On Corda's 'Clarification' of Schwarzschild's Solution

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Abstract

A paper by C. Corda (*A Clarification on the Debate on "the Original Schwarzschild Solution''*, *EJTP* 8, **No. 25** (2011) 65-82) purports equivalence of Schwarzschild's original solution (1916) and Hilbert's subsequent solution (1917), the latter commonly but incorrectly called 'Schwarzschild's solution'. The derivation of Schwarzschild's actual solution by Corda is, in fact, a copy of Schwarzschild's original derivation with only changes in notation and equation numbering. It adds nothing new to the problem. Corda's subsequent arguments on gravitational collapse follow those advanced by Misner, Thorne, and Wheeler for Hilbert's solution, in their book 'Gravitation', and suffer thereby from the very same shortfalls. Consequently, Corda has failed to prove his alleged equivalence of the Schwarzschild and Hilbert solutions. Moreover, it is not difficult to prove that these are not equivalent. Furthermore, all methods employed to otherwise 'extend' Droste's solution into Hilbert's solution thus producing a black hole constitute a violation of the rules of pure mathematics and are invalid.

Keywords: Schwarzschild solution; Black holes; Metric extensions; General Relativity.

1 Introduction

One hundred years ago, on the 13th of January 1916, Karl Schwarzschild communicated his solution to Einstein's gravitational field for a `mass point'. Then, less than five years ago, on the 25th of May 2011, a paper by Christian Corda was published which reproduced most of Schwarzschild's paper. In a prelude on page 70 of his paper, Corda stated,

> *"In our approach we will suppose again that a(r,t) = 0, but, differently from the standard analysis, we will assume that the length of the circumference centred in the origin of the coordinate system is not 2πr. We release an apparent different physical assumption, i.e. that arches of circumference are deformed by the presence of mass of the central body M. Note that this different physical hypothesis permits to circumnavigate the Birkhoff Theorem [4] which leads to the 'standard Schwarzschild solution' [3]."*

Then, before launching into his modifications of Schwarzschild's equations, Corda stated, again on page 70,

> *"Then, we proceed assuming -mr², where m is a generic function to be determined in order to obtain the length of circumferences centred in the origin of the coordinate system are not 2πr. In other words, m represents a measure of the deviation from 2πr of circumferences centred in the origin of the coordinate system."*

Corda's special generic function *m=m(r)* plays no special role since it already appeared in Schwarzschild's paper as *G=G(r)*, where it deformed nothing.

Schwarzschild's solution [1] for Einstein's equations $R_{\mu\nu} = 0$ is,

$$ds^2 = \left(1-\frac{\alpha}{R}\right)dt^2 - \left(1-\frac{\alpha}{R}\right)^{-1} dR^2 - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (1.1)$$
$$R = \left(r^3 + \alpha^3\right)^{1/3}, \quad 0 \le r$$

Hilbert's solution [2] is,

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \left(1-\frac{2m}{r}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \qquad (1.2)$$
$$0 \le r$$

Corda [3] has argued that Schwarzschild's original solution (1916) is equivalent to Hilbert's solution (1917)[1]. It is evident that Schwarzschild's solution is singular (i.e. undefined) only at $r = 0$. Prima facie, Hilbert's solution seems to be singular at $r = 2m$ and at $r = 0$, by which the black hole originated. In Eq.(1.1), $r = \sqrt{x^2 + y^2 + z^2}$, where *x*, *y*, *z* are Cartesian coordinates for Euclidean 3-space, and so *r* is the radius of a Euclidean sphere centred at the origin of coordinates. Nonetheless neither *R* nor *r* is the radius of anything or even a distance in Eq.(1.1)[2] and similarly *r* in Eq.(1.2) is neither radius nor distance therein. Now let *r′* be the radius of a Euclidean sphere. It is routinely claimed that in Eq.(1.2), $r = r' = \sqrt{x^2 + y^2 + z^2}$ (e.g. Einstein [4]), from which the black hole was constructed. This is incorrect because in (1.2), $r = \sqrt{x_0^2 + y_0^2 + z_0^2}$ + $\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ = $r + r'$ where $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ = $2m$ is the distance of the centre of a Euclidean sphere from the origin of coordinates and *r′* its radius, which has been explained elsewhere [5-7]. Only when $x_0=y_0=z_0=0=r_0$ does $r = r' = \sqrt{x^2 + y^2 + z^2}$. When a sphere initially centred at the origin of coordinates is moved, it takes its centre with it, and the position of the Euclidean sphere is specified by the

[1] In Eq.(1.1) *c*=1, in Eq.(1.2) *c*=1 and Newton's gravitational constant *G*=1.

[2] Although Eq.(1.1) and Eq.(1.2) are spherically symmetric they non-Euclidean. Hence the radius in Eq.(1.1) and Eq.(1.2) is not *r*. The radius must be calculated from the non-Euclidean equations, so that the Euclidean radial distance corresponds to the different non-Euclidean radial distance. There are two different but related spaces involved; Euclidean space and non-Euclidean space.

coordinates of its centre (x_0,y_0,z_0) so that $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ whereupon the radius *r'* of the sphere is no longer given by $r = r' = \sqrt{x^2 + y^2 + z^2}$, but by $r' = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$. The intrinsic geometry of a sphere is not altered by changing its position and so its radius does not change with a change of position. When Hilbert set r^2 as the coefficient of $\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$ in the derivation of his solution, he unwittingly shifted the centre of Schwarzschild's Euclidean sphere from $r=r_0=0$ to the coordinates (x_0,y_0,z_0) at the distance $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} = 2m$ from the origin of coordinates, mistakenly thinking the centre still at $r=0$. Hilbert shifted Schwarzschild's Euclidean sphere but left its centre behind. The result was fantastic. David Hilbert had separated the Euclidean sphere from its centre and even placed the centre outside the sphere. Consequently, Hilbert's solution is not equivalent to Schwarzschild's, and black hole theory violates the rules of pure mathematics. Accordingly, Corda has incorrectly argued that in (1.1), $-\alpha \le r$, in order to produce two singularities in the fashion of Hilbert's solution leading to the black hole: Corda incorrectly argues that $-\alpha \le \sqrt{x^2 + y^2 + z^2}$ in (1.1).

2 Notation Manipulation

Schwarzschild used the following notation in his paper: *F*, *G*, *H*, x_1, x_2, x_3, x_4, f_1, f_2, f_3, where *F*, *G*, *H*, f_1, x_1, f_2, f_3 are functions of $r = \sqrt{x^2 + y^2 + z^2}$, and $x_1 = r^3/3$, $x_2 = -\cos\theta$, $x_3 = \varphi$, $x_4 = t$.

Corda used the following notation in his paper: *l*, *m*, *h*, *X*, *Y*, *Z*, *t*, *A*, *B*, *C*, where *l*, *m*, *h*, *X*, *A*, *B*, *C* are functions of $r = \sqrt{x^2 + y^2 + z^2}$.

That Corda's derivation of Schwarzschild's solution is merely a point by point copy of Schwarzschild's is demonstrated by comparisions:

<u>Schwarzschild</u>

$$ds^2 = Fdt^2 - \left(G + Hr^2\right)dr^2 - Gr^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{6}$$

$$x_1 = \frac{r^3}{3}, \quad x_2 = -\cos\theta, \quad x_3 = \varphi \tag{7}$$

$$ds^2 = Fdx_4^2 - \left(\frac{G}{r^4} + \frac{H}{r^2}\right)dx_1^2 - Gr^2\left[\frac{dx_2^2}{1 - x_2^2} + dx_3^2\left(1 - x_2^2\right)\right] \tag{8}$$

<u>Corda</u>

$$ds^2 = hdr^2 - mr^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) + ldt^2 \tag{5}$$

$$X = \frac{r^3}{3}, \quad Y = -\cos\theta, \quad Z = \varphi \tag{6}$$

$$ds^2 = ldt^2 + \frac{h}{r^4}dX^2 - mr^2\left[\frac{dY^2}{1-Y^2} + dZ^2\left(1-Y^2\right)\right] \qquad (7)$$

Inspection reveals that Corda's equations (5), (6) and (7) are precisely Schwarzschild's equations (6), (7) and (8) respectively, with the following change of notation: $l=F$, $h=-(G+Hr^2)$, $m=G$, $X=x_1$, $Y=x_2$, $Z=x_3$, $t=x_4$. In his Eq.(5) Corda has also rearranged the order of the components of Schwarzschild's Eq.(6).

At his Eqs.(8) Corda sets $A \equiv -h/r^4$, $B \equiv mr^2$, $C \equiv l$, and rewrites his Eq.(7) as,

$$ds^2 = Cdt^2 - AdX^2 - B\frac{dY^2}{1-Y^2} - BdZ^2\left(1-Y^2\right) \qquad (10)$$

Compare this to Schwarzschild's Eq.(9),

$$ds^2 = f_4 dx_4^2 - f_1 dx_1^2 - f_2\frac{dx_2^2}{1-x_2^2} - f_2 dx_3^2\left(1-x_2^2\right) \qquad (9)$$

Corda's Eq.(10) is just Schwarzschild's Eq.(9) relabelled with $C=f_4$, $t=x_4$, $A=f_1$, $X=x_1$, $B=f_2=f_3$ (since $f_2=f_3$ in Schwarzschild's analysis), $Y=x_2$ and $Z=x_3$.

Corda then goes on to the Riemann-Christoffel symbols of the second kind at his Eqs.(11). They are the same as Schwarzschild's, except for Corda's relabelling, and the fact that Schwarzschild [1] did not number them. Since there are ten such pairs of symbols, only a few are listed by way of example:

Schwarzschild

$$\Gamma^1_{11} = -\frac{1}{2}\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}, \quad \Gamma^1_{22} = +\frac{1}{2}\frac{1}{f_1}\frac{\partial f_2}{\partial x_1}\frac{1}{1-x_2^2}, \quad \Gamma^3_{31} = -\frac{1}{2}\frac{1}{f_2}\frac{\partial f_2}{\partial x_1} \quad \cdots$$

Corda

$$\Gamma^X_{XX} = -\frac{1}{2A}\frac{\partial A}{\partial X}, \quad \Gamma^X_{YY} = \frac{1}{2A}\frac{\partial B}{\partial X}\frac{1}{1-Y^2}, \quad \Gamma^Z_{ZX} = -\frac{1}{2B}\frac{\partial B}{\partial X} \quad \cdots \qquad (11)$$

wherein Corda set $A=f_1$, $X=x_1$, $B=f_2$, $Y=x_2$, $Z=x_3$. He also rearranged the order of the ten Riemann-Christoffel symbols appearing in Schwarzschild's paper.

Equations (13), (14), (15), and (16) in Corda's paper are precisely Schwarzschild's equations (b), (a), (c) and (d) respectively, rearranged to equate to zero, and embellished with Corda's relabelling:

Schwarzschild

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}\right)=\frac{1}{2}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}\right)^2+\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2+\frac{1}{2}\left(\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)^2 \qquad \text{(a)}$$

Corda

$$\frac{\partial}{\partial X}\left(\frac{1}{A}\frac{\partial A}{\partial X}\right)-\frac{1}{2}\left(\frac{1}{A}\frac{\partial A}{\partial X}\right)^2-\left(\frac{1}{B}\frac{\partial B}{\partial X}\right)^2-\frac{1}{2}\left(\frac{1}{C}\frac{\partial C}{\partial X}\right)^2=0 \qquad (14)$$

Schwarzschild

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_2}{\partial x_1}\right)=2+\frac{1}{f_1 f_2}\left(\frac{\partial f_2}{\partial x_1}\right)^2 \qquad \text{(b)}$$

Corda

$$\frac{\partial}{\partial X}\left(\frac{1}{A}\frac{\partial B}{\partial X}\right)-2-\frac{1}{AB}\left(\frac{\partial B}{\partial X}\right)^2=0 \qquad (13)$$

Schwarzschild

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_4}{\partial x_1}\right)=\frac{1}{f_1 f_4}\left(\frac{\partial f_4}{\partial x_1}\right)^2 \qquad \text{(c)}$$

Corda

$$\frac{\partial}{\partial X}\left(\frac{1}{A}\frac{\partial C}{\partial X}\right)-\frac{1}{AC}\left(\frac{\partial C}{\partial X}\right)^2=0 \qquad (15)$$

Schwarzschild

$$\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}+\frac{2}{f_2}\frac{\partial f_2}{\partial x_1}+\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}=0 \qquad \text{(d)}$$

Corda

$$\frac{1}{A}\frac{\partial A}{\partial X}+\frac{2}{B}\frac{\partial B}{\partial X}+\frac{1}{C}\frac{\partial C}{\partial X}=0 \qquad (16)$$

Corda's relabelling is clear, notwithstanding his rearrangement of Schwarzschild's equations to equal zero: $X=x_1$, $A=f_1$, $B=f_2$, $C=f_4$.

Corda's Eqs. (17) and (18) are relabelled copies of Schwarzschild's equations (*c'*) and (*c''*),

Schwarzschild

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)=\frac{1}{f_1 f_4}\frac{\partial f_1}{\partial x_1}\frac{\partial f_4}{\partial x_1} \qquad \text{(c')}$$

$$\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}=\alpha f_1 \qquad (\alpha \textit{ integration constant}) \qquad \text{(c'')}$$

Corda

$$\frac{\partial}{\partial X}\left(\frac{1}{C}\frac{\partial C}{\partial X}\right)=\frac{1}{AC}\frac{\partial A}{\partial X}\frac{\partial C}{\partial X} \qquad (17)$$

$$\frac{1}{C}\frac{\partial C}{\partial X}=\alpha A \qquad (18)$$

"(*where α is an integration constant*)"

and his Eqs. (19) and (20) relabelled copies of Schwarzschild's next two but unnumbered equations, with Schwarzschild's latter equation multiplied by −1:

Schwarzschild

$$\frac{\partial}{\partial x_1}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}+\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)=\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2+\frac{1}{2}\left(\frac{1}{f_1}\frac{\partial f_1}{\partial x_1}+\frac{1}{f_4}\frac{\partial f_4}{\partial x_1}\right)^2$$

$$-2\frac{\partial}{\partial x_1}\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)=3\left(\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}\right)^2$$

Corda

$$\frac{\partial}{\partial X}\left(\frac{1}{A}\frac{\partial A}{\partial X}+\frac{1}{C}\frac{\partial C}{\partial X}\right)=\left(\frac{1}{B}\frac{\partial B}{\partial X}\right)^2+\frac{1}{2}\left(\frac{1}{A}\frac{\partial A}{\partial X}+\frac{1}{C}\frac{\partial C}{\partial X}\right)^2 \qquad (19)$$

$$2\frac{\partial}{\partial X}\left(\frac{1}{B}\frac{\partial B}{\partial X}\right)=-3\left(\frac{1}{B}\frac{\partial B}{\partial X}\right)^2 \qquad (20)$$

The relabelling is $X=x_1$, $A=f_1$, $C=f_4$, $B=f_2$. Note that Corda's Eq.(20) is Schwarzschild's second unnumbered equation multiplied through by -1.

Corda's Eqs.(21) and (22) are numbered and relabelled copies of Schwarzschild's very next two unnumbered equations,

Schwarzschild

$$\frac{1}{f_2}\frac{\partial f_2}{\partial x_1}=\frac{2}{3x_1+\rho}$$

$$f_2 = \lambda(3x_1 + \rho)^{2/3}$$ (*λ integration constant*)

Corda

$$\frac{1}{B}\frac{\partial B}{\partial X} = \frac{2}{3X + b} \tag{21}$$

$$B = d(3X + b)^{2/3} \tag{22}$$

"(*where d is an integration constant*)"

Once again Corda's relabelling of Schwarzschild's unnumbered equations is clearly evident: $B=f_2$, $X=x_1$, $d=\lambda$, $b=\rho$. To set $d=1$ Corda next invokes the same condition that Schwarzschild applied at his Eq.(10) to fix his $\lambda=1$:

Schwarzschild

$$f_2 = (3x_1 + \rho)^{2/3} \tag{10}$$

Corda

$$B = (3X + b)^{2/3} \tag{23}$$

The copy precedure continues, with Corda's reproduction of Schwarzschild's derivation in modified raiment. Indeed, immediately after his Eq.(23) Corda presents an unnumbered equation. The corresponding equation in Schwarzschild's paper is also unnumbered. They are as follows:

Schwarzschild

$$\frac{\partial f_4}{\partial x_1} = \alpha f_1 f_2 = \frac{\alpha}{f_2^{\,2}} = \frac{\alpha}{(3x_1 + \alpha)^{4/3}}$$

Corda

$$\frac{\partial C}{\partial X} = \alpha AB = \frac{\alpha}{B^2} = \alpha(3X + b)^{-\frac{4}{3}}$$

Here the relabelling is also obvious.

Corda's Eqs.(24) and (25) are again just relabelled copies of Schwarzschild's Eqs.(11) and (12) respectively:

Schwarzschild

$$f_4 = 1 - \alpha(3x_1 + \rho)^{-1/3} \tag{11}$$

$$f_1 = \frac{(3x_1 + \rho)^{-4/3}}{1 - \alpha(3x_1 + \rho)^{-1/3}} \tag{12}$$

Corda

$$C = 1 - \alpha(3X + b)^{-\frac{1}{3}} \tag{24}$$

$$A = \frac{(3X + b)^{-\frac{4}{3}}}{1 - \alpha(3X + b)^{-\frac{1}{3}}} \tag{25}$$

The relabelling is, $C=f_4$, $A=f_1$, $X=x_1$, $b=\rho$.

Corda's Eq.(26) is a relabelled copy of Schwarzschild's Eq.(13) and his three Eqs.(27) are relabelled combinations of Schwarzschild's unnumbered equations immediately following the latter's Eq.(13), as follows:

Schwarzschild

$$\rho = \alpha^3 \tag{13}$$

$$f_1 = \frac{1}{R^4}\frac{1}{1 - \alpha / R}, \quad f_2 = f_3 = R^2, \quad f_4 = 1 - \alpha / R$$
$$R = (3x_1 + \rho)^{1/3} = \left(r^3 + \alpha^3\right)^{1/3}$$

Corda

$$b = \alpha^3 \tag{26}$$

$$A = \left(r^3 + \alpha^3\right)^{-\frac{4}{3}}\left[1 - \alpha\left(r^3 + \alpha^3\right)^{-\frac{1}{3}}\right]^{-1}$$
$$B = \left(r^3 + \alpha^3\right)^{\frac{2}{3}} \tag{27}$$
$$C = 1 - \alpha\left(r^3 + \alpha^3\right)^{-\frac{1}{3}}$$

Clearly, from Schwarzschild's Eq.(13) Corda (Eq.(26)) has relabelled $b=\rho$ and from Schwarzschild's unnumbered equations, Corda (Eqs.(27)) relabelled $A=f_1$, $B=f_2$, $C=f_4$.

At his Eq.(28) Corda has simply reproduced Schwarzschild's Eq.(14)[3] wherein he has explicitly substituted Schwarzschild's $R = \left(r^3 + \alpha^3\right)^{1/3}$, and altered the sequence of terms in the metric:

[3]See Eq.(1) herein.

Schwarzschild

$$ds^2 = (1-\alpha/R)dt^2 - \frac{dR^2}{1-\alpha/R} - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad R = \left(r^3+\alpha^3\right)^{1/3} \tag{14}$$

Corda

$$ds^2 = \left[1-\frac{\alpha}{\left(r^3+\alpha^3\right)^{\frac{1}{3}}}\right]dt^2 - \left(r^3+\alpha^3\right)^{\frac{2}{3}}\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \\ - \frac{d\left(r^3+\alpha^3\right)^{\frac{2}{3}}}{1-\frac{\alpha}{\left(r^3+\alpha^3\right)^{\frac{1}{3}}}} \tag{28}$$

Then, according to the standard practice of cosmology, Corda assigned, at his Eq.(30), by means of Newton's expression for escape speed, $\alpha=r_g=2GM/c^2$, and also, according to common cosmology practice, set c=1 and G=1 so that $\alpha=r_g=2M$, where M is alleged to be the mass that is the source of a gravitational field:

$$ds^2 = \left[1-\frac{r_g}{\left(r^3+r_g^3\right)^{\frac{1}{3}}}\right]dt^2 - \left(r^3+r_g^3\right)^{\frac{2}{3}}\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \\ - \frac{d\left(r^3+r_g^3\right)^{\frac{2}{3}}}{1-\frac{r_g}{\left(r^3+r_g^3\right)^{\frac{1}{3}}}} \tag{30}$$

Corda then remarks,

> *"Historically, the line-element (30) represents `the original Schwarzschild solution' to Einstein field equations as it has been derived for the first time by Karl Schwarzschild in [3] with a slight different analysis."*

There are two additional problems here: (1) Schwarzschild [1] did not assign $\alpha=r_g=2M$ in his solution, as even a cursory reading of his original paper attests; (2) Corda's 'analysis' is nothing but a reproduction of Schwarzschild's derivation. Corda's "*slight different analysis*" is different only by his relabelling of the variables, functions and constants of Schwarzschild, with some rearrangement of terms and different numbering of equations, but otherwise precisely the same in every other respects by virtue of it being a reproduction.

At his Eq.(31) Corda relabelled Schwarzschild's 'auxiliary quantity' *R*,

Schwarzschild

$$R = \left(r^3 + \alpha^3\right)^{1/3}$$

Corda

$$\hat{r} = \left(r^3 + r_g^3\right)^{\frac{1}{3}} \tag{31}$$

Here Corda has set $\hat{r} = R$ and r_g=α, where r_g=2*M*.

Finally, at his Eq.(32), Corda presents Schwarzschild's solution (the latter's Eq.(14)), again embellished with α=r_g, renumbering, and rearrangement of terms,

Schwarzschild

$$ds^2 = (1-\alpha/R)dt^2 - \frac{dR^2}{1-\alpha/R} - R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \quad R = \left(r^3 + \alpha^3\right)^{1/3} \tag{14}$$

Corda

$$ds^2 = \left(1 - \frac{r_g}{\hat{r}}\right)dt^2 - \hat{r}^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) - \frac{d\hat{r}^2}{1-\frac{r_g}{\hat{r}}} \tag{32}$$

wherein $\hat{r} = \left(r^3 + r_g^3\right)^{\frac{1}{3}}$, $r_g = \alpha$.

Since Corda's derivation of Schwarzschild's actual solution is nothing but a relabelled point by point copy of Schwarzschild's derivation, he has contributed nothing new to the solution of the problem.

3 The spherical surface

The surface embedded in Schwarzschild's metric is,

$$ds^2 = R^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{3.1}$$

This expression is an instance of the First Fundamental Quadratic Form of a surface. The intrinsic geometry of a surface is entirely independent of any space it which it might be embedded. The most important feature of the intrinsic geometry of a surface is its Gaussian curvature[4]. The Gaussian curvature *K* of the above surface is easily calculated at $K=1/R^2$ [7-11,14]. Herein is the true meaning of *R*. Hence, in Hilbert's metric $K=1/r^2$. Since *K* is a positive constant the surface (3.1) is a spherical surface [11,14]. Since they are non-Euclidean, *r* is neither the radius nor even a distance in the

[4] The Theorema Egregium of Gauss.

metrics of Schwarzschild or Hilbert. The 'Schwarzschild radius' is not the radius of anything in these metrics. Yet cosmology maintains that the 'Schwarzschild radius' or 'gravitational radius' (i.e. $r=r_g=2m$ in Hilbert's solution) is the radius of the event horizon of a black hole of mass m. On page 79 of his paper Corda conforms to cosmology:

> *"This assumption enables the origin of the coordinate system to be not a single point, but a spherical surface having radius equal to the gravitational radius, i.e. the surface of the Schwarzschild sphere. … In fact, a coordinate transform that transfers the origin of the coordinate system, which is the surface having radius equal to the gravitational radius, in a non-dimensional material point in the core of the black hole, re-obtains the solution re-adapted by Hilbert."*

4 Corda's deformed circumference

Corda makes much of his function $m=m(r)$:

Corda

$$m = \frac{\left(r^3 + \alpha^3\right)^{2/3}}{r^2} \tag{29}$$

On page 70 of his paper [3] he writes,

> *"… we will assume that the length of the circumference centred in the origin of the coordinate system is not 2πr. We release an apparent different physical assumption, i.e. that arches of circumference are deformed by the presence of mass of the central body M."*

On page 73 he writes,

> *"Hence, we understand that the assumption to locate the mathematical singularity of the function A at X = 0 coincides with the physical condition that the length of the circumference centred in the origin of the coordinate system is* $2\pi\left(r^3 + \alpha^3\right)^{\frac{1}{3}}$, *which is different from the value 2πr. This is the apparent fundamental physical difference between this solution and the `standard Schwarzschild solution' (1), i.e. the one enabled by Hilbert ..."*

However, there is nothing either physically or mathematically meaningful in Corda's circumference assumption for the simple fact that he has confounded r in Schwarzschild's solution and r in Hilbert's solution as being the radial distance[5] in both. Moreover, Corda's m is nothing but a

[5]It is standard in cosmology to erroneously treat r in Hilbert's solution as radial distance, despite having numerous 'definitions' for it, as is particularly evident in the notion of the

relabelling of Schwarzschild's G, i.e. $m(r) = G(r)$, noted in §2 above, and so it already appears in Schwarzschild's Eq.(8), where it does not deform anything. This is easily seen by the fact that the length of a closed geodesic in the spherical surface embedded in Schwarzschild's metric is given by,

$$C = 2\pi R \tag{4.1}$$

which is indifferent to the functional form that R takes [5,6,10,11]. If $R(r)=r$ then $C=2\pi r$; if $R = \left(r^3 + \alpha^3\right)^{1/3}$ then $C = 2\pi\left(r^3 + \alpha^3\right)^{1/3}$; if $R = (\alpha - r)$ then $C = 2\pi(\alpha - r)$. In Eq.(4.1) the quantity R does not care how its value is assigned, and neither does C[6].

Corda's function m deforms nothing, because it is simply the ratio of the Gaussian curvatures of two different spherical surfaces. Recall that the Gaussian curvature K_M of the spherical surface in Minkowski's metric is $K_M=1/r^2$ and the Gaussian curvature K_S of the corresponding spherical surface in Schwarzschild's metric is $K_S=1/R^2=1/\left(r^3 + \alpha^3\right)^{2/3}$. Then,

$$m = \frac{\left(r^3 + \alpha^3\right)^{2/3}}{r^2} = \frac{R^2}{r^2} = \frac{K_M}{K_S} = G \tag{4.2}$$

which has nothing to do with a deformation of a circumference.

5 Gravitational collapse

In §3 of his paper, Corda mimics the analysis of Misner, Thorne and Wheeler [12], which he admits he reproduces, except for replacing their r with Schwarzschild's $R = \left(r^3 + \alpha^3\right)^{1/3}$, wherein Corda sets $\alpha=r_g=2M$ (the 'Schwarzschild' or 'gravitational' radius). He has therefore not added anything new to the issue. In any event, mathematical and seemingly physical arguments for the extension of Schwarzschild's solution to $-\alpha \le r = \sqrt{x^2 + y^2 + z^2}$ are easily proven to require violation of the rules of pure mathematics, and so they are invalid.

6 The infinite equivalence class

The important issues are the invariant curvature scalars and the infinite equivalence class from which all admissible equivalent specific expressions of the solution for $R_{\mu\nu}=0$[7] are obtained.

The quantity R in Schwarzschild's solution and the quantity r in Hilbert's solution can be replaced by any analytic function $R_c(r)$ without violating $R_{\mu\nu}=0$ or spherical symmetry. However, not simply any analytic function of r is permissible for a solution to Einstein's 'gravitational field' $R_{\mu\nu}=0$.

'Schwarzschild radius' (i.e. the radius of the 'event horizon'); but it is in fact neither a radius nor a distance in the these metrics (see for example, [14].

[6] Of course, the range on r in all cases must also be correctly specified.

[7] Einstein's 'Field Equations of Gravitation in the Absence of Matter' [13, §14].

Satisfaction of the field equations is a necessary but insufficient condition. For example, replace Hilbert's r with $R_c(r)=e^r$. The resulting metric is singular only at $r=\ln(2m)$. At $r=0$ nothing special happens; on the unproven assumption that $r<\ln(2m)$ is permissible. But $R_c(r)=e^r$ is itself impermissible because the resulting metric is not asymptotically flat. The infinite equivalence class of permissible analytic functions $R_c(r)$ must be ascertained.

The metric for Minkoswki spacetime in Cartesian coordinates is,

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{6.1}$$

where c is the speed of light in vacuo[8]. Changing to spherical coordinates, Eq.(6.1) becomes,

$$ds^2 = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{6.2}$$

which is spherically symmetric. The spatial section of Eq.(6.1) is,

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{6.1b}$$

which is Euclidean. The spatial section of Eq.(6.2) is,

$$ds^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{6.2b}$$

which is spherically symmetric about $r=0$ for three-dimensional Euclidean space.

Since at least 1896 it has been known that the most general metrical ground-form with spherical symmetry for three-dimensional space is,

$$ds^2 = A^2dR_c^2 + R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) \tag{6.3}$$

where $R_c=R_c(r)$, $A=A(R_c(r))$, and $r=\sqrt{x^2+y^2+z^2}$ is the r in Eq.(6.2b). Eq.(6.3) is, in general, non-Euclidean. Note that Eqs. (6.1b), (6.2b) and (6.3) are positive definite, i.e. the signs of the coefficients of the differential elements are all positive[9]. Note also that if $R_c(r)=r$ and $A(r)=1$, then the Euclidean Eq.(6.2b) is recovered.

The required infinite equivalence class must satisfy the following fundamental requirements, in accordance with Einstein's [4] prescription:

1. it must be static,
2. it must be spherically symmetric,
3. it must satisfy $R_{\mu\nu}=0$,
4. it must be asymptotically flat.

[8] That is, $c=2.998\times10^8$m/s.

[9] The signature is fixed at (+,+,+).

The infinite equivalence class has been derived elsewhere [5,6,10,11,14-16], and it is given by,

$$ds^2 = \left(1-\frac{\alpha}{R_c}\right)dt^2 - \left(1-\frac{\alpha}{R_c}\right)^{-1} dR_c^2 - R\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$
$$R_c = \left(\left|r-r_0\right|^n + \alpha^n\right)^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+ \tag{6.4}$$

Here α is a positive real constant, r_0 and n entirely arbitrary constants. Note that the spatial section of Eq.(6.4) is precisely of the form of Eq.(6.3), generalised from Eq.(6.2b). Choosing r_0=0, n=3, $r \geq r_0$ produces Schwarzschild's solution Eq.(1.1). Choosing r_0=0, n=1, $r \geq r_0$ produces Brillouin's solution [17]. Choosing $r_0=\alpha$, n=1, $r \geq r_0$ produces Droste's solution [18]. However, Hilbert's solution is not an element of the infinite set (6.4), and thus it is not equivalent to Schwarzschild's solution, contrary to Corda's claims. Note that Hilbert's solution is specifically an alleged 'extension' of Droste's solution, and hence invalid by the very choice of Droste's solution (since selection of Droste's solution places the centre of a Euclidean sphere at the point (x_0,y_0,z_0) at the distance $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2} = \alpha$ from the origin of coordinates for Minkowski space). The reader can easily verify for himself that Eq.(6.4) satisfies all four conditions required by Einstein.

Since every element of Eq.(6.4) is equivalent, then if any element thereof can be `extended' to form a black hole, then every element must be extendible in the same fashion. Conversely, if any element of Eq.(6.4) cannot be extended, then none can be extended, on account of equivalence. It is evident that no element of Eq.(6.4) can be extended to form a black hole because the latter requires that in Eq.(6.4), $0 \leq R_c$, which is impossible because $|r-r_0|^n$ is never less than zero. The metric Eq.(6.4) is singular at only one point, $r=r_0$. To amplify this fact, set r_0=0, n=2, such that $R_c = \left(r^2 + \alpha^2\right)^{1/2}$. Then the resulting metric is well defined for all real values of r, except r=0. Note that $r^2 \geq 0$ and therefore $R_c \geq \alpha$ necessarily. Since this particular equivalent metric cannot be extended to produce a black hole, no element of Eq.(6.4) can be extended to produce a black hole, due to equivalence. Consequently, neither Droste's solution can be extended to yield Hilbert's 'solution' nor Schwarzschild's solution be extended to $-\alpha \leq r = \sqrt{x^2 + y^2 + z^2}$. Hilbert's solution is invalid and so the black hole is invalid. The theory of black holes requires that $|r-r_0|^n<0$, which is a violation of the rules of pure mathematics, and so the theory of black holes is false.

The role of r is now apparent. Consider a sphere of radius $r'=|r-r_0|$ in Euclidean 3-space (i.e. the spatial section of Minkowski spacetime) such that r and r_0 lie on the same radial line through the origin of coordinates r=0. Then the centre of the sphere is located at a point with Cartesian

coordinates (x_0,y_0,z_0) at a distance $r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ from the origin of the coordinate system. The equation of this sphere in Cartesian coordinates is,

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = (r - r_0)^2 = r'^2 \qquad (6.5)$$

Only when $x_0=y_0=z_0=0=r_0$, is the sphere centred at the origin of the coordinate system. In any case, the radius of the sphere is $|r-r_0|$ no matter where in Euclidean 3-space the sphere is centred, as illustrated in Fig.1[10].

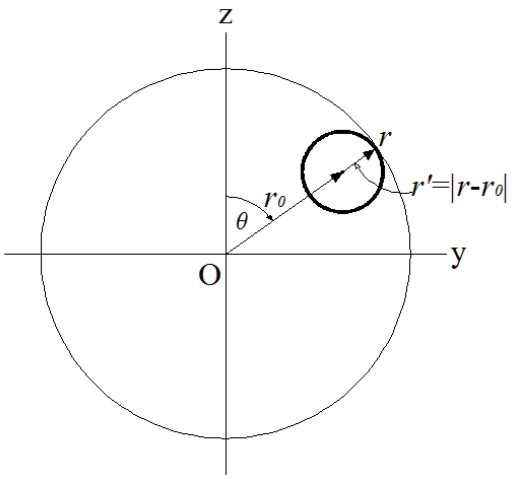

Fig. 1: A Euclidean sphere of radius r' centred at $r=r_0$ (Cartesian coordinates (x_0,y_0,z_0) at a distance $r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ from the origin O; r and r_0 lying in the same radial line through O). Schwarzschild centred at the origin O but Hilbert, by setting the coefficient of ($d\theta^2+\sin^2\theta d\varphi^2$) at r^2, unwittingly centred at $r_0=\alpha=2m$ (see Eq.(6.4)), mistakenly thinking he had centred at $r=0$, thereby separating the sphere from its centre, misplacing its centre at $r=0$. In other words, Hilbert unwittingly moved Schwarzschild's Euclidean sphere in Minkowski space to $r=r_0$ leaving its centre behind at $r=0$. The large circle denotes a sphere relative to the origin of coordinates. The small circle denotes the sphere actually involved in Hilbert's solution. Thus, when $r'=0$, $r=r_0$, the latter misinterpreted as the radius of a black hole event horizon, with $r=0$ misinterpreted as an infinitely dense 'physical' singularity of a black hole, where spacetime curvature is 'infinite'*. Note that as r' grows its sphere engulfs the origin O and beyond, so none of Euclidean 3-space is left out. Cosmology however, seeks to stop at $r=0$, all the while attempting to drive the centre of the sphere r' down to $r=0$ when in fact it is at $r_0=\alpha=2m$. When $r=0$ in Hilbert's solution, the radius $r' = r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ and the centre of the sphere at (x_0,y_0,z_0), as Eq.(6.5) attests. Hilbert confounded the origin of the coordinate system for the centre of the Euclidean sphere, and so his solution is invalid.

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*There are forces in General Relativity, but gravity is not one of them, because it is 'spacetime curvature'. Infinite spacetime curvature is therefore infinite gravity.

[10] The third dimension x has been suppressed for ease of illustration.

The radius $|r\text{-}r_0|$ in Euclidean 3-space (i.e. the spatial section of Minkowski spacetime) is mapped by Eq.(6.4) into the corresponding radius R_p in Schwarzschild non-Euclidean 3-space (i.e. the spatial section of Schwarzschild spacetime) [5,6,10,11,14-16].

$$R_p = \int \frac{dR_c}{\sqrt{1-\frac{\alpha}{R_c}}} = \sqrt{R_c\left(R_c-\alpha\right)} + \alpha \ln\left(\frac{\sqrt{R_c}+\sqrt{R_c-\alpha}}{\sqrt{\alpha}}\right)$$
$$R_c = \left(\left|r-r_0\right|^n + \alpha^n\right)^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+ \tag{6.6}$$

Note that $R_p(r_0) = 0 \; \forall r_0 \; \forall n$ (the centre of the non-Euclidean sphere is located at $R_p(r_0)$), although the metric itself is undefined there (and only there). Also note that $R_c(r_0) = \alpha \; \forall r_0 \; \forall n$.

Following §3 above, $R_c(r)$ maps the the Gaussian curvature $K=1/|r\text{-}r_0|^2$ of the spherical surface of a sphere centred anywhere in Minkowski spacetime, into the corresponding Gaussian curvature $K = 1/R_c^2$ of the spherical surface embedded in Schwarzschild spacetime.

Hence, in general, by Eq.(6.4) and Eq.(6.6), the radius $|r\text{-}r_0|$ of a Euclidean sphere centred at the point (x_0,y_0,z_0) in Minkowski spacetime is mapped into the corresponding radius R_p of a non-Euclidean sphere centred at $R_p(r_0)=0$ in Schwarzschild spacetime, and the Gaussian curvature $1/|r\text{-}r_0|^2$ of the spherical surface embedded in Minkowski spacetime is mapped to the corresponding Gaussian curvature $1/R_c^2$ of the spherical surface embedded in Schwarzschild spacetime. From this it follows that choosing $R_c(r)=r$ (i.e. $r_0=\alpha=2m$ and $n=1$ in Eq.(6.4)) shifts the centre of the Euclidean sphere associated with Hilbert's solution, to a point (x_0,y_0,z_0) at a distance $\sqrt{x_0^2+y_0^2+z_0^2} = 2m$ from the origin of coordinates of the associated Euclidean space and so the radius of that Euclidean sphere is zero when $r=r_0=2m$, then making $R_p(r_0)=0$ in Schwarzschild spacetime. Note that r and r_0 are just distances (scalars) from the origin of coordinates and so vector notation is not required. Indeed, considering vectors $\mathbf{r}$ and $\mathbf{r_0}$, if they are collinear then the radius r' of the Euclidean sphere is $r'=|\mathbf{r}\text{-}\mathbf{r_0}| = |r\text{-}r_0|$ where $r=|\mathbf{r}|$ and $r_0=|\mathbf{r_0}|$. Consequently, when Hilbert set the coefficient of $(d\theta^2+\sin^2\theta d\varphi^2)$ to r^2, he unwittingly moved the centre of Schwarzschild's Euclidean sphere from $r=0$ to $r=2m$ so that instead of $r = r' = \sqrt{x^2+y^2+z^2}$ he should have obtained $r = \sqrt{x_0^2+y_0^2+z_0^2} + \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} = r_0+r'$ so that when the radius r' of the Euclidean sphere, centred at the point (x_0,y_0,z_0), is zero, i.e. when $x=x_0$, $y=y_0$, $z=z_0$, $r = r_0 = \sqrt{x_0^2+y_0^2+z_0^2}$. Hilbert confounded the radius r' of the Euclidean sphere with the radius r from the origin of the coordinate system of the associated Euclidean space. This is amplified by the fact that since r and r_0 lie on the very same radial line through the origin of coordinates,

$$\sqrt{x^2+y^2+z^2} = \sqrt{x_0^2+y_0^2+z_0^2} + \sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} \quad (6.6)$$

so that when $x=x_0$, $y=y_0$, $z=z_0$, Eq.(6.6) reduces to the *identity*,

$$\sqrt{x_0^2+y_0^2+z_0^2} = \sqrt{x_0^2+y_0^2+z_0^2}$$

in which case,

$$r = r_0 = \sqrt{x_0^2+y_0^2+z_0^2}$$

because the radius r' of the Euclidean sphere has now converged to $r'=0$ at the centre of the sphere, located at the point (x_0,y_0,z_0) at the distance $r = r_0 = \sqrt{x_0^2+y_0^2+z_0^2}$ from the origin of the coordinate system. The origin of the coordinate system of the spatial section of Minkowski spacetime is always at $r=0$ but the centre of the Euclidean sphere need not be there - it can be placed anywhere, as Eq.(6.4) and Eq.(6.5) permit, illustrated in Fig.1.

If $\mathbf{r}$ and $\mathbf{r_0}$ are not collinear, then the radius vector $\mathbf{r'}$ of the Euclidean sphere is $\mathbf{r'}=(\mathbf{r}-\mathbf{r_0})$ the radius of that sphere is $r'=|\mathbf{r'}|=|\mathbf{r}-\mathbf{r_0}|$ (see Fig.2), and all other related expressions become, trivially, scalar functions of a vector variable:

$$R_p = \int \frac{dR_c}{\sqrt{1-\frac{\alpha}{R_c}}} = \sqrt{R_c(R_c-\alpha)} + \alpha \ln\left(\frac{\sqrt{R_c}+\sqrt{R_c-\alpha}}{\sqrt{\alpha}}\right) \quad (6.7)$$

$$R_c = (|\mathbf{r}-\mathbf{r_0}|^n+\alpha^n)^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+$$

Then as $\mathbf{r} \to \mathbf{r_0}$, $\mathbf{r'} \to \mathbf{O}$ and $r' \to 0$.

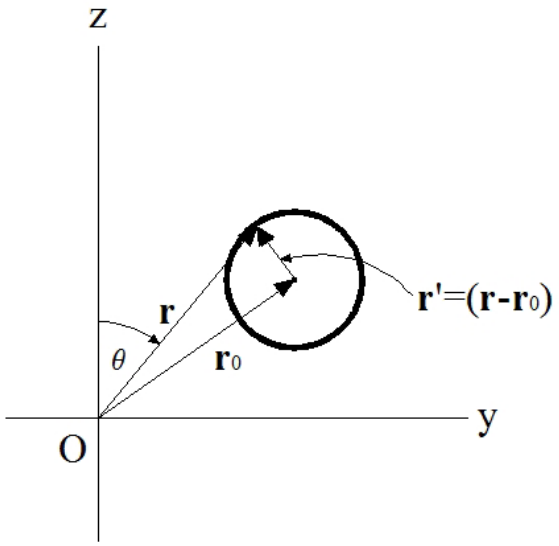

Fig. 2: The radius vector $\mathbf{r'}$ of the Euclidean sphere centred at the pointed end of the position vector $\mathbf{r_0}$ is $\mathbf{r'}=(\mathbf{r}-\mathbf{r_0})$ and the radius of that sphere is $r'=|\mathbf{r}-\mathbf{r_0}|$. As the vector $\mathbf{r} \to \mathbf{r_0}$ the radius vector of the Euclidean sphere $\mathbf{r'} \to \mathbf{O}$ and so the radius $r' \to 0$.

Note that $|r\text{-}r_0|$ means that (r-r_0) may be ≥0 or ≤0, or both. In fact, when r=0 the radius of the Euclidean sphere is $r'=|\text{-}r_0| = r_0=\sqrt{x_0^2 + y_0^2 + z_0^2}$. By elementary analytic geometry, r can take values less than zero because -r is the radius pointing in the opposite direction to r.

Consider Schwarzschild's unnumbered equation, $3x_1=\alpha^3-\rho$. To eliminate the integration constant ρ he had two obvious choices, (a) $\rho=\alpha^3$ or (b) $\rho=0$. Schwarzschild chose (a) "*In order that this discontinuity coincides with the origin*" [1]. If he chose (b) he would have obtained Droste's solution [18], in which case the centre of the Euclidean sphere is moved along any radial line to the point (x_0,y_0,z_0), at the distance $r = r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$ from the origin of coordinates (as depicted in Fig.1).

It follows that all the scalar curvature invariants are finite - contrary to cosmology, there are no curvature singularities anywhere [5-11,14,15,21].

7 Isotropic coordinates

The infinite equivalence class for Schwarzschild spacetime can be expressed in isotropic coordinates [14,15,19][11]:

$$ds^2 = \frac{\left(1-\frac{\alpha}{4R_c}\right)^2}{\left(1+\frac{\alpha}{4R_c}\right)^2}dt^2 - \left(1+\frac{\alpha}{4R_c}\right)^4 dR_c^2 - \left(1+\frac{\alpha}{4R_c}\right)^4 R_c^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

$$R_c = \left[\, |r-r_0|^n + \left(\frac{\alpha}{4}\right)^n \right]^{1/n}, \quad r, r_0 \in \Re, \quad n \in \Re^+$$

The radius R_p is given by,

$$R_p = \int \left(1+\frac{\alpha}{4R_c}\right)^2 dR_c = R_c + \frac{\alpha}{2}\ln\left(\frac{4R_c}{\alpha}\right) - \frac{\alpha^2}{8R_c} + \frac{\alpha}{4}$$

Note that $R_p(r_0) = 0\ \forall r_0\ \forall n$ and $R_c(r_0) = \alpha\ \forall r_0\ \forall n$. Clearly no element of this infinite equivalence class can be 'extended' to produce a black hole, as amplified by the case r_0=0, n=2.

A similar result is obtained for the Reissner-Nordström solution in isotropic coordinates [14,15,19]. In no case can a black hole solution be obtained, for the reason that it constitutes a violation of the rules of pure mathematics in the same fashion as for Hilbert's solution.

[11] Here c=1.

8 Other black holes

The Kerr-Newman solution subsumes the Kerr, Reissner-Nordström, and Schwarzschild solutions. In similar fashion the Kerr-Newman configuration requires an infinite equivalence class for its solution. The infinite equivalence class is [14,15,20]:

$$ds^2 = -\frac{\Delta - a^2 \sin^2\theta}{\rho^2} dt^2 - \frac{2a \sin^2\theta\left(R_c^2 + a^2 - \Delta\right)}{\rho^2} dt d\varphi +$$

$$+ \frac{\left(R_c^2 + a^2\right)^2 - a^2 \Delta \sin^2\theta}{\rho^2} \sin^2\theta\, d\varphi^2 + \frac{\rho^2}{\Delta} dR_c^2 + \rho^2 d\theta^2$$

$$\Delta = R_c^2 - \alpha R_c + a^2 + q^2, \quad \rho^2 = R_c^2 + a^2 \cos^2\theta, \quad R_c = \left(\left|r - r_0\right|^n + \xi^n\right)^{1/n}$$

$$\xi = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - q^2 - a^2 \cos^2\theta}, \quad a^2 + q^2 < \frac{\alpha^2}{4}, \quad r, r_0 \in \Re, \quad n \in \Re^+$$

Note that no element of this infinite equivalence class can be extended to produce a black hole, once again amplified by the case r_0=0, n=2. Consequently there is no black hole. If there is no 'angular momentum', a=0 and the Reissner-Nordström solution is recovered. If charge q=0 the Kerr solution is recovered. If a=0 and q=0 the Schwarzschild solution is recovered. In all cases, contrary to cosmology, there are curvature singularities nowhere [14].

9 Conclusions

Corda's derivation of Schwarzschild's solution is merely a point by point reproduction of Schwarzschild's derivation, embellished with relabelling of Schwarzschild's variables and functions, renumbering of his equations, rearrangement of elements of his equations, and numbering of his unnumbered equations. Corda has thereby not advanced anything new to the solution of the problem.

Corda's claim that Schwarzschild's solution and Hilbert's solution are equivalent is demonstrably false. Droste's solution and Brillouin's solution are equivalent to Schwarzschild's solution. The solution to $R_{\mu\nu}$=0 requires an infinite equivalence class in order to provide for all permissible 'transformations of coordinates'. Hilbert's solution is not an element of the infinite equivalence class and is therefore invalid, amplified by the fact that Hilbert's solution is an alleged extension of Droste's solution, which cannot be extended by its very selection. The 'extension' of Droste's solution to Hilbert's solution to produce a black hole constitutes a violation of the rules of pure mathematics and so it is invalid. It is from the extension of Droste's solution that Hilbert enabled the black hole. Hence, the theory of black holes is invalid.

Corda's claim that "*the length of the circumference centred in the origin of the coordinate system is not 2πr*" is false because the length of a closed geodesic in a spherical surface *always* has the form 2π*R*, being indifferent to how *R* is assigned a value. Corda's assertion that *r* in Hilbert's solution

and Schwarzschild's solution is radial distance therein, although standard cosmology, is demonstrably false. The 'Schwarzschild radius' is not the radius of anything, or even a distance, in the Schwarzschild metric or Hilbert's metric. It is therefore not the radius of a black hole event horizon.

Corda's [3] conclusion that "*… Hilbert was not wrong but they are definitely wrong the authors who claim that 'the original Schwarzschild solution' implies the non existence of BHs*'" is incorrect. The black hole is a product of violations of the rules of pure mathematics and therefore false. Schwarzschild's solution does not permit a black hole. Since the Michell-Laplace dark body does not share the properties of the black hole, it is not a black hole [14,22]. Hence, there is no legitimate mathematical theory of black holes.

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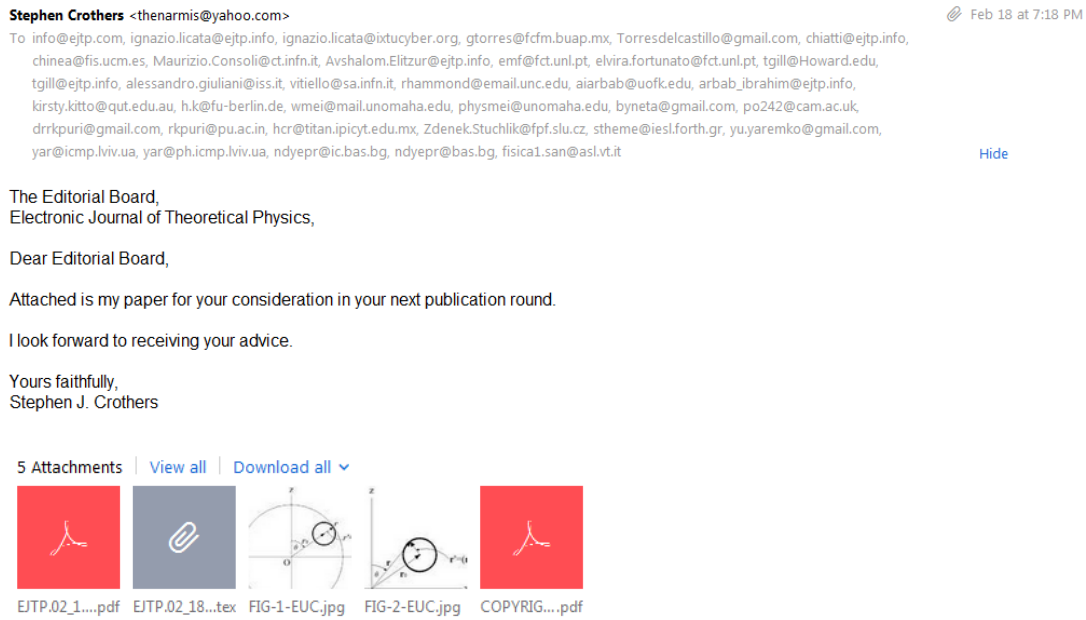
NOTE: This paper was received by all the editors of the Electronic Journal of Theoretical Physics (EJTP) on the 18th of February 2016. Subsequent request to all editors for status was ignored. On the 17th of March 2016 the Editorial Board of EJTP rejected this paper on the grounds that no editors were interested in the subject matter. Appended below is the entire correspondence. This paper is to be published: *Hadronic Journal* Vol. 39, 2016.



Stephen Crothers <thenarmis@yahoo.com> Feb 18 at 7:18 PM

To info@ejtp.com, ignazio.licata@ejtp.info, ignazio.licata@ixtucyber.org, gtorres@fcfm.buap.mx, Torresdelcastillo@gmail.com, chiatti@ejtp.info, chinea@fis.ucm.es, Maurizio.Consoli@ct.infn.it, Avshalom.Elitzur@ejtp.info, emf@fct.unl.pt, elvira.fortunato@fct.unl.pt, tgill@Howard.edu, tgill@ejtp.info, alessandro.giuliani@iss.it, vitiello@sa.infn.it, rhammond@email.unc.edu, aiarbab@uofk.edu, arbab_ibrahim@ejtp.info, kirsty.kitto@qut.edu.au, h.k@fu-berlin.de, wmei@mail.unomaha.edu, physmei@unomaha.edu, byneta@gmail.com, po242@cam.ac.uk, drrkpuri@gmail.com, rkpuri@pu.ac.in, hcr@titan.ipicyt.edu.mx, Zdenek.Stuchlik@fpf.slu.cz, stheme@iesl.forth.gr, yu.yaremko@gmail.com, yar@icmp.lviv.ua, yar@ph.icmp.lviv.ua, ndyepr@ic.bas.bg, ndyepr@bas.bg, fisica1.san@asl.vt.it Hide

The Editorial Board,
Electronic Journal of Theoretical Physics,

Dear Editorial Board,

Attached is my paper for your consideration in your next publication round.

I look forward to receiving your advice.

Yours faithfully,
Stephen J. Crothers

5 Attachments | View all | Download all

EJTP.02_1....pdf EJTP.02_18...tex FIG-1-EUC.jpg FIG-2-EUC.jpg COPYRIG....pdf

On the 18th of February 2016 I received an acknowledgment from a person named Ammar, from email address *info@ejtp.com* and copied to another editor named Ignazio Licata at email address *ignazio.licata@ejtp.info*

"*Dear Stephen*
We received your manuscript and will send it to experts for review for more investigations, and ask Corda for clarification.
P. S. Please use our online system for future submissions, this is not a proper way to send the manuscript to all editors!!! We have our regular systematic channel for paper's review.
Yours
CC to EJTP editor."

Stephen Crothers <thenarmis@yahoo.com> Mar 12 at 6:31 PM
To EJTP journal Hide
CC Ignazio Licata, ignazio.licata@ixtucyber.org, gtorres@fcfm.buap.mx, Torresdelcastillo@gmail.com, chiatti@ejtp.info, chinea@fis.ucm.es, Maurizio.Consoli@ct.infn.it, Avshalom.Elitzur@ejtp.info, emf@fct.unl.pt, elvira.fortunato@fct.unl.pt, tgill@Howard.edu, tgill@ejtp.info, alessandro.giuliani@iss.it, vitiello@sa.infn.it, rhammond@email.unc.edu, aiarbab@uofk.edu, arbab_ibrahim@ejtp.info, kirsty.kitto@qut.edu.au, h.k@fu-berlin.de, wmei@mail.unomaha.edu, physmei@unomaha.edu, byneta@gmail.com, po242@cam.ac.uk, drrkpuri@gmail.com, rkpuri@pu.ac.in, hcr@titan.ipicyt.edu.mx, Zdenek.Stuchlik@fpf.slu.cz, stheme@iesl.forth.gr, yu.yaremko@gmail.com, yar@icmp.lviv.ua, yar@ph.icmp.lviv.ua, ndyepr@ic.bas.bg, ndyepr@bas.bg, fisica1.san@asl.vt.it Hide

Dear EJTP editors,

Please advise as to the status of my paper; "On Corda's 'Clarification' of Schwarzschild's solution".

I note that in his acknowledgement of receipt of my paper on 18 February, your processing official Ammar complained of my sending my paper to all of you. The very serious nature of the matter is such that all of you had to be made aware of it, and be kept abreast of developments.

Yours faithfully,
Stephen J. Crothers
PS. I do not intend to wait indefinitely for your answer.

Stephen Crothers <thenarmis@yahoo.com> Mar 17 at 2:07 PM
To EJTP journal Hide
CC Ignazio Licata, ignazio.licata@ixtucyber.org, gtorres@fcfm.buap.mx, Torresdelcastillo@gmail.com, chiatti@ejtp.info, chinea@fis.ucm.es, Maurizio.Consoli@ct.infn.it, Avshalom.Elitzur@ejtp.info, emf@fct.unl.pt, elvira.fortunato@fct.unl.pt, tgill@Howard.edu, tgill@ejtp.info, alessandro.giuliani@iss.it, vitiello@sa.infn.it, rhammond@email.unc.edu, aiarbab@uofk.edu, arbab_ibrahim@ejtp.info, kirsty.kitto@qut.edu.au, h.k@fu-berlin.de, wmei@mail.unomaha.edu, physmei@unomaha.edu, byneta@gmail.com, po242@cam.ac.uk, drrkpuri@gmail.com, rkpuri@pu.ac.in, hcr@titan.ipicyt.edu.mx, Zdenek.Stuchlik@fpf.slu.cz, stheme@iesl.forth.gr, yu.yaremko@gmail.com, yar@icmp.lviv.ua, yar@ph.icmp.lviv.ua, ndyepr@ic.bas.bg, ndyepr@bas.bg, fisica1.san@asl.vt.it Hide

The Editors,
Electronic Journal of Theoretical Physics,

Dear Editors,

Your Author and sometime Guest Editor, Mr. C. Corda, in consequence of the paper I sent you, has brought the following new article to my attention:

Corda, C., Confusion and Crackpottery by Mr. Stephen J. Crothers,

http://vixra.org/abs/1603.0235

In view of this latest development I request your immediate advice as to your decision on my paper. You have had it for a month, which is ample time to make a decision.

Yours faithfully,
Stephen J. Crothers

On the 17th of March 2016 I received a rejection advice from the EJTP Editor-in-Chief, Ignazio Licata, and EJTP Founder and Director Ammar Sakaji. They told me this:

"Kind Crothers,
We had you asked to make submissions and requests correctly, and not to the whole editorial board. This attitude is very intrusive. Anyway..... the `serious nature of the matter' is precisely the point that differentiates us.
In pre-review phase we have concluded that the question of "Schwarzschild's solution" is not of interest for EJTP. No editor has shown the slightest interest in a review process. We recommend that you submit your article to a journal focused on the gravity themes, or in history of Physics. We must also specify that the "dialectic" question between you and Corda can remain serenely in the repositories, not concern us.
Cordially,
Ammar Sakaji,
EJTP Founder and Director
Ignazio Licata
EJTP Editor in Chief"