Observation on the numbers 4p^2-2p-1 where p and 2p-1 are primes

Abstract. In this paper I observe that many numbers of the form 4*p^2 – 2*p – 1, where p and 2*p – 1 are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = n*d – n + 1; (iii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = n*d + n – 1, and I make few related notes.

Observation:

Many numbers of the form N = 4*p^2 – 2*p – 1, where p and 2*p – 1 are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = n*d – n + 1; (iii) they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = n*d + n - 1.

Verifying the observation:
(true for the first 27 odd primes p for which 2*p – 1 is also prime)

Note that if p is prime of the form 10*k + 9 than the least prime factor of N is 5 and obviously then N respects the condition (ii) or (iii).

Also note that if d is equal to 11 and Q is of the form 10k + 1 is obviously respected condition (ii).

: for p = 3, N = 29, prime;
: for p = 7, N = 181, prime;
: for p = 19, N divisible by 5;
: for p = 31, N = 19*199 and 199 = 11*19 – 10;
: for p = 37, N = 11*491, d = 11 and Q = 10k + 1;
: for p = 79, N divisible by 5;
: for p = 97, N = 37441, prime;
: for p = 139, N divisible by 5;
: for p = 157, N = 29*3389 and 3389 = 121*29 – 120;
: for p = 199, N divisible by 5;
: for p = 211, N = 11*16151, d = 11 and Q = 10k + 1;
: for p = 229, N divisible by 5;
: for p = 271, N = 293221, prime;
: for p = 307, N = 89*4229 and 4229 = 47*89 + 46;
: for p = 331, N = 29*15089 and 15089 = 503*29 + 502;

: for p = 337, N = 453601, prime;
: for p = 367, N = 11*48911, d = 11 and Q = 10k + 1;
: for p = 379, N divisible by 5;
: for p = 439, N divisible by 5;
: for p = 499, N divisible by 5;
: for p = 547, N = 1195741, prime;
: for p = 577, N = 241*5521 and 5521 = 23*241 – 22;
: for p = 601, N = 19*75979 and 75979 = 4221*19 – 4220;
: for p = 607, N = 11*133871, d = 11 and Q = 10k + 1;
: for p = 619, N divisible by 5;
: for p = 661, N = 131*13331 and 13331 = 101*131 + 100.

Notes:

: Some numbers of this form meet another condition, i.e. they are equal to d*Q, where d is the least prime factor and Q the product of the others, and Q = (n*d – n + m)/m, or respectively Q = (n*d + n – m)/m. An example: for p = 691, N = 149*12809 and 12809 = (427*149 + 427 - 5)/5;

: Some numbers of this form meet yet another condition, i.e. they are equal to d*Q, where d is the least prime factor and Q the product of the others, and the number Q – d + 1 is prime or respectively the number Q + d - 1 is prime. An example: for p = 727, N = 139*15199 and 15199 – 139 + 1 = 15061, prime.