

$$\ell = \{(x, y) \in K^2 \mid x \odot a \oplus y \odot b = c,\ a \neq 0_K \text{ or } b \neq 0_K\} \tag{1}$$

called the straight line associated with corps K.

Equations $x \odot a \oplus y \odot b = c$, *called equations of the straight line* ℓ. *Sets of straight lines connected to the body K mark* $\mathcal{L}$. It is evidently that

$$\mathcal{P} \cap \mathcal{L} = \varnothing .$$

Definition 2.3. *Will say that the point* $P=(\alpha, \beta) \in \mathcal{P}$ *is incident to straight line (1), if its coordinates verify equation of* ℓ,

This means that if it is true equation $\alpha \odot a \oplus \beta \odot b = c$. This fact write down

$$P \in \ell.$$

Defined in this way is an incidence relations

$$\mathcal{I} \subseteq \mathcal{P} \times \mathcal{L},$$

such that $\forall (P, \ell)$, $\mathcal{P}\mathcal{I}\mathcal{L} \Leftrightarrow P \in \ell$. So even here, when pionts *P* is incidents with straight line ℓ, we will say otherwise point P is located at straight line ℓ, or straight line ℓ *passes by points P.*

It is thus obtained, connected to the corps ***K*** a incidence structure $\mathcal{A}=(\mathcal{P}, \mathcal{L}, \mathcal{I})$. Our intention is to study it.

According to (1), a straight line ℓ its having the equation

$$x \odot a \oplus y \odot b = c, \text{ where } a \neq 0_K \text{ or } b \neq 0_K . \tag{2}$$

Condition (2) met on three cases: 1) $a \neq 0_K$ and $b = 0_K$; 2) $a = 0_K$ and $b \neq 0_K$; 3) $a \neq 0_K$ and $b \neq 0_K$, that allow the separation of the sets $\mathcal{L}$ the straight lines of its three subsets $\mathcal{L}_0$, $\mathcal{L}_1$, $\mathcal{L}_2$ as follows:

$$\boldsymbol{\mathcal{L}_0} = \{\ell \in \mathcal{L} \mid x \odot a \oplus y \odot b = c,\ a \neq 0_K \text{ and } b = 0_K\}; \tag{3}$$

$$\boldsymbol{\mathcal{L}_1} = \{\ell \in \mathcal{L} \mid x \odot a \oplus y \odot b = c,\ a = 0_K \text{ and } b \neq 0_K\}; \tag{4}$$

$$\boldsymbol{\mathcal{L}_2} = \{\ell \in \mathcal{L} \mid x \odot a \oplus y \odot b = c,\ a \neq 0_K \text{ and } b \neq 0_K\}. \tag{5}$$

Otherwise, subset $\boldsymbol{\mathcal{L}_0}$ is a sets of straight lines $\ell \in \mathcal{L}$ with equation

$$x \odot a = c, \text{ where } a \neq 0_K \Leftrightarrow x = d, \text{ where } d = c \odot a^{-1}; \qquad (3')$$

subset $\boldsymbol{\mathcal{L}_1}$ is a sets of straight lines $\ell \in \mathcal{L}$ with equation

$$y \odot b = c, \text{ where } b \neq 0_K \Leftrightarrow y = f, \text{ where } f = c \odot b^{-1}; \qquad (4')$$

Whereas subset $\boldsymbol{\mathcal{L}_2}$ is a sets of straight lines $\ell \in \mathcal{L}$ with equation

$$x \odot a \oplus y \odot b = c, \text{where } a \neq 0_K \text{ and } b \neq 0_K \Leftrightarrow y = x \odot k \oplus g; \qquad (5')$$

$$\text{where } k = (-1_K) \odot a \odot b^{-1} \neq 0_K, \; g = c \odot b^{-1};$$

Hence the

- a straight line $\ell \in \boldsymbol{\mathcal{L}_0}$ is completely determined by the element $d \in K$ such that its equation is $x = d$,
- a straight line $\ell \in \boldsymbol{\mathcal{L}_1}$ is completely determined by the element $f \in K$ such that its equation is $y = f$ and
- a straight line $\ell \in \boldsymbol{\mathcal{L}_2}$ is completely determined by the elements $k \neq 0_K, g \in K$ such that its equation is $y = x \odot k \oplus g$.

From the above it is clear that $\Pi = \{\boldsymbol{\mathcal{L}_0}, \boldsymbol{\mathcal{L}_1}, \boldsymbol{\mathcal{L}_2}\}$ is a separation of the sets of straight lines $\boldsymbol{\mathcal{L}}$.

THEOREM 2.1. **For every two distinct points P, $Q \in \mathcal{P}$, there exist only one straight line $\ell \in \mathcal{L}$ that passes in those two points.**

***Proof*.** Let $P=(p_1, p_2)$ and $Q=(q_1, q_2)$. Fact that $P \neq Q$ means

$$(p_1, p_2) \neq (q_1, q_2). \qquad (6)$$

Based on (6) we distinguish three cases:

1) $p_1 = q_1$ and $p_2 \neq q_2$;
2) $p_1 \neq q_1$ and $p_2 = q_2$;
3) $p_1 \neq q_1$ and $p_2 \neq q_2$;

Let's be straight line $\ell \in \mathcal{L}$, yet unknown, according to (2), having the equation $x \odot a \oplus y \odot b = c$, where $a \neq 0_K$ *or* $b \neq 0_K$.

Consider the **case 1)** $p_1 = q_1$ and $p_2 \neq q_2$. From the fact P, $Q \in \ell$ we have:

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ q_1 \odot a \oplus q_2 \odot b = c \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \oplus q_2 \odot b \end{cases}$$

But $p_1 = q_1$ and $p_2 \neq q_2$, so, from the fact that $(K,\oplus)$ is abelian group, by Definition 1.4, we get

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_2 \odot b = q_2 \odot b \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ (p_2 - q_2) \odot b = 0_K \end{cases}$$

From above, according to Theorem 1.1, corps K is complete ring, so with no divisor 0_K, results

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ b = 0_K \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a = c \\ b = 0_K \end{cases}$$

From this

a) if $p_1 = 0_K$, we get

$$\begin{cases} 0_K \odot a = c \\ b = 0_K\ (a \neq 0_K) \end{cases} \Leftrightarrow \begin{cases} c = 0_K \\ b = 0_K\ (a \neq 0_K) \end{cases}.$$

According to this result, equation (2) takes the form $x \odot a = 0_K$, where $a \neq 0_K$, otherwise

$$x = 0_K \tag{7}$$

(since, being $a \neq 0_K$, it is element of group $(K^*, \odot)$, so
$x \odot a = 0_K \Leftrightarrow x = 0_K \odot a^{-1} \Leftrightarrow x = 0_K$).

b) if $p_1 \neq 0_K$, and p_1 is element of group $(K^*, \odot)$, exists p_1^{-1}, that get the results:

$$\begin{cases} a = p_1^{-1} \odot c \\ b = 0_K\ (a \neq 0_K) \end{cases}$$

under which, equation (2) in this case take the form

$$x \odot p_1^{-1} \odot c = c, \text{ where } a \neq 0_K \Leftrightarrow x \odot p_1^{-1} = 1_K \Leftrightarrow x = p_1 \tag{7'}$$

Here it is used the right rules simplifying in the group $(K^*, \odot)$, with $c \neq 0_K$, because $a = p_1^{-1} \odot c$ and $a \neq 0_K$.

For two cases (7) and (7') notice that, when $p_1 = q_1$ and $p_2 \neq q_2$, there exists a unique straight line ℓ with equation $x = d$ of the form (3'), so a line $\ell \in \mathcal{L}_0$.

Case 2) $p_1 \neq q_1$ and $p_2 = q_2$ is an analogous way and achieved in the conlusion and in this case there exists a unique straight line ℓ with equation $y = f$ of the form (4'), so a line $\ell \in \mathcal{L}_1$.

Consider now the **case 3)** $p_1 \neq q_1$ and $p_2 \neq q_2$. From the fact $P, Q \in \ell$ we have:

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ q_1 \odot a \oplus q_2 \odot b = c \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \oplus q_2 \odot b \end{cases}. \tag{8}$$

The second equation can be written in the form

$$(p_1 - q_1) \odot a = (q_2 - p_2) \odot b, \text{ that bearing } a \neq 0_K \text{ and } b \neq 0_K \tag{9}$$

Regarding to the coordinates of point P we distinguish these four cases:

a) $p_1 = 0_K = p_2$. This bearing $q_1 \neq 0_K$ and $q_2 \neq 0_K$. In this conditions (8) take the form

$$\begin{cases} c = 0_K \\ a = -q_1^{-1} \odot q_2 \odot b \end{cases}.$$

According to this result, equation (2) take the form $x \odot (-q_1^{-1} \odot q_2 \odot b) \oplus y \odot b = 0_K$, where , according (9), $b \neq 0_K$. So, by the properties of group we have:

$$[x \odot (-q_1^{-1} \odot q_2) \oplus y] \odot b = 0_K \Leftrightarrow -x \odot (q_1^{-1} \odot q_2) \oplus y = 0_K \odot b^{-1} \Leftrightarrow$$

$$y = x \odot (q_1^{-1} \odot q_2), \text{ where } q_1^{-1} \odot q_2 \neq 0_K \tag{10}$$

b) $p_1 = 0_K \neq p_2$. This bearing $q_1 \neq 0_K$. In this conditions, system (8) take the form

$$\begin{cases} p_2 \odot b = c \\ q_1 \odot a = q_1^{-1} \odot (p_2 - q_2) \odot b \end{cases}.$$

–This result, give the equation (2) the form $x \odot [q_1^{-1} \odot (p_2 - q_2)] \odot b \oplus y \odot b = c$, where besides $c \neq 0_K$, by (9), the $b \neq 0_K$. So, by the properties of group we have:

$$x \odot [q_1^{-1} \odot (p_2 - q_2)] \odot b \oplus y \odot b = c \Leftrightarrow [x \odot q_1^{-1} \odot (p_2 - q_2) \oplus y] \odot b = c \Leftrightarrow$$

$$[x \odot q_1^{-1} \odot (p_2 - q_2) \oplus y] \odot p_2^{-1} \odot c = c \Leftrightarrow$$
$$[x \odot q_1^{-1} \odot (p_2 - q_2) \oplus y] \odot p_2^{-1} = 1_K \Leftrightarrow$$
$$x \odot [q_1^{-1} \odot (p_2 - q_2)] \oplus y = p_2 \Leftrightarrow$$
$$y = x \odot [q_1^{-1} \odot (p_2 - q_2)] \oplus p_2, \quad \text{where } q_1^{-1} \odot (p_2 - q_2) \neq 0_K \qquad (11)$$

c) $p_1 \neq 0_K = p_2$. This bearing $q_2 \neq 0_K$, and the system (8) take the form

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ (p_1 - q_1) \odot a = (q_2 - p_2) \odot b \end{cases}.$$

In a similar way *b*) it is shown that equation (2) take the form

$$y = x \odot [(q_1 - p_1)^{-1} \odot q_2] \oplus p_1 \odot (p_1 - q_1)^{-1} \odot q_2$$
$$\text{where } (q_1 - p_1)^{-1} \odot q_2 \neq 0_K . \qquad (12)$$

d) $p_1 \neq 0_K$ and $p_2 \neq 0_K$. We distinguish four subcases:

***d*₁)** $q_1 = 0_K = q_2$. From the system (8) we have

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ q_1 \odot a \oplus q_2 \odot b = 0_K \end{cases} \Rightarrow c = 0_K \text{ and } a = -p_1^{-1} \odot p_2 \odot b$$

After e few transformations equation (2) take the form

$$y = x \odot (p_1^{-1} \odot p_2), \text{ where } p_1^{-1} \odot p_2 \neq 0_K \qquad (13)$$

***d*₂)** $q_1 = 0_K \neq q_2$. From the system (8) we have

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_2 \odot b \end{cases} \Rightarrow q_2 \odot b = c \Rightarrow c = 0_K$$

and $a = p_1^{-1} \odot (q_2 - p_2) \odot b$, where $b = q_2^{-1} \odot c$.

After e few transformations equation (2) take the form

$$y = x \odot [p_1^{-1} \odot (p_2 - q_2)] \oplus q_2, \text{ ku } p_1^{-1} \odot (p_2 - q_2) \neq 0_K \qquad (14)$$

***d*₃)** $q_1 \neq 0_K = q_2$. In this conditions (8) bearing

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \end{cases} \Rightarrow q_1 \odot a = c \Rightarrow c \neq 0_K$$

and $b = p_2^{-1} \odot (q_1 - p_1) \odot a$, where $a = q_1^{-1} \odot c$.

After e few transformations equation (2) take the form

$$y = x \odot [q_1^{-1} \odot (p_1 - q_1) \odot p_2] \oplus q_1 \odot (q_1 - p_1)^{-1} \odot p_2,$$
$$\text{where } q_1^{-1} \odot (p_1 - q_1) \odot p_2 \neq 0_K \qquad (15)$$

***d*₄)** $q_1 \neq 0_K$ and $q_2 \neq 0_K$. If $c = 0_K$ system (8) have the form

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = 0_K \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \oplus q_2 \odot b \end{cases}.$$

After e few transformations results that the equation (2) have the form

$$y = x \odot [q_1^{-1} \odot (p_1 - q_1)^{-1} \odot (p_2 - q_2)],$$
$$\text{where } q_1^{-1} \odot (p_1 - q_1)^{-1} \odot (p_2 - q_2) \neq 0_K \quad (16)$$

If $c \neq 0_K$, system (8), by multiplying both sides of his equations with c^{-1}, this is transform as follows:

$$\begin{cases} p_1 \odot a \oplus p_2 \odot b = c \\ p_1 \odot a \oplus p_2 \odot b = q_1 \odot a \oplus q_2 \odot b \end{cases} \Leftrightarrow \begin{cases} p_1 \odot a_1 \oplus p_2 \odot b_1 = 1_K \\ p_1 \odot a_1 \oplus p_2 \odot b_1 = q_1 \odot a_1 \oplus q_2 \odot b_1 \end{cases}$$

From this equation (2) take the form

$$y = x \odot [(p_1 - q_1)^{-1} \odot (p_2 - q_2)] \oplus b_1^{-1},$$
$$\text{where } (p_1 - q_1)^{-1} \odot (p_2 - q_2) \neq 0_K \quad (17)$$

As conclusion, from the four cases (14), (15), (16) and (17), we notice that, when $p_1 \neq q_1$ and $p_2 \neq q_2$, there exists an unique straight line ℓ with equation $y = x \odot k \oplus g$ of the form (5'), so a line $\ell \in \mathcal{L}_2$.

THEOREM 2.2. **For a point $P \in \mathcal{P}$ and a straight line $\ell \in \mathcal{L}$ such that $P \notin \ell$ exists only one straight line $r \in \mathcal{L}$ passing the point P, and such that $\ell \cap r = \varnothing$.**

Proof. Let it be $\boldsymbol{P} = (p_1, p_2)$. We distinguish cases:

- **a)** $p_1 = 0_K$ and $p_2 = 0_K$;
- **b)** $p_1 \neq 0_K$ and $p_2 = 0_K$;
- **c)** $p_1 = 0_K$ and $p_2 \neq 0_K$;
- **d)** $p_1 \neq 0_K$ and $p_2 \neq 0_K$;

The straight line, still unknown r, let us have equation

$$x \odot \alpha \oplus y \odot \beta = \gamma, ku \ \ \alpha \neq 0_K \ ose \ \ \beta \neq 0_K \quad (18)$$

For straight line ℓ, we distinguish these cases: **1)** $\ell \in \mathcal{L}_0$**; 2)** $\ell \in \mathcal{L}_1$**; 3)** $\ell \in \mathcal{L}_2$

Case 1) $\ell \in \mathcal{L}_0$**.** In this case it has equation $x = d$.

The fact that $P = (p_1, p_2) \notin \ell$, It brings to $p_1 \neq d$. But the fact that $\ell \cap r = \varnothing$, it means that there is no point $Q \in \mathcal{P}$, that $Q \in \ell$ and $Q \in r$, otherwise is this true

$$\forall Q \in \mathcal{P}, Q \notin \ell \cap r. \quad (19)$$

In other words there is no system solution

$$\begin{cases} x = d \neq p_1 \\ x \odot \alpha \oplus y \odot \beta = \gamma \end{cases}. \quad (19')$$

since $P \in r$, that brings

$$p_1 \odot \alpha \oplus p_2 \odot \beta = \gamma, \text{ where } \alpha \neq 0_K \ and \ \beta \neq 0_K \quad (20)$$

In case **a)** $p_1 = 0_K$ and $p_2 = 0_K$, from (20) it turns out that $\gamma = 0_K$,

Then equation (18) take the form

$$x \circledcirc \alpha \oplus y \circledcirc \beta = 0_K, \text{ where } \alpha \neq 0_K \; or \; \beta \neq 0_K$$

- If $\alpha \neq 0_K \; ore \; \beta = 0_K$, equation (18) take the form
$$x \circledcirc \alpha = 0_K \Leftrightarrow x = 0_K$$
Determined so a straight line r with equation $x = 0_K$, that passing point $P = (0_K, 0_K)$, for which the system (19') no solution, after his appearance:
$$\begin{cases} x = d \neq 0_K \\ \\ x = 0_K \end{cases}$$
- If $\alpha = 0_K \; or \; \beta \neq 0_K$, equation (18) take the form
$$y \circledcirc \beta = 0_K \Leftrightarrow y = 0_K$$
that defines a straight line r_1. In this case system (19') take the form
$$\begin{cases} x = d \neq 0_K \\ \\ y = 0_K \end{cases},$$
which solution point $Q = (d, 0_K) \in \ell \cap r_1$. This proved that straight line r_1 It does not meet the demand $\ell \cap r_1 = \emptyset$.
- If $\alpha \neq 0_K \; ose \; \beta \neq 0_K$, equation (18) take the form
$$y \circledcirc \beta = -x \circledcirc \alpha \Leftrightarrow y = x \circledcirc (-\alpha \circledcirc \beta^{-1})$$
that defines a straight line r_2. In this case system (19') take the form
$$\begin{cases} x = d \neq 0_K \\ \\ y = x \circledcirc (-\alpha \circledcirc \beta^{-1}) \end{cases},$$
which solution point $R = (d, -d \circledcirc \alpha \circledcirc \beta^{-1}) \in \ell \cap r_2$. Also straight line r_2 it does not meet the demand $\ell \cap r_1 = \emptyset$.

In this way we show that, when$\ell \in \boldsymbol{\mathcal{L}_0}$ exist just a straight line r, whose equation is

$$x = 0_K$$

that satisfies the conditions of Theorem.

Conversely proved Theorem 2.2 is true for cases **2)** $\ell \in \boldsymbol{\mathcal{L}_1}$ dhe **3)** $\ell \in \boldsymbol{\mathcal{L}_2}$ **.**

THEOREM 2.3**. In the incidence structure $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$) connected to the corp *K*, there exists three points not in a straight line**.

Proof. From Proposition 1.5, since the corp K is unitary ring, this contains 0_K and $1_K \in K$, such that $0_K \neq 1_K$. It is obvious that the points $P = (0_K, 0_K), Q = (1_K, 0_K)$ and $R = (0_K, 1_K)$ are different points pairwise distinct $\mathcal{P}$. Since $P \neq Q$, and $0_K \neq 1_K$, by the case 2) of the proof of Theorem 2.1, results that the straight line $PQ \in \mathcal{L}_1$, so it have equation of the form $y = f$. Since $P \in PQ$ results that $f = 0_K$. So equation of PQ is $y = 0_K$. Easily notice that the point $R \notin PQ$.

Three Theorems 2.1, 2.2, 2.3 shows that an incidence structure **$\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$)** connected to the corp ***K*,** satisfy three axioms **A1, A2, A3** of Definition 1.2 of an afine plane. As consequence we have

THEOREM 2.4. **An incidence structure $\mathcal{A}$=($\mathcal{P}$, $\mathcal{L}$, $\mathcal{I}$) connected to the corp *K* is an afine plane connected with that corp.**

REFERENCES.

[1] **KRİSTAQ FİLİPİ (**2013**).** Algjebra Abstrakte. Botimi I. Shtepia Botuese Migeralb. Tiranë. ISBN 978-9928-07-147-7.

[2] **KRİSTAQ FİLİPİ (**2015**)**. Algjebra dhe Gjeometria. (Ribotim). Shtepia Botuese Edlora Tiranë. ISBN 978-9928-4270-0-7.

[3] **F. SADIKI** (2015). Modele Strukturash Algjebrike Ternare Në Gjeometrinë Projektive. Disertacion Për Marrjen E Gradës Shkencore "Doktor". Tiranë.

[4] **FRANCİS BORCEUX (2014).** An Axiomatic Approach to Geometry(Geometric Trilogy I&II). Springer International Publishing Switzerland. ISBN 978-3-319-01729-7.

[5] **MARCEL BERGER (**1987**).** Geometry I. Springer-Verlag Berlin Heidelberg. ISBN 978-3-540-11658-5.

[6] **MARCEL BERGER (**1987**).** Geometry II. Springer-Verlag Berlin Heidelberg. ISBN 978-3-540-17015-0.

[7] **NATHAN JACOBSON (1953).** Lectures in Abstract Algebra. Volume II. Linear Algebra. ISBN 0-387-90123-X Springer-Verlag New York Heidelberg Berlin.

[8] **DAVİD HİLBERT (1900).** Les principes fondamentaux de la géométrie. Paris, Gauthier-Villars, Imprimeur-Libraire.

[9] **GEORGE A. JENNİNGS (**1994**).** Modern Geometry with Applications. Springer-Verlag New York, Inc. ISBN 0-387-94222-X Springer-Verlag New York Berlin Heidelberg.

[10] **REY CASSE (**2006**).** Projective Geometry: An Introduction. Published in the United States by Oxford University Press Inc., New York. ISBN 0–19–929885–8.

[11] **AGUSTÍ REVENTÓS TARRİDA (**2011**).** Affine Maps,Euclidean Motions and Quadrics. Springer-Verlag London Dordrecht Heidelberg New York.ISBN 978-0-85729-709-9.

[12] **K.W. GRUENBERG AND A. J. WEİR** (1977). Linear Geometry (2nd Edition). (Graduate texts in mathematics ; vol 49). Springer-Verlag, New York Inc. ISBN 0-387-90227-9.

[13] **F. BUEKENHOUT, (EDİTORS)(1995).** HANDBOOK OF INCIDENCE GEOMETRY. Elsevier Science B.V. ISBN: 0 444 88355 X .

[14] **ELİSABETH KUİJKEN (2003).** A Study of Incidence Structures and Codes related to Regular Two-Graphs. Proefschrift voorgelegda an de Faculteit Wetenschappen tot het behalen van de graad van Doctor in de Wetenschappen richting Wiskunde.

[15] **IVAN KOLÁŘ (1966).** Geometric structure with Hilbert's axioms of incidence and order. Institute of Mathematics AS CR.Časopis oro pěstování matematiky, roc, Praha

[16] **MİCHELE AUDİN** (2002). Geometry. Institut de Recherche Mathematique Avancee, Universite Louis Pasteur et CNRS, 7 rue Rene Descartes, 67084 Strasbourg cedex, France.

[17] **G.ERİC MOORHOUSE. (2007)** Incidences Geometry.