Quantum-inspired teleportation.

Michail Zak
Jet Propulsion Laboratory California Institute of Technology
Pasadena, CA 91109

Abstract.

Based upon quantum-inspired entanglement in quantum-classical hybrids, a simple algorithm for instantaneous transmissions of *non-intentional* messages (chosen at random) to remote distances is proposed. A special class of situations when such transmissions are useful is outlined. Application of such a quantum-inspired teleportation, i.e. instantaneous transmission of conditional information on remote distances for *security* of communications is discussed. Similarities and differences between quantum systems and quantum-classical hybrids are emphasized.

1. Introduction.

This paper was motivated by recent discovery and experimental verification of the most fundamental and still mysterious phenomenon in quantum mechanics: quantum entanglement. Formally, quantum entanglement as well as associated with it quantum non-locality follows from the Schrödinger equation; however, its physical meaning is still under extensive discussions. The most attractive aspect of quantum entanglement, in terms of a new quantum technology, is associated with instantaneous transmission of messages on remote distances known as teleportation. However, practical applications of this effect are restricted by the postulate adopted by many authors that these messages cannot deliver any *intentional* information. That is why all the entanglement-based communication algorithms must include a classical channel. The degree of usefulness of entanglement-based communication technology *without any classical channels* has been discussed in [1,2]. The paradigm discussed there is the following. Let us assume that agents *A* and *B* possess a set of *N* particles (say, electrons), that are in a one-to-one correspondence such that each pair is entangled; and suppose that the agent *A* performs a sequence of measurements: one particle per unit time-step. Each measurement performed by the agent *A* has two equally probable outcomes. In case of electrons, these outcomes can be spin-up (+) or spin down (-). If (+) and (-) are converted by the agent into the movements along an axis to the right or to the left, respectively, the sequence of the agent's measurement can be interpreted as a symmetric unrestricted random walk. Hence, by performing these measurements, the agent *A* selected (randomly) one trajectory out of 2^N equally probable trajectories of the corresponding random walk. Due to entanglement, the agent *B* instantaneously receives this trajectory (after performing simultaneously the same type of measurements). This paradigm is easily generalizable to *n* entangled agents if each of them has a set of *N* particles entangled pairwise with the similar particles of all the other agents. The usefulness of such entanglement– based communications has been discussed in [1,2]. It has been demonstrated there how a randomly chosen message can deliver non-intentional, but useful, information under special conditions that include a preliminary agreement between the sender and the receiver. The conditional-message paradigm has been extended by applying the entanglement-based correlations to an active system represented by a collection of intelligent agents. The problem of behavior of intelligent agents correlated by identical random messages in a decentralized way has its own significance: it simulates evolutionary behavior of biological and social systems correlated only via simultaneous sequences of unexpected events. As shown in [1], under the condition that the agents have certain preliminary knowledge about each other, the whole system can exhibit emergent phenomena such as topological self-organization, inverse diffusion; it also can perform transmission of conditional information, decentralized coordination, cooperative computing, competitive games.

However, the main obstacle to further progress in the quantum-based instantaneous transmission of *conditional* information is the same as those that is to progress of quantum computing: the hardware implementations.

The basic idea of this paper is to implement instantaneous transmission of *conditional* information on remote distances *via quantum-classical hybrid* that preserves superposition of random solutions, while allowing one to measure its state variables using classical methods. In other words, such a hybrid system reinforces the advantages and minimizes limitations of both quantum and classical characteristics. The formal mathematical difference between quantum and classical mechanics is better pronounced in the Madelung (rather than the Schrödinger) equation. Two factors contribute to this difference: the scale of the system introduced through the Planck constant and the topology of the Madelung equations that include the feedback (in the form of the quantum potential) from the Liouville equation to the Hamilton-Jacobi equation. Ignoring the scale factor as well as the concrete form of the feedback, we concentrated upon preserving the topology while varying the types of the feedbacks. A general approach to the choice of the feedback was introduced and discussed in [3]. More specific feedbacks linked to the behavioral models of Livings were presented in [4-8]. In this paper we are concerned only with information capabilities of the proposed model disregarding possible physical interpretations.

2. Destabilizing effect of Liouville feedback.

We will start with derivation of an auxiliary result that illuminates departure from Newtonian dynamics. For mathematical clarity, we will consider here a one-dimensional motion of a unit mass under action of a force *f* depending upon the dimensionless *velocity* *v* and time *t*

$$\dot{v} = f(v,t), \tag{1}$$

If initial conditions are not deterministic, and their probability density is given in the form

$$\rho_0 = \rho_0(V), \qquad where \quad \rho \geq 0, \quad and \quad \int_{-\infty}^{\infty} \rho dV = 1 \tag{2}$$

while ρ is a *single- valued* function, then the evolution of this density is expressed by the corresponding Liouville equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial v}(\rho f) = 0 \tag{3}$$

The solution of this equation subject to initial conditions and normalization constraints (2) determines probability density as a function of *V* and *t* : $\rho = \rho(V,t)$.

In order to deal with the constraint (2), let us integrate Eq. (3) over the whole space assuming that $\rho \to 0$ at $|V| \to \infty$ and $|f| < \infty$. Then

$$\frac{\partial}{\partial t}\int_{-\infty}^{\infty} \rho dV = 0, \qquad \int_{-\infty}^{\infty} \rho dV = const, \tag{4}$$

Hence, the constraint (3) is satisfied for $t > 0$ if it is satisfied for $t = 0$.

Let us now specify the force f as a feedback from the Liouville equation

$$f(v,t) = \varphi[\rho(v,t)] \tag{5}$$

and analyze the motion after substituting the force (5) into Eq.(1)

$$\dot{v} = \varphi[\rho(v,t)], \tag{6}$$

This is a fundamental step in our approach. Although the theory of ODE does not impose any restrictions upon the force as a function of space coordinates, the Newtonian physics does: equations of motion are never coupled with the corresponding Liouville equation. Moreover, it can be shown that such a coupling leads to non-Newtonian properties of the underlying model. Indeed, substituting the force f from Eq. (5) into Eq. (4), one arrives at the *nonlinear* equation for evolution of the probability density

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial V}\{\rho \varphi[\rho(V,t)]\} = 0 \tag{7}$$

Let us now demonstrate the destabilizing effect of the feedback (5). For that purpose, it should be noted that the derivative $\partial \rho / \partial v$ must change its sign, at least once, within the interval $-\infty < v < \infty$, in order to satisfy the normalization constraint (2).

But since

$$Sign\frac{\partial \dot{v}}{\partial v} = Sign\frac{d\varphi}{d\rho}Sign\frac{\partial \rho}{\partial v} \tag{8}$$

there will be regions of *v* where the motion is unstable, and this instability generates randomness with the probability distribution guided by the Liouville equation (8). It should be noticed that the condition (9) may lead to exponential or polynomial growth of *v* (in the last case the motion is called neutrally stable, however, as will be shown below, it causes the emergence of randomness as well if prior to the polynomial growth, the Lipchitz condition is violated)..

3. **Non-classical effects**.

Prior to introduction of the superposition phenomenon, we will demonstrate additional non-classical effects displayed by the solutions to Eqs. (6) and (7) when

$$\varphi(\rho) = \frac{1}{2}\rho,$$

and therefore

$$\dot{v} = \frac{1}{2}\rho \tag{9}$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{2}\frac{\partial}{\partial V}(\rho^2) = 0 \tag{10}$$

The solution of Eq. (10) subject to the initial conditions $\rho_0(V)$ and the normalization constraint (2) is given in the following implicit form [9]

$$\rho = \rho_0(\lambda), \qquad V = \lambda + \rho_0(\lambda)t \tag{11}$$

This solution subject to the initial conditions and the normalization constraint, describes propagation of initial distribution of the density $\rho_0(V)$ with the speed *V* that is proportional to the values of this density, i.e. the higher values of ρ propagates faster than lower ones. As a result, any compressive part of the wave, where the propagation velocity is a decreasing function of *V*, ultimately "breaks" to give a triple-valued (but still continuous) solution for $\rho(V,t)$. Eventually, this process leads to the formation of strong discontinuities that are related to propagating jumps of the probability density. In the theory of nonlinear waves, this phenomenon is known as the formation of a shock wave. Thus, as follows from the solution (11), a single-valued continuous probability density spontaneously transforms into a triple-valued, and then, into discontinuous distribution. In aerodynamical application of Eq. (10), when ρ stands for the gas density, these phenomena are eliminated through the model correction: at the small neighborhood of shocks, the gas viscosity ν cannot be ignored, and the model must include the term describing dissipation of mechanical energy. The corrected model is represented by the Burgers' equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{2}\frac{\partial}{\partial V}(\rho^2) = \nu \frac{\partial^2 \rho}{\partial V^2} \tag{12}$$

As shown in [9], this equation has continuous single-valued solution (no matter how small is the viscosity ν), and that provides a perfect explanation of abnormal behavior of the solution to Eq. (10). Similar correction can be applied to the case when ρ stands for the *probability* density if one includes Langevin forces $\Gamma(t)$ into Eq. (9)

$$\dot{v} = \rho + \sqrt{\nu}\Gamma(t), \quad <\Gamma(t)>= 0, \qquad <\Gamma(t)\Gamma(t')>= 2\delta(t-t') \tag{13}$$

Then the corresponding Fokker-Planck equation takes the form (28). It is reasonable to assume that small random forces of strength $\sqrt{\nu} << 1$ are always present, and that protects the mathematical model (9), (10) from singularities and multi-valuedness in the same way as it does in the case of aerodynamics.

It worth noticing that Eq. (12) can be obtained from Eq. (9) in which random force is replaced by additional Liouville feedback

$$\dot{v} = \rho - \nu \frac{\partial}{\partial V}\ln \rho, \qquad \nu > 0, \tag{14}$$

An interesting non-classical property of a solution of this equation is decrease of entropy.
Indeed,

$$\frac{\partial H}{\partial t} = -\frac{\partial}{\partial t}\int_{-\infty}^{\infty} \rho \ln \rho dV = -\int_{-\infty}^{\infty} \dot{\rho}(\ln \rho + 1)dV = \int_{-\infty}^{\infty} \frac{\partial}{\partial V}(\rho^2)\ln(\rho+1)dV$$
$$= [\,\Big|_{-\infty}^{\infty} \rho^2(\ln\rho + 1) - \int_{-\infty}^{\infty} \rho dV] = -1 < 0 \tag{15}$$

Obviously, presence of small diffusion, when $\nu << 1$, does not change the inequality (15) during certain period of time. (However, eventually, for large times, diffusion takes over, and the inequality (15) is reversed). It is easily verifiable that the solution to Eq. (12) satisfies the constraint (2) if the corresponding initial condition does, [9].

4. Emergence of superposition.

Let us concentrate now on the solution of the system (14) and (12) remembering that it is a particular case of the system (6), (7)

$$\dot{v} = \zeta\rho - \nu \frac{\partial}{\partial V} \ln \rho, \qquad \zeta > 0, \quad [\zeta] = \frac{1}{\sec}, \qquad [\nu] = \frac{1}{\sec}, \tag{16}$$

$$\frac{\partial \rho}{\partial t} + \zeta \frac{\partial}{\partial V}(\rho^2) = \nu \frac{\partial^2 \rho}{\partial V^2}, \tag{17}$$

subject to a single-hump initial condition

$$\rho_0(V) = A\delta(V) \quad at \qquad t = 0, \quad A = const. \tag{18}$$

where A is the initial area of the hump, and

$$v(t = 0) = v_0 \tag{19}$$

The variable v in Eq. (32) is a dimensionless velocity $v \to v / v_0$, and the "Reynolds" number

$$R = \zeta \frac{A}{2\nu} \tag{20}$$

We will be interested in the solution of the system (16), (17) for the case of large Reynolds number

$$R \to \infty, \ and \qquad \zeta >> \nu \tag{21}$$

In this case, Eq. (16) can be simplified by omitting the "viscose" term

$$\dot{v} = \zeta\rho, \quad \zeta > 0 \tag{22}$$

However, omitting the last term in Eq. (17) would lead to qualitative changes outlined above, and in particular, it would prevent us to start with the sharp initial conditions (18).

We will start with the solution to Eq. (17). It is different from the standard Burger's equation only by a physical interpretation of the variable ρ that is now a probability density (instead of density of a gas), but as shown in [8], the constraint (2) is satisfied automatically if it is satisfied for the initial condition (18). Thus, the solution to Eq.(17) subject to the conditions (2), (18) and (21) reads

$$\rho = \frac{V}{2\zeta t} \ in \qquad 0 < V < \sqrt{4A\zeta t} \quad and \qquad \rho = 0 \quad outside \tag{23}$$

The solution has a shock of probability density

$$[\rho] = \sqrt{\frac{A}{2\zeta t}} \qquad at \qquad V = \sqrt{4A\zeta t} \tag{24}$$

Substituting the solution (23) into equation (22) one obtains

$$\dot{v} = \frac{v}{t} \quad in \qquad 0 < v < \sqrt{4A\zeta t} \tag{25}$$

and

$$\dot{v} = 0 \quad in \qquad v > \sqrt{4A\zeta t} \tag{26}$$

whence

$$v = Ct \quad in \qquad 0 < v < \sqrt{4A\zeta t} \tag{27}$$

$$v = C \quad in \qquad v > \sqrt{4A\zeta t} \tag{28}$$

We will be interested here only in the region of Eqs. (25) and (27). Here C is an arbitrary constant. Since *v=0* at *t=0* for any value of C, the solution (27) is consistent with the sharp initial condition (18). For a fixed C, the solution (27) is *unstable* since

$$\frac{d\dot{v}}{dv} = \frac{1}{2t} > 0 \tag{29}$$

and therefore, an initial error always grows generating *randomness* whose probability is controlled by Eq.(17) . Initially, at *t=0*, this growth is of infinite rate since the Lipschitz condition at this point is violated

$$\frac{d\dot{v}}{dv} \to \infty \qquad at \qquad t \to 0 \tag{30}$$

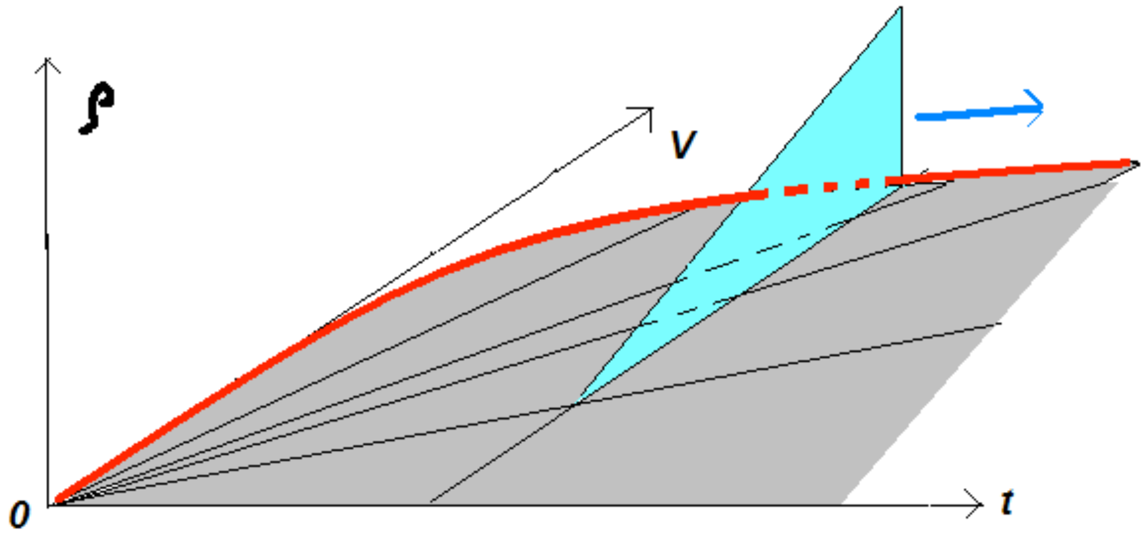

Figure 1. The triangular shock wave of probability density and samples of associated stochastic process.

Considering first Eq. (25) at fixed *C* as a sample of the underlying stochastic process (23), and then varying *C(ω=V)* (where ω is a variable running over different samples of the stochastic process) , one arrives at the whole ensemble characterizing that process, (see Fig. 1). As follows from Fig. 1, the stochastic process converges to the attractor represented by the curve (24) on the *V-t* plane where the shock of the *probability* density occurs (see the red line in Fig. 1).

Thus we arrived at another non-classical effect that is similar to quantum superposition.

Remark. For more mathematical details see Appendix 1.

5. Emergence of entanglement via global constraint.

Let us consider *n* coupled systems

$$\dot{v}_i = \zeta_i \rho(v_1,...v_n,t), \qquad \zeta_i > 0, \quad [\zeta_i] = \frac{1}{\sec}, \qquad i = 1.2,...n. \tag{31}$$

$$\frac{\partial \rho(V_1,...V_n,t)}{\partial t} + \sum_{i=1}^{n} \zeta_i \frac{\partial}{\partial V_i}[\rho^2(V_1,...V_n,t)] = 0, \qquad i = 1,2,...n \tag{32}$$

subject to the initial conditions similar to

$$\rho_i^{\,0}(V_i) = A\delta(V_i) \;\; at \qquad t = 0, \quad A = const. \tag{33}$$

The system is subjected to the following (n-1) independent global constraints

$$\frac{\dot{v}_i}{\dot{v}_j} = \frac{\zeta_i}{\zeta_j} \tag{34}$$

It is important to emphasize that although $\dot{v}_i$ and $\dot{v}_j$ are random (as it was demonstrated for the one-dimensional case), their ratio is deterministic: it is just a number. This is another non-classical effect. Indeed, in theory of stochastic processes, two random functions are considered equal if they have the same statistical invariants, but their point-to-point equalities are not required (although it can happen with a vanishingly small probability). As will be demonstrated below, the *diversion of determinism into randomness via instability (due to a Liouville feedback), and then conversion of randomness to determinism via entanglement* is the fundamental non-classical paradigm that may lead to instantaneous transmission of *conditional* information on remote distance that will be discussed below.

Remark. For more mathematical details see Appendix 2.

6. Instantaneous transmission of *conditional* information on remote distance.

Let us consider *n* observers, and assume that each of them gets a copy of the system (31), (32) and runs it separately. Although they run identical systems, the outcomes of even synchronized runs may be different since the solutions of these systems are random. However, the global constraint (34) must be satisfied. Therefore, if the observer #1 (the sender) made a measurement of the acceleration $\dot{v}_1$ at $t=T$, then the receiver, by measuring the corresponding acceleration $\dot{v}_i$ at the same instant $t=T$ and using the constraint (34), can reconstruct the acceleration $\dot{v}_1$ that was measured by the sender. It should be emphasized that if a receiver decides to measure directly the acceleration $\dot{v}_1$ at $t=T,$ he may get a wrong value since the accelerations are random, and only their ratios are deterministic. Obviously, the transmission of this knowledge is instantaneous as soon as the measurements have been performed. In addition to that, the distance between the observers is irrelevant since the *x*-coordinate does not enter the governing equations. However, the Shannon information transmitted is ***zero***. Indeed, none of the senders can control the outcomes of their measurements since they are random; in other words, the senders cannot transmit ***intentional*** messages. Nevertheless, based upon the transmitted knowledge, they can *coordinate* their actions based upon conditional information: if the observer #1 knows his own measurements, he can fully determine the measurements of the others.

It should be noticed that the transmission procedure described above can be simplified since Eq. (32) does not depend upon Eqs. (31), and therefore, it can be solved prior to the transmission. Then the sender and each of the receivers have to run the system (31) into which the prepared solution

$$\rho = \rho(v_1,...v_n,t) \qquad (35)$$

is to be substituted.

But even this procedure can be further simplified. Indeed, turning to the constraint (34), and setting the initial conditions

$$v_i(t=0)=0 \qquad (36)$$

one finds

$$v_i = \frac{\zeta_i}{\zeta_j} v_j, \qquad (37)$$

Now the sender (the observer #1) can express all the variables $v_{i\neq 1}$ via v_1, then substitute them into Eq. (35) and run only one ODE

$$\dot{v}_1 = \zeta_1 \hat{\rho}(v_1), \qquad \hat{\rho} = \rho(V_1 \mid V_i = \frac{\zeta_i}{\zeta_1} V_1)\rho_1(V_1) \qquad at \qquad V_j = v_j \qquad (38)$$

to obtain the solution

$$v_1 = v_1(t) \qquad (39).$$

This solution is random, (see Fig. 1), and therefore, the sender cannot generate an intentional message.

Meanwhile, the receiver (the observer #*k*) has to run the following ODE

$$\dot{v}_k = \zeta_k \hat{\rho}(v_k), \qquad \hat{\rho}_1 = \rho(\frac{\zeta_k}{\zeta_1} v_k, \frac{\zeta_k}{\zeta_2} v_k, ... \frac{\zeta_k}{\zeta_n} v_k), \qquad (40)$$

to get the random solution

$$v_k = v_k(t) \qquad (41)$$

that is entangled with any sample of the solutions to Eq. (38). Based upon this entanglement expressed by Eq. (37), the receiver reconstructs the signal generated by the sender as following

$$v_1 = \frac{\zeta_1}{\zeta_k} v_k, \qquad (42)$$

The information capacity of the proposed channel can be evaluated if one turns to Eq. (35). Based upon this evaluation, each receiver can introduce the conditional entropy, mutual information, etc.

It should be emphasized again that *the origin of entanglement of all the observers is the joint probability density that couples their actions*, and such a constraint does not exist in Newtonian mechanics.

Several properties of the proposed correlation should be emphasized. First, there is no centralized source, or a sender of the signal since each receiver can become a sender as well. Indeed, an observer receives a signal by performing certain measurements synchronized with the measurements of the others. Thereby the signal uniformly and simultaneously distributed over the observers in a decentralized way. Second, the signals transmit no intentional information that would favor one agent over another. Third, all the sequence of signals received by different observers are not only statistically equivalent, but also point-by-point identical. Fourth, it is important to assume that each agent knows that the other agent simultaneously receives the identical signals. Finally, the sequences of the signals are true random so that no agent could predict the next step with the probability different from those described by the density (35). It turns out that under these quite general assumptions, the entangled observers-agents can perform non-trivial tasks that include transmission of conditional information from one agent to another, simple paradigm of cooperation, etc. The problem of behavior of intelligent agents correlated by identical random messages in a decentralized way has its own significance: it simulates evolutionary behavior of biological and social systems correlated only via simultaneous sensoring sequences of unexpected events. In order to justify the usefulness of the proposed correlation paradigm, consider an earthquake which is represented by some sequence of totally unpredictable jolts. All the "agents" (humans, animals) receive these unexpected signals simultaneously, and from that moment their activity became correlated and organized: they run to shelter, turn off pipelines, etc.

Let us discuss possible application of the entanglement introduced above to *security* of communications. It is always a temptation to simulate any new quantum or quantum-inspired phenomenon by classical tools. In the case of entanglement such a possibility was excluded from the very beginning since this is a non-local phenomenon that does not have any classical equivalents. However, one can argue that actually the system under consideration becomes classical as soon as the message is received and interpreted by the agent; therefore, instead of entanglement-based correlations between the agents, one can generate a pool of samples of stochastic processes in advance, make copies and distribute them over the agents, so that any two agents to be correlated would have identical records of "random" messages. However, there is a fundamental flaw in such an implementation, since in that case the whole scenario of the agent's evolution is fully predetermined, and someone (for instance, those who generated, copied and distributed the messages) can know this scenario in advance. In principle, each agent also can find out his future messages since the knowledge about this future has already existed. The difference between the entangled and classical cases is similar to that between real-time and pre-recorded TV programs: in the first case, future is unpredictable, while in the second case "future" has already happened, although the viewer may not know about that. In a more practical sense, the difference between the quantum and classical implementations becomes important when the communications between the agents are supposed to be confidential: in the classical case, the confidential information, in principle, is available long before it is needed, and that makes such communications less secure. In order to illustrate a security aspect of the proposed algorithm, suppose that a sender possesses N different messages, which he can choose only at random with equal probability, and assume that any of these messages allows each receiver to achieve his goal as long as the secrecy of the message is preserved. (For instance, if a military attack can be conducted in many different ways, the most important is the secrecy of the selected strategy.) Then from the viewpoint of Shannon information, the transmission of such a message is useless. However, if one is asked what the chance is that the message can be decoded by a wild guess, the answer will be: $1/N$. This means that the number of equally acceptable (but randomly chosen) messages is proportional to the degree of secrecy of the transmission, and that represents the value of this transmission. Actually, the sender *coordinates* and *synchronizes* the actions of the receivers (regardless of the origin of the message itself) and preserves the secrecy of the communications by making the choice of his message random. It should be emphasized again that the whole procedure makes sense only under the condition that a receiver can use any of these messages to achieve the same objective, but nobody else must know what kind of message has been received.

The most effective way of implementation of the proposed quantum-classical hybrid is by means of analog devices such as VLSI chips used for neural net's analog simulations, [9].

7. Discussion and conclusion.

Based upon quantum-inspired entanglement in quantum-classical hybrids, a simple algorithm for quantum-inspired teleportation, i.e. for instantaneous transmissions of *non-intentional* messages (chosen at random) to remote distances is proposed. A special class of situations when such transmissions are useful is outlined. Application of instantaneous transmission of conditional information on remote distances for *security* of communications is discussed. Similarities and differences between quantum systems and quantum-classical hybrids are emphasized. It has been demonstrated that quantum-classical hybrid preserves the topology of the Schrödinger equation (in the Madelung form), but replaces the quantum potential with other, specially selected, function of probability density, (Fig. 2)

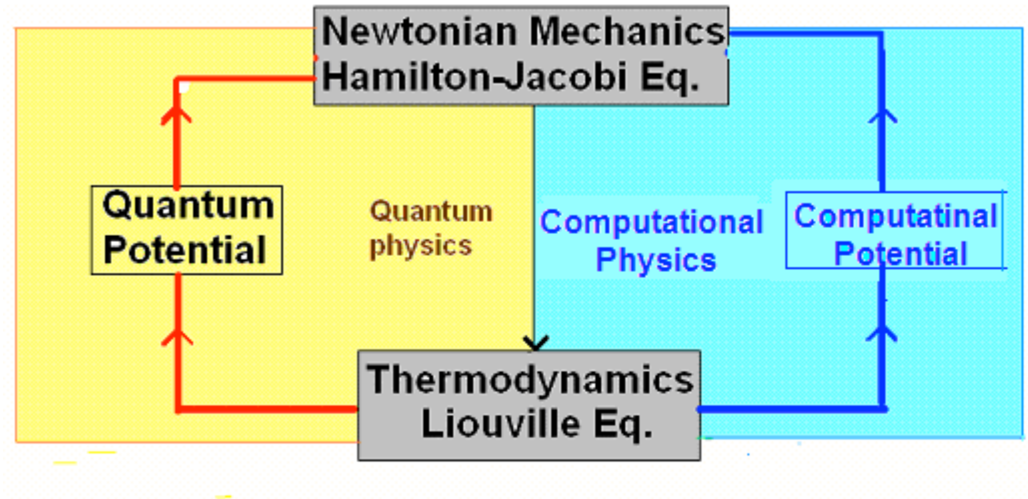

Figure 2. Classical physics, Quantum physics, and Quantum-Classical Hybrid.

It has been shown that two fundamental similarities with quantum mechanics are due to the Liouville feedback that introduces the probability density into the equations of motion. The role of the probability-based feedback is twofold: first, it creates a transition from determinism to randomness, and that imitates quantum superposition; second, it entangles random samples of the stochastic process thereby allowing one to convert randomness back to determinism. Despite such a quantum-like characteristic, the hybrid can be of classical scale, and all the measurements can be performed classically. Indeed, measurement in the hybrid systems has more differences than similarities with quantum systems. The most important difference is that this system is of classical scale, and it does not interact (in a quantum way) with the measurement procedure. (Indeed, the proposed hybrid system can be implemented by means of analog devices such as VLSI chips used for neural net's analog simulations). As a result, the solution can be observed as a function of time describing the evolution of a measured state variable during the whole duration of measurement. Detailed analysis of similarities and differences between a quantum system and a quantum-classical hybrid were performed in [8].

Acknowledgment

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Appendix 1.

The solution (23)-(28) to the system (16), (17) subject to the initial conditions (18) is, strictly speaking, valid for *t>0* excluding a vanishingly small period $0 \le t < 2\nu / A\zeta$. Indeed, within this period, the probability density ρ is still close to the delta-function, and the diffusion term in Eq. (17) dominates over the non-linearity. Therefore, for this period, the system (16), (17) has the form

$$\dot{v} = -\nu \frac{\partial}{\partial v} \ln \rho, \qquad \text{(A1)}$$

$$\frac{\partial \rho}{\partial t} = \nu \frac{\partial^2 \rho}{\partial V^2}, \qquad \text{(A2)}$$

The solution of Eq. (A2) subject to the sharp initial condition is

$$\rho = \frac{1}{2\sqrt{\pi \nu t}} \exp(-\frac{V^2}{4\nu t}), \qquad \text{(A3)}$$

Substituting this solution into Eq. (A1) at *V=v* one arrives at the differential equation with respect to *v(t)*

$$\dot{v} = \frac{v}{2t} \qquad \text{(A4)}$$

whence

$$v = C\sqrt{t}, \qquad \text{(A5)}$$

where *C* is an arbitrary constant. Since *v=0* at *t=0* for any value of *C*, the solution (A4) is consistent with the sharp initial condition for the solution (A3) of the corresponding Liouvile equation (A2) (that takes the form of the Fokker-Planck equation). The solution (A3) describes the simplest irreversible motion: it is characterized by the "beginning of time" where all the trajectories intersect (that results from the violation of Lipschitz condition at *t=0,* Fig.A1), while the backward motion obtained by replacement of *t* with *(-t)* leads to imaginary values of velocities. One can notice that the probability density (A3) possesses the same properties.

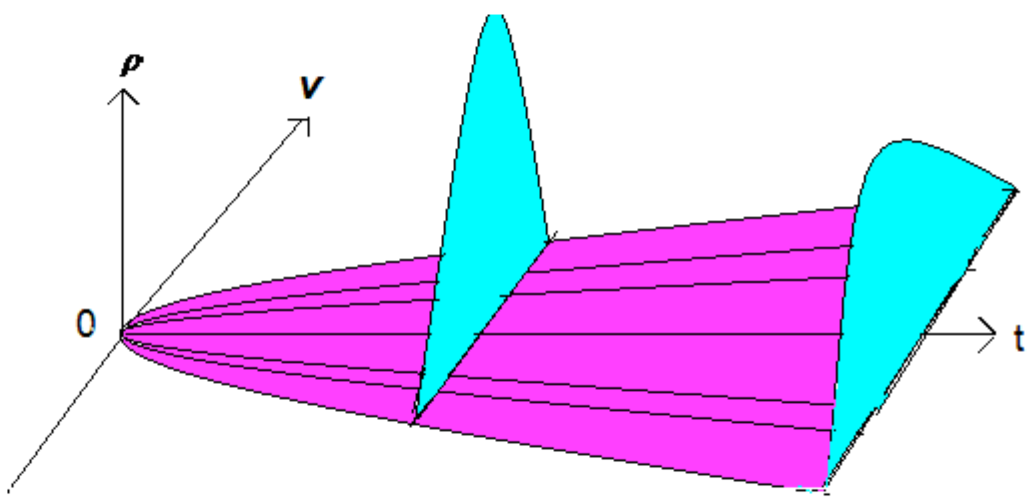

Figure A1. Origin of superposition.

For a fixed *C*, the solution (A5) is *unstable* since

$$\frac{d\dot{v}}{dv} = \frac{1}{2t} > 0, \qquad \text{(A6)}$$

and therefore, an initial error always grows generating *randomness.* Initially, at $t=0$, this growth is of infinite rate since the Lipschitz condition at this point is violated

$$\frac{d\dot{v}}{dv} \to \infty \qquad at \qquad t \to 0 \tag{A7}$$

This type of instability has been introduced and analyzed in [11]. Considering first Eq. (A5) at fixed C as a sample of the underlying stochastic process (A3), and then varying C, one arrives at the whole ensemble characterizing that process, (see Fig. A1). One can verify that, as follows from Eq. (A3), [12], the expectation and the variance of this process are, respectively

$$MV = 0, \qquad DV = 2\nu t \tag{A8}$$

The same results follow from the ensemble (A5) at $-\infty \le C \le \infty$. Indeed, the first equality in Eq. (A8) results from symmetry of the ensemble with respect to $v=0$; the second one follows from the fact that

$DV \propto v \propto t$. Thus, the solution to Eq. (17) starts with the form (A3), and only after $t > A/2\nu\xi$ it takes the form (23). Similarly, the solution to eq. (16) starts with the form (A5), and after $t > A/2\nu\xi$ it takes the form (27), (28).

Appendix 2.

As in a one-dimensional case discussed in Appendix 1, during a small initial period $0 \le t < 2\nu/A\xi$, the system (31), (32) should be written in the form similar to (A1), (A2). However, in order to capture the effect of a multi-dimensionality, it will be sufficient to start with $n=2$

$$\dot{v}_1 = -a_{11}\nu \frac{\partial}{\partial v_1}\ln\rho - a_{12}\nu \frac{\partial}{\partial v_2}\ln\rho, \tag{A9}$$

$$\dot{v}_2 = -a_{21}\nu \frac{\partial}{\partial v_1}\ln\rho - a_{22}\nu \frac{\partial}{\partial v_2}\ln\rho, \tag{A10}$$

$$\frac{\partial\rho}{\partial t} = a_{11}\nu \frac{\partial^2\rho}{\partial V^2} + (a_{12} + a_{21})\nu \frac{\partial^2\rho}{\partial V_1 \partial V_2} + a_{22}\nu \frac{\partial^2\rho}{\partial V_2}, \tag{A11}$$

The solution to Eq. (A11) has a closed form

$$\rho = \frac{1}{\sqrt{2\pi\nu \det[\hat{a}_{ij}]t}} \exp(-\frac{1}{4t} b_{ij} V_i V_j), \qquad i = 1,2. \tag{A12}$$

Here

$$[b_{ij}] = [\hat{a}_{ij}]^{-1} \text{ where } \hat{a}_{11} = a_{11}, \hat{a}_{22} = a_{22}, \hat{a}_{12} = \hat{a}_{21} = a_{12} + a_{21},\ \hat{a}_{ij} = \hat{a}_{ji}, b_{ij} = b_{ji} \tag{A13}$$

Substituting the solution (A12) into Eqs. (A9) and (A10), one obtains

$$\dot{v}_1 = \frac{b_{11}v_1 + b_{12}v_2}{2t} \tag{A14}$$

$$\dot{v}_2 = \frac{b_{21}v_1 + b_{22}v_2}{2t} \tag{A15}$$

Eliminating t from these equations, one arrives at an ODE in the configuration space

$$\frac{dv_2}{dv_1} = \frac{b_{21}v_1 + b_{22}v_2}{b_{11}v_1 + b_{12}v_2}, \qquad v_2 \to 0 \quad at \qquad v_1 \to 0, \tag{A16}$$

This is a classical singular point treated in text books on ODE.
Its solution depends upon the roots of the characteristic equation

$$\lambda^2 - 2b_{12}\lambda + b^2{}_{12} - b_{11}b_{22} = 0 \tag{A17}$$

Since both the roots are real in our case, let us assume for concreteness that they are of the same sign, for instance, $\lambda_1 = 1, \quad \lambda_2 = 1$. Then the solution to Eq. (A16) is represented by the family of straight lines

$$v_2 = \widetilde{C} v_1, \qquad \widetilde{C} = const. \tag{A18}$$

Substituting this solution into Eq. (A14), yields

$$v_1 = C(b_{11} + \widetilde{C} b_{12})\sqrt{t} \tag{A19}$$

Thus, the solution to Eq. (A9) is represented by a two-parametrical family of curves as expected. The solution to Eq. (A10) can be written in a similar form.

The solution for n-dimensional case of Eq. (A11) can be written in the form similar to (A12)

$$\rho = \frac{1}{\sqrt{(2\pi)^{n/2}\nu \det[\hat{a}_{ij}]t}} \exp(-\frac{1}{4t} b_{ij} V_i V_j), \quad i = 1,2...n \tag{A20}$$

The solutions (A14),(A19), and (A20) valid for a vanishingly small period $0 \le t < 2\nu / A\zeta$. After $t > 2\nu / A\zeta$, one should proceed with solutions to Eqs. (31) and (32).

It should be noticed that the availability of the solution (A20) is important. Indeed, as discussed in Section 6, the solution to Eq. (32) must be provided to the observers prior to their communication. But since (32) is an n-dimensional non-linear PDE, its solution, most probably, should be obtained only numerically, and the singularities at *t=0* may be lost. Therefore, the solution (A20) has to be included in the numerical solution as the onset of the superposition.