Divide Beal's Conjecture into Several Parts Gradually to Prove Beal's Conjecture

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Introduction: The Beal's Conjecture was discovered by Andrew Beal in 1993. Later the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society. Yet it is still both unproved and un-negated a conjecture hitherto.

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Abstract

In this article, we first classify A, B and C according to their respective odevity, and thereby get rid of two kinds from $A^X+B^Y=C^Z$. Then, affirmed the existence of $A^X+B^Y=C^Z$ in which case A, B and C have at least a common prime factor by certain of concrete equalities. After that, proved $A^X+B^Y\neq C^Z$ in which case A, B and C have not any common prime factor by the mathematical induction with the aid of the symmetric law of positive odd numbers after divide the inequality in four. Finally, reached a conclusion that the Beal's conjecture holds water via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

Keywords: Beal's conjecture; indefinite equation; inequality; odevity; mathematical induction; symmetric law of positive odd numbers.

The Proof

The Beal's Conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

We consider the limits of values of aforesaid A, B, C, X, Y and Z as given requirements for hinder concerned indefinite equations and inequalities.

First we classify A, B and C according to their respective odevity, and thereby remove following two kinds from $A^X+B^Y=C^Z$.

1. If A, B and C all are positive odd numbers, then A^X+B^Y is an even number, yet C^Z is an odd number, so there is only $A^X+B^Y\neq C^Z$ due to an odd number $\neq$ an even number.

2. If any two of A, B and C are positive even numbers, yet another is a positive odd number, then when A^X+B^Y is an even number, C^Z is an odd number; yet when A^X+B^Y is an odd number, C^Z is an even number, so there is only $A^X+B^Y\neq C^Z$ due to an odd number $\neq$ an even number.

Thus, we merely continue to have two kinds of $A^X+B^Y=C^Z$ under the given requirements as follows.

1. A, B and C all are positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number.

For indefinite equation $A^X+B^Y=C^Z$ of satisfying aforementioned either set of qualifications, in fact, it has many sets of solution with A, B and C which are positive integers. Let us here instance two concrete equalities respectively to explain this proposition.

When A, B and C all are positive even numbers, if let A=B=C=2 and X=Y $\geq$3, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $2^X+2^X=2^{X+1}$. Obviously indefinite equation $A^X+B^Y=C^Z$ at the here has a set of solution with A, B and C which are positive integers 2, 2 and 2, and that A, B and C have common prime factor 2.

In addition, if let A=B=162, C=54, X=Y=3 and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $162^3+162^3=54^4$. So indefinite equation $A^X+B^Y=C^Z$ at the here has a set of solution with A, B and C which are positive integers 162, 162 and 54, and that A, B and C have common prime factors 2 and 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3 and Z=5, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $3^3+6^3=3^5$. So indefinite equation $A^X+B^Y=C^Z$ at the here has a set of solution with A, B and C which are positive integers 3, 6 and 3, and that A, B and C have common prime factor 3.

In addition, if let A=B=7, C=98, X=6, Y=7 and Z=3, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $7^6+7^7=98^3$. So indefinite equation $A^X+B^Y=C^Z$ at the here has a set of solution with A, B and C which are positive integers 7, 7 and 98, and that A, B and C have common prime factor 7.

Therefore, indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either set of qualifications is able to hold water, but A, B and C must have at least a common prime factor.

By now, if we can prove that there is only $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor, then the conjecture is tenable definitely.

Since A, B and C have common prime factor 2 when A, B and C all are positive even numbers, so these circumstances that A, B and C have not a common prime factor can only occur in which case A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not a common prime factor, then any two of them have not a common prime factor either, because in case any two have a common prime factor, namely A^X+B^Y, C^Z-A^X or C^Z-B^Y has a common prime factor, yet C^Z, B^Y or A^X has not the common prime factor, then it will directly lead up to $A^X+B^Y \neq C^Z$, $C^Z-A^X \neq B^Y$ or $C^Z-B^Y \neq A^X$ according to the unique factorization theorem of natural number.

Unquestionably, let following two inequalities add together to replace $A^X+B^Y \neq C^Z$ under the given requirements plus the set of qualifications that A, B and C are two positive odd numbers and a positive even number without a common prime factor, this is possible categorically.

1. $A^X+B^Y \neq 2^Z G^Z$ under the given requirements plus the set of qualifications that A and B are two positive odd numbers, G is a positive integer, and that A, B and 2G have not a common prime factor.

2. $A^X+2^YD^Y \neq C^Z$ under the given requirements plus the set of qualifications that A and C are two positive odd numbers, D is a positive integer, and that A, C and 2D have not a common prime factor.

For aforesaid $A^X+B^Y \neq 2^ZG^Z$, when G=1, it is exactly $A^X+B^Y \neq 2^Z$. When G>1: if G is an odd number, then the inequality changes not, namely it is still $A^X+B^Y \neq 2^ZG^Z$; if G is an even number, then the inequality is expressed by $A^X+B^Y \neq 2^W$ or $A^X+B^Y \neq 2^WH^Z$, where H is an odd number ≥3, and W is an integer > Z.

Without doubt, $A^X+B^Y \neq 2^W$ can represent $A^X+B^Y \neq 2^Z$, and $A^X+B^Y \neq 2^WH^Z$ can represent $A^X+B^Y \neq 2^ZG^Z$, where H is an odd numbers ≥3, and W is an integer ≥3. So $A^X+B^Y \neq 2^ZG^Z$ is expressed by two inequalities as follows.

(1) $A^X+B^Y \neq 2^W$, where A and B are positive odd numbers without a common prime factor, and X, Y and W are integers ≥3.

(2) $A^X+B^Y \neq 2^WH^Z$, where A, B and H are positive odd numbers without a common prime factor, X, Y, Z and W are integers ≥3, and H ≥3.

Again go back to aforementioned $A^X+2^YD^Y \neq C^Z$ to say, when D=1, it is exactly $A^X+2^Y \neq C^Z$. When D>1: if D is an odd number, then the inequality changes not, namely it is still $A^X+2^YD^Y \neq C^Z$; if D is an even number, then the inequality is expressed by $A^X+2^W \neq C^Z$ or $A^X+2^WR^Y \neq C^Z$, where R is an odd number ≥3, and W is an integer >Y.

Without doubt, $A^X+2^W \neq C^Z$ can represent $A^X+2^Y \neq C^Z$, and $A^X+2^W R^Y \neq C^Z$ can represent $A^X+2^Y D^Y \neq C^Z$, where R is an odd number ≥3, and W is an integer ≥3. So $A^X+2^Y D^Y \neq C^Z$ is expressed by two inequalities as follows.

(3) $A^X+2^W \neq C^Z$, where A and C are positive odd numbers without a common prime factor, and X, W and Z are integers ≥3.

(4) $A^X+2^W R^Y \neq C^Z$, where A, R and C are positive odd numbers without a common prime factor, X, Y, Z and W are integers ≥3, and R ≥3.

We regard the limits of values of A, B, C, H, R, X, Y, Z and W in above-listed four inequalities plus their co-prime relation in each of inequalities as known requirements, thereinafter.

Thus it can be seen, the proof of $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor is changed to prove the existence of above-listed four inequalities under the known requirements. Such being the case, we shall first prove $A^X+B^Y \neq 2^W$ and $A^X+B^Y \neq 2^W H^Z$. For this purpose, we must beforehand expound circumstances and terminologies relating to the proof.

First let us divide all positive odd numbers into two kinds, i.e. Φ and Ω. Namely the form of Φ is 1+4n, and the form of Ω is 3+4n, where n ≥ 0. Odd numbers of Φ and Ω respectively arrange orderly as the follows.

Φ: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61…1+4n …

Ω: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63…3+4n …

Besides, we likewise use symbol Φ to denote one of Φ, and likewise use symbol Ω to denote one of Ω in the sequence of non-concrete odd numbers and on formulations concerning the symmetry of odd numbers.

After that, let us list from small to large positive odd numbers plus $2^W H^Z$ among them below, where H is an odd number ≥1, and W, Z ≥ 3. Also label the belongingness of each of odd numbers alongside.

1^W∈Φ, 3∈Ω; 5∈Φ, 7∈Ω, (2^3), 9∈Φ, 11∈Ω, 13∈Φ, 15∈Ω, (2^4), 17∈Φ, 19∈Ω, 21∈Φ, 23∈Ω, 25∈Φ, 3^3∈Ω, 29∈Φ, 31∈Ω, (2^5), 33∈Φ, 35∈Ω, 37∈Φ, 39∈Ω, 41∈Φ, 43∈Ω, 45∈Φ, 47∈Ω, 49∈Φ, 51∈Ω, 53∈Φ, 55∈Ω, 57∈Φ, 59∈Ω, 61∈Φ, 63∈Ω, (2^6), 65∈Φ, 67∈Ω, 69∈Φ, 71∈Ω, 73∈Φ, 75∈Ω, 77∈Φ, 79∈Ω, 3^4∈Φ, 83∈Ω, 85∈Φ, 87∈Ω, 89∈Φ, 91∈Ω, 93∈Φ, 95∈Ω, 97∈Φ, 99∈Ω, 101∈Φ, 103∈Ω, 105∈Φ, 107∈Ω, 109∈Φ, 111∈Ω, 113∈Φ, 115∈Ω, 117∈Φ, 119∈Ω, 121∈Φ, 123∈Ω, 5^3∈Φ, 127∈Ω, (2^7), 129∈Φ, 131∈Ω, 133∈Φ, 135∈Ω, 137∈Φ, 139∈Ω, 141∈Φ, 143∈Ω, 145∈Φ, 147∈Ω, 149∈Φ, 151∈Ω, 153∈Φ, 155∈Ω, 157∈Φ, 159∈Ω, 161∈Φ, 163∈Ω, 165∈Φ, 167∈Ω, 169∈Φ, 171∈Ω, 173∈Φ, 175∈Ω, 177∈Φ, 179∈Ω, 181∈Φ, 183∈Ω, 185∈Φ, 187∈Ω, 189∈Φ, 191∈Ω, 193∈Φ, 195∈Ω, 197∈Φ, 199∈Ω, 201∈Φ, 203∈Ω, 205∈Φ, 207∈Ω, 209∈Φ, 211∈Ω, 213∈Φ, 215∈Ω, ($2^3 \times 3^3$), 217∈Φ, 219∈Ω, 221∈Φ, 223∈Ω, 225∈Φ, 227∈Ω, 229∈Φ, 231∈Ω, 233∈Φ, 235∈Ω, 237∈Φ, 239∈Ω, 241∈Φ, 3^5∈Ω, 245∈Φ, 247∈Ω, 249∈Φ, 251∈Ω, 253∈Φ, 255∈Ω, (2^8), 257∈Φ, 259∈Ω, 261∈Φ, 263∈Ω, 265∈Φ, 267∈Ω, 269∈Φ, 271∈Ω …

By this token, the permutation of positive odd numbers from small to large has infinitely many cycles of Φ plus Ω, to wit Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω ⋯

Next, list orderly many kinds of odd numbers which have a common odd number as the base number, and label the belongingness of each of them alongside.

$1^1 \in \Phi$,	$3^1 \in \Omega$,	$5^1 \in \Phi$,	$7^1 \in \Omega$,	$9^1 \in \Phi$,	$11^1 \in \Omega$,
$1^2 \in \Phi$,	$3^2 \in \Phi$,	$5^2 \in \Phi$,	$7^2 \in \Phi$,	$9^2 \in \Phi$,	$11^2 \in \Phi$,
$1^3 \in \Phi$,	$3^3 \in \Omega$,	$5^3 \in \Phi$,	$7^3 \in \Omega$,	$9^3 \in \Phi$,	$11^3 \in \Omega$,
$1^4 \in \Phi$,	$3^4 \in \Phi$,	$5^4 \in \Phi$,	$7^4 \in \Phi$,	$9^4 \in \Phi$,	$11^4 \in \Phi$,
$1^5 \in \Phi$,	$3^5 \in \Omega$,	$5^5 \in \Phi$,	$7^5 \in \Omega$,	$9^5 \in \Phi$,	$11^5 \in \Omega$,
$1^6 \in \Phi$,	$3^6 \in \Phi$,	$5^6 \in \Phi$,	$7^6 \in \Phi$,	$9^6 \in \Phi$,	$11^6 \in \Phi$,
…	…	…	…	…	…
$13^1 \in \Phi$,	$15^1 \in \Omega$,	$17^1 \in \Phi$,	$19^1 \in \Omega$,	$21^1 \in \Phi$,	$23^1 \in \Omega$ …
$13^2 \in \Phi$,	$15^2 \in \Phi$,	$17^2 \in \Phi$,	$19^2 \in \Phi$,	$21^2 \in \Phi$,	$23^2 \in \Phi$ …
$13^3 \in \Phi$,	$15^3 \in \Omega$,	$17^3 \in \Phi$,	$19^3 \in \Omega$,	$21^3 \in \Phi$,	$23^3 \in \Omega$ …
$13^4 \in \Phi$,	$15^4 \in \Phi$,	$17^4 \in \Phi$,	$19^4 \in \Phi$,	$21^4 \in \Phi$,	$23^4 \in \Phi$ …
$13^5 \in \Phi$,	$15^5 \in \Omega$,	$17^5 \in \Phi$,	$19^5 \in \Omega$,	$21^5 \in \Phi$,	$23^5 \in \Omega$ …
$13^6 \in \Phi$,	$15^6 \in \Phi$,	$17^6 \in \Phi$,	$19^6 \in \Phi$,	$21^6 \in \Phi$,	$23^6 \in \Phi$ …
…	…	…	…	…	…

From above-listed many kinds of odd numbers, we are not difficult to see that odd numbers whereby each of Φ as a base number belong still within Φ; odd numbers which every even power of Ω forms belong within Φ;

and odd numbers which every odd power of Ω forms belong within Ω, i.e. $\Phi^X \in \Phi$, $\Omega^{2n} \in \Phi$ and $\Omega^{2n-1} \in \Omega$, where $X \geq 1$ and $n \geq 1$.

In other words, odd numbers whose exponents are even numbers belong within Φ, and odd numbers which odd powers of Φ form belong within Φ; yet odd numbers which odd powers of Ω form belong within Ω.

Also two adjacent odd numbers which have an identical exponent or a common odd number as the base number except for 1 are an even number apart, and that such even numbers are getting greater and greater along with two exponents or two base numbers are getting greater and greater.

Altogether, odd numbers of odd exponents plus even exponents are exactly all odd numbers of Φ plus Ω. Yet odd numbers whose exponents ≥ 3 are merely a part of all odd numbers, and this part is dispersed among all odd numbers, thus the part odd numbers conform to the symmetric law of positive odd numbers we shall follow to define.

We put even numbers like $2^{W-1}H^Z$ at set places among the sequence of positive odd numbers, and that regard each of $2^{W-1}H^Z$ as a symmetric center of positive odd numbers concerned, where H is an odd number ≥ 1, $W \geq 3$ and $Z \geq 3$. Then, odd numbers on the left side of $2^{W-1}H^Z$ and odd numbers near $2^{W-1}H^Z$ on the right side of $2^{W-1}H^Z$ are one-to-one bilateral symmetries at the number axis or in the sequence of natural numbers. For example, if we regard 2^{W-1} as a symmetric center, then $2^{W-1}-1 \in \Omega$ and $2^{W-1}+1 \in \Phi$, $2^{W-1}-3 \in \Phi$ and $2^{W-1}+3 \in \Omega$, $2^{W-1}-5 \in \Omega$ and $2^{W-1}+5 \in \Phi$, $2^{W-1}-7 \in \Phi$

and $2^{W-1}+7\in\Omega$ etc are one-to-one bilateral symmetry respectively.

We regard one-to-one bilateral symmetries between odd numbers of Φ and odd numbers of Ω at the number axis or in the sequence of natural numbers for symmetric center $2^{W-1}H^Z$ as the symmetric law of positive odd numbers.

The symmetric law of positive odd numbers indicates that for symmetric center $2^{W-1}H^Z$, it can only symmetrize one of Φ and one of Ω, yet can not symmetrize two of Φ or two of Ω.

After regard some $2^{W-1}H^Z$ as a symmetric center, from this $2^{W-1}H^Z$ starts out, both there are finitely many cycles of Ω plus Φ leftwards until $\Omega=3$ with $\Phi=1$, and there are infinitely many cycles of Φ plus Ω rightwards in the sequence of nature numbers.

According to the symmetric law of positive odd numbers, two distances from a symmetric center to bilateral symmetric Φ and Ω are two equilong segments at the number axis or two identical odd differences in the sequence of natural numbers.

Thus, the sum of every pair of bilateral symmetric odd numbers Φ and Ω is equal to the double of the even number as the symmetric center. Yet over the left, a sum of two non-symmetric odd numbers is unequal to the double of the even number as the symmetric center absolutely.

In other words, after regard a certain $2^{W-1}H^Z$ as a symmetric center, not just can only symmetrize Φ and Ω, but also this $2^W H^Z$ as the sum of two odd numbers can only be got from the addition of bilateral symmetric Φ and Ω. Please, you pay attention to such a conclusion, because it will be considered as an important basis that concerns the proof.

Before do the proof, it is necessary to define two terminologies, namely for a positive odd number, if its exponent is greater than or equal to 3, then we term the odd number an odd number of the greater exponent; if its exponent is equal to 1 or 2, then we term the odd number an odd number of the smaller exponent.

Pursuant to preceding related basic concepts, thereinafter, we shall prove aforesaid four inequalities by the mathematical induction with the aid of the symmetric law of positive odd numbers, one by one.

Firstly, Let us regard 2^{W-1} as a symmetric center of positive odd numbers concerned to first prove $A^X+B^Y\neq 2^W$ under the known requirements.

(1) When W-1=2, bilateral symmetric odd numbers on two sides of symmetric center 2^2 are listed as follows.

1^3, 3, (2^2), 5, 7

Obviously, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^2. So we get $A^X+B^Y\neq 2^3$ under the known requirements.

When W-1=3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of symmetric center 2^{W-1} are listed as follows successively.

1^6, 3, 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

From above-listed odd numbers plus 2^{W-1}, we are not difficult to see that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{W-1}, where W-1=3, 4, 5 and 6. So there are $A^X+B^Y\neq 2^4$, $A^X+B^Y\neq 2^5$, $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$ under the known requirements.

(2) Suppose that when W-1=K with K≥ 6, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^K. That is to say, suppose $A^X+B^Y\neq 2^{K+1}$ under the known requirements.

(3) Prove that when W-1=K+1, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}. In other words, it needs us to prove $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

Proof * We have known that odd numbers whereby 2^{W-1} including 2^K plus 2^{K+1} as a symmetric center conform to the symmetric law of positive odd numbers. Let us now list the form of permutation of odd numbers whereby 2^{K+1} including 2^K as a symmetric center as follows.

1^{K+1}, 3, 5, 7, …**Φ, Ω, Φ, Ω,** (2^K),**Φ, Ω, Φ, Ω,** …**Φ, Ω, Φ, Ω,** …**Φ, Ω, Φ, Ω,** (2^{K+1}),**Φ, Ω, Φ, Ω,**…**Φ, Ω, Φ, Ω,** …**Φ, Ω, Φ, Ω, Φ, Ω, Φ, Ω,**…**Φ, Ω, Φ, Ω.**

Since every pair of bilateral symmetric odd numbers for symmetric center 2^{W-1} belongs to Φ and Ω, so two differences from 2^{W-1} to bilateral symmetric Φ and Ω are an identical odd number actually.

In reality, all odd numbers whereby 2^K as a symmetric center are exactly odd numbers on the left side of symmetric center 2^{K+1}. Thus, for odd numbers whereby 2^{K+1} as a symmetric center, their a half retains still original places after move symmetric center to 2^{K+1} from 2^K, and the half lies on the left side of 2^{K+1}, while another half is formed from 2^{K+1} plus each of odd numbers whereby 2^K as the symmetric center, and the half lies on the right side of 2^{K+1}.

Suppose that A^X and B^Y are a pair of bilateral symmetric odd numbers for symmetric center 2^K, then there is $A^X+B^Y=2^{K+1}$ according to the preceding conclusion about the double of $2^{W-1}H^Z$ as symmetric center.

Since there are not two odd numbers of the greater exponent on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^K according to second step of the mathematical induction, thus let tentatively A^X as an odd number of the greater exponent, and B^Y as an odd number of the smaller exponent, i.e. let $X \geq 3$, and $Y = 1$ or 2.

By now, let B^Y plus 2^{K+1} makes B^Y+2^{K+1}. Please, see a simple illustration at the number axis as the follows.

A^X+2^{K+1} B^Y+2^{K+1}

1, 3... A^X 2^K B^Y 2^{K+1} $2^{K+2}-B^Y$ 3×2^K $2^{K+2}-A^X$ 2^{K+2}

Since there is only $A^X+B^Y \neq 2^{K+1}$ under the known requirements according to second step of the mathematical induction, therefore there is inevitably $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 or 2. And that it has $B^Y+2^{K+1}=A^X+2B^Y=2^{K+2}-A^X$ further. Evidently A^X and $2^{K+2}-A^X$ are a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1} due to $A^X+(2^{K+2}-A^X)=2^{K+2}$. So A^X and A^X+2B^Y in the case are bilateral symmetric odd numbers for symmetric center 2^{K+1}, and that it has $A^X+(2^{K+2}-A^X)=A^X+(A^X+2B^Y)=2^{K+2}$ under the known requirements except for Y, and Y=1 or 2. Of course, A^X and A^X+2B^Y in the case are a pair of bilateral symmetric Φ and Ω for symmetric center 2^{K+1} still.

But then, there is only $A^X+B^Y\neq 2^{K+1}$ under the known requirements, thus it has $A^X+[A^X+2B^Y]=2[A^X+B^Y]\neq 2^{K+2}$.

In any case, A^X+2B^Y is a positive odd number, so let $A^X+2B^Y=D^E$, where E expresses the greatest common divisor of exponents of distinct prime divisors of D, and D is a positive odd number, then we get $A^X+[A^X+2B^Y]=A^X+D^E\neq 2^{K+2}$ under the known requirements.

That is to say, no matter what positive integer which E equals and no matter what positive odd number which D equals from $A^X+2B^Y=D^E$ under the known requirements, there is $A^X+D^E\neq 2^{K+2}$ invariably. Namely A^X and D^E in which case $A^X+2B^Y=D^E$ under the known requirements are not two bilateral symmetric odd numbers for symmetric center 2^{K+1}.

Whereas A^X and D^E in which case $A^X+2B^Y=D^E$ under the known requirements except for Y and Y=1 or 2 are indeed a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}, and from this, we get $A^X+(A^X+2B^Y)=2^{K+2}$ according to the preceding conclusion reached. Such being the case, provided slightly change the evaluation of any letter of A^X+2B^Y, then it at once is not original that A^X+2B^Y under the known requirements except for Y and Y=1 or 2, naturally, now it lies not on the place of the symmetry of A^X either. Namely A^X and A^X+2B^Y under the known requirements are not bilateral symmetric odd numbers for symmetric center 2^{K+1} because the value of Y has changed, i.e. from Y=1 or 2 to Y $\geq$3, thus there is $A^X+ [A^X+2B^Y]=A^X+D^E\neq 2^{K+2}$ under the known requirements according to the preceding conclusion about the double of $2^{W-1}H^Z$ as the symmetric center. In addition, A^X was supposed as any positive odd number of the greater exponent on the left side of symmetric center 2^{K+1}, and there is $A^X+B^Y=2^{K+1}$ under the known requirements except for Y and Y=1 or 2, thereby it has $A^X+2B^Y=2^{K+1}+B^Y$. Thus it can be seen, A^X+2B^Y i.e. D^E lies on the right side of symmetric center 2^{K+1}.

For inequality $A^X+D^E\neq 2^{K+2}$, let us substitute D therein by B, since B and D can express any identical positive odd number, and substitute Y for E where E $\geq$3, since Y$\geq$3.

Consequently, we obtain $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

In the proof, if B^Y is an odd number of the greater exponent, then A^X is surely an odd number of the smaller exponent, yet a conclusion concluded on the premise is one and the same with $A^X+B^Y\neq 2^{K+2}$ under the known requirements really.

If A^X and B^Y are a pair of bilateral symmetric odd numbers of the smaller exponents for symmetric center 2^K, then whether A^X and A^X+2B^Y, or B^Y and B^Y+2A^X are still a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}. But, no matter what positive odd number which A^X+2B^Y or B^Y+2A^X equal, it can not turn the pair of bilateral symmetric odd numbers into two odd numbers of the greater exponents, because A^X or B^Y in the pair is not an odd number of the greater exponent originally.

To sum up, we have proven that when W-1=K+1 with K ≥6, there is only $A^X+B^Y\neq 2^{K+2}$ under the known requirements. In other words, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}.

Apply the preceding way of doing, we can continue to prove that when W-1=K+2, K+3…up to every integer >K+3, there are merely $A^X+B^Y\neq 2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$ … up to $A^X+B^Y\neq 2^W$ under the known requirements.

Secondly, Let us successively prove $A^X+B^Y\neq 2^W H^Z$ under the known requirements, and point out H ≥3 at the here emphatically.

(1) When H=1, $2^{W-1}H^Z$ to wit 2^{W-1}, we have proven $A^X+B^Y\neq 2^W$ under the known requirements in the preceding section. Namely there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center 2^{W-1}.

(2) Suppose that when H=J, and J is an odd number ≥1, $2^{W-1}H^Z$ to wit

$2^{W-1}J^Z$, there is only $A^X+B^Y\neq 2^WJ^Z$ under the known requirements. Namely suppose that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}J^Z$.

(3) Prove that when H=K with K=J+2, $2^{W-1}H^Z$ to wit $2^{W-1}K^Z$, there is only $A^X+B^Y\neq 2^WK^Z$ under the known requirements too. Namely prove that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}K^Z$.

Proof * We known that after regard $2^{W-1}H^Z$ as a symmetric center, the sum of every pair of bilateral symmetric odd numbers is equal to 2^WH^Z, yet a sum of two odd numbers of no symmetry is unequal to 2^WH^Z absolutely.

In addition, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}J^Z$. Namely there is only $A^X+B^Y\neq 2^WJ^Z$ under the known requirements according to second step of the mathematical induction.

Such being the case, let us suppose that A^X and B^Y are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}J^Z$, also tentatively let Y ≥ 3, and X=1 or 2, then there is surely $A^X+B^Y=2^WJ^Z$.

On the other, after regard $2^{W-1}K^Z$ as a symmetric center, B^Y and $2^WK^Z-B^Y$ are a pair of bilateral symmetric odd numbers due to $B^Y+(2^WK^Z-B^Y)=2^WK^Z$ according to the preceding conclusion about the double of $2^{W-1}H^Z$ as the symmetric center.

By now, let A^X plus $2^W(K^Z-J^Z)$ makes $A^X+2^W(K^Z-J^Z)$, also $A^X+2^W(K^Z-J^Z) = A^X+2^WK^Z-2^WJ^Z=2^WK^Z-(2^WJ^Z-A^X) =2^WK^Z-B^Y$ under the known requirements except for X, and X=1 or 2, due to $A^X+B^Y=2^WJ^Z$ in the case.

Now that there is $A^X+2^W(K^Z-J^Z)=2^WK^Z-B^Y$ under the known requirements except for X, and X=1 or 2; in addition B^Y and $2^WK^Z-B^Y$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, then B^Y and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, thus we get $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ under the known requirements except for X, and X=1 or 2.

Of course, B^Y and $A^X+2^W(K^Z-J^Z)$ in the case are still a pair of bilateral symmetric Φ and Ω for symmetric center $2^{W-1}K^Z$.

From $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W(K^Z-J^Z)$ and preceding supposed $A^X+B^Y\neq 2^WJ^Z$ under the known requirements, we get $B^Y+[A^X+2^W(K^Z-J^Z)]= [A^X+B^Y]+2^WK^Z-2^WJ^Z \neq 2^WK^Z$ under the known requirements. Thus it can be seen, B^Y and $A^X+2^W(K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ because the sum which B^Y plus $A^X+2^W(K^Z-J^Z)$ makes is not equal to 2^WK^Z.

It is obvious that $A^X+2^W(K^Z-J^Z)$ in the aforesaid two cases expresses two disparate odd numbers, due to $X \geq 3$ in one and X=1 or 2 in another.

From $A^X+2^W(K^Z-J^Z) =2^WK^Z-(2^WJ^Z-A^X)$ and $2^WJ^Z-A^X\neq B^Y$ under the known requirements, we get $A^X+2^W(K^Z-J^Z) \neq 2^WK^Z-B^Y$.

In any case, $A^X+2^W(K^Z-J^Z)$ is a positive odd number, thus let $A^X+2^W(K^Z-J^Z)$

$=F^V$, where V expresses the greatest common divisor of exponents of distinct prime divisors of F, and F is a positive odd number, so there is $F^V \neq 2^W K^Z - B^Y$ due to $A^X+2^W(K^Z-J^Z) \neq 2^W K^Z - B^Y$ under the known requirements. Namely there is $B^Y+F^V \neq 2^W K^Z$ under the known requirements.

Since B^Y and $A^X+2^W (K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, and $B^Y+ [A^X+2^W (K^Z-J^Z)] =2^W K^Z$ under the known requirements except for X and X=1 or 2, according to the conclusion reached previously. Such being the case, provided slightly change the evaluation of any letter of $A^X+2^W (K^Z-J^Z)$, then it at once is not original that $A^X+2^W (K^Z-J^Z)$ under the known requirements except for X and X=1 or 2, naturally, now it lies not on the place of the symmetry of B^Y either. Namely B^Y and $A^X+2^W (K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ because the value of X has changed, i.e. from X=1 or 2 to $X \geq 3$, thereby there is $B^Y+[A^X+2^W(K^Z-J^Z)] \neq 2^W K^Z$ under the known requirements according to the preceding conclusion about the double of $2^{W-1}H^Z$ as the symmetric center. Namely there is $B^Y+F^V \neq 2^W K^Z$ under the known requirements due to $A^X+2^W (K^Z-J^Z) =F^V$.

For inequality $B^Y+F^V \neq 2^W K^Z$, let us substitute F therein by A, since A and F can express any identical positive odd number, and substitute X for V where $V \geq 3$, since $X \geq 3$.

Consequently, we obtain $A^X+B^Y \neq 2^W K^Z$ under the known requirements.

In the proof, if A^X is an odd number of the greater exponent, then B^Y is surely an odd number of the smaller exponent, yet a conclusion concluded on the premise is one and the same with $A^X+B^Y\neq 2^W K^Z$ under the known requirements really.

If A^X and B^Y are a pair of bilateral symmetric odd numbers of the smaller exponents for symmetric center $2^{W-1}J^Z$, then whether B^Y and $A^X+2^W(K^Z-J^Z)$, or A^X and $B^Y+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ too. But, no matter what positive odd number which $A^X+2^W(K^Z-J^Z)$ or $B^Y+2^W(K^Z-J^Z)$ equal, it can not turn the pair of bilateral symmetric odd numbers into two odd numbers of the greater exponents, since B^Y or A^X in the pair is not an odd number of the greater exponent originally.

To sum up, we have proven $A^X+B^Y\neq 2^W K^Z$ with K=J+2 under the known requirements. Namely when H=J+2, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}(J+2)^Z$.

Apply the above-mentioned way of doing, we can continue to prove that when H=J+4, J+6… up to every odd number > J+6, there are merely $A^X+B^Y\neq 2^W(J+4)^Z$, $A^X+B^Y\neq 2^W(J+6)^Z$… up to $A^X+B^Y\neq 2^W H^Z$ under the known requirements, and point out $H\geq 3$ at the here emphatically.

Thirdly, We shall apply the reduction to absurdity on the premise by the mathematical induction to prove $A^X+2^W\neq C^Z$ under the known requirements.

And let $A^X<2^W$, $C^Z>2^W$ in this proof.

(1) When W=3, 4, 5, 6 and 7, bilateral symmetric odd numbers on two sides of symmetric center 2^3, 2^4, 2^5, 2^6 or 2^7 are listed successively below.

1^7, 3, 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127, (2^7), 129, 131, 133, 135, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 159, 161, 163, 165, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225,227, 229, 231, 233, 235, 237, 239, 241, 3^5, 245, 247, 249, 251, 253, 255.

There is 1^7 on the left side of 2^3; There is 1^7 on the left side of 2^4; There are 1^7 and 3^3 on the left side of 2^5; There are $1^7$and 3^3 on the left side of 2^6; There are 1^7, 3^3, 3^4 and 5^3 on the left side of 2^7.

It is observed that $1^3+2^3\neq C^Z$; $1^7+2^4\neq C^Z$; $1^7+2^5\neq C^Z$, $3^3+2^5\neq C^Z$; $1^7+2^6\neq C^Z$, $3^3+2^6\neq C^Z$; $1^7+2^7\neq C^Z$, $3^3+2^7\neq C^Z$, $3^4+2^7\neq C^Z$ and $5^3+2^7\neq C^Z$ under the known requirements i.e. $A^X+2^3\neq C^Z$, $A^X+2^4\neq C^Z$, $A^X+2^5\neq C^Z$, $A^X+2^6\neq C^Z$ and $A^X+2^7\neq C^Z$ under the known requirements.

(2) Suppose that when W=K with K $\geq$7, there is only $A^X+2^K\neq C^Z$ under the known requirements.

(3) Prove that when W=K+1, there is only $A^X+2^{K+1}\neq C^Z$ under the known requirements too.

Proof* Known that there is $A^X+2^K \neq C^Z$ under the known requirements, and C^Z is an odd number of the greater exponent, so A^X+2^K is an odd number of the smaller exponent.

Let us regard $3\times2^{K-1}$ as a symmetric center, then A^X+2^K and $2^{X+1}-A^K$ are a pair of bilateral symmetric odd numbers due to $(A^X+2^K)+(2^{X+1}-A^K)=3\times2^K$ according to the preceding conclusion about the double of $2^{W-1}H^Z$ as the symmetric center. Since A^X+2^K is an odd number of the smaller exponent, so $2^{K+1}-A^K$, both could be an odd number of the greater exponent, and could be an odd number of the smaller exponent, according to the proven result that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}H^Z$ where W and Z ≥3, and H is an odd number ≥3.

Again regard 2^{K+1} as a symmetric center, then A^X+2^{K+1} and $2^{K+1}-A^X$ are a pair of bilateral symmetric odd numbers due to $(A^X+2^{K+1})+(2^{K+1}-A^X)=2^{K+2}$ according to the preceding conclusion about the double of $2^{W-1}H^Z$ as the symmetric center. If A^X+2^{K+1} is an odd number of the greater exponent, then $2^{K+1}-A^X$ can only be an odd number of the smaller exponent, according to the proven result that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center 2^{W-1} where W ≥3.

Thus it can be seen, deduced an exponent of odd number $2^{K+1}-A^X$ from supposed A^X+2^{K+1} as an odd number of the greater exponent and deduced the exponent of odd number $2^{K+1}-A^X$ from known condition i.e. $A^X+2^K \neq C^Z$ are inconsistent, so A^X+2^{K+1} can only be an odd number of the smaller exponent, just can satisfy $2^{K+1}-A^X$ that both could be an odd number of the greater exponent, and could be an odd number of the smaller exponent. Please, see also following a simple illustration at the number axis.

2^K $\quad A^X+2^K$ $\quad 3 \times 2^{K-1}$ $\quad 2^{K+1}-A^X$ $\quad 2^{K+1}$ $\quad A^X+2^{K+1}$

Now that A^X+2^{K+1} is an odd number of the smaller exponent, yet C^Z which is greater than 2^{K+1} is an odd number of the greater exponent.

Therefore, we get $A^X+2^{K+1} \neq C^Z$.

Apply the preceding way of doing, we can continue to prove that when W=K+2, K+3…up to every integer >K+3, there are merely $A^X+2^{K+2} \neq C^Z$, $A^X+2^{K+3} \neq C^Z$ … up to $A^X+2^W \neq C^Z$ under the known requirements.

Fourthly, Let us apply the reduction to absurdity on the premise by the mathematical induction to last prove $A^X+2^W R^Y \neq C^Z$ under the known requirements. And let $A^X<2^W R^Y$, $C^Z>2^W R^Y$ in this proof.

(1) When R=1, $2^W R^Y$ to wit 2^W, we have proven $A^X+2^W \neq C^Z$ under the known requirements in the preceding section.

(2) Suppose that when R=J, and J is an odd number ≥1, $2^W R^Y$ to wit $2^W J^Y$, there is only $A^X+2^W J^Y \neq C^Z$ under the known requirements.

(3) Prove that when R=K with K=J+2, $2^W R^Y$ to wit $2^W K^Y$, there is only $A^X+2^W K^Y \neq C^Z$ under the known requirements too.

Proof* Known that there is $A^X+2^W J^Y \neq C^Z$ under the known requirements, and C^Z is an odd number of the greater exponent, so $A^X+2^WJ^Y$ is an odd number of the smaller exponent.

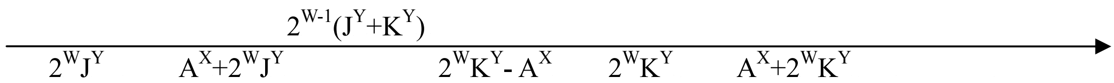
Let us regard $2^{W-1}(J^Y+K^Y)$ as a symmetric center, then $A^X+2^WJ^Y$ and $2^WK^Y-A^X$ are a pair of bilateral symmetric odd numbers due to $(A^X+2^WJ^Y)+(2^WK^Y-A^X)=2^W(J^Y+K^Y)$ according to the preceding conclusion about the double of $2^{W-1}H^Z$ as the symmetric center. Here, what we must point out is that J^Y+K^Y contains an odd factor ≥3 since $J^Y+K^Y \neq 2^V$ with V≥3, similarly hereinafter.

Since $A^X+2^WJ^Y$ is an odd number of the smaller exponent, so $2^WK^Y-A^K$, both could be an odd number of the greater exponent, and could be an odd number of the smaller exponent, according to the proven result that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}H^Z$ where W≥3, Z ≥3, and H is an odd number ≥3.

Again regard 2^WK^Y as a symmetric center, then $A^X+2^WK^Y$ and $2^WK^Y-A^X$ are a pair of bilateral symmetric odd numbers due to $(A^X+2^WK^Y)+(2^WK^Y-A^X)=2^{W+1}K^Y$ according to the preceding conclusion about the double of $2^{W-1}H^Z$ as the symmetric center. If $A^X+2^WK^Y$ is an odd number of the greater exponent, then $2^WK^Y-A^X$ can only be an odd number of the smaller exponent according to the proven result that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}H^Z$ where W ≥3, Z ≥3, and H is an odd number ≥3.

Thus it can be seen, deduced an exponent of $2^W K^Y - A^X$ from supposed $A^X+2^W K^Y$ as an odd number of the greater exponent and deduced the exponent of $2^W K^Y - A^X$ from known condition, i.e. $A^X+2^W J^Y \neq C^Z$ are inconsistent, so $A^X+2^W K^Y$ can only be an odd number of the smaller exponent, just can satisfy $2^W K^Y - A^X$ that both could be an odd number of the greater exponent, and could be an odd number of the smaller exponent.

Please, see also following a simple illustration at the number axis.



Now that $A^X+2^W K^Y$ is an odd number of the smaller exponent, yet C^Z which is greater than $2^W K^Y$ is an odd number of the greater exponent.

Therefore, we get $A^X+2^W K^Y \neq C^Z$, i.e. $A^X+2^W (J+2)^Y \neq C^Z$.

Apply the preceding way of doing, we can continue to prove that when R=J+4, J+6…up to every integer>K+6, there are merely $A^X+2^W (J+4)^Y \neq C^Z$, $A^X+2^W (J+6)^Y \neq C^Z$…up to $A^X+2^W R^Y \neq C^Z$ under the known requirements.

To sun up, we have proven every kind of $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor.

In addition, in the beginning of this article, we have proven that $A^X+B^Y=C^Z$ under the given requirements plus the qualification that A, B and C have at least a common prime factor has many sets of solution with A, B and C which are positive integers.

Last, let $A^X+B^Y=C^Z$ under the given requirements as compared $A^X+B^Y \neq C^Z$ under the given requirements, we inevitably reach such a conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the given requirements is that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal's conjecture holds water.

PS. If the Beal's conjecture by the proof is tenable, then let X=Y=Z, so indefinite equation $A^X+B^Y=C^Z$ is transformed into $A^X+B^X=C^X$. In addition, divide three terms of $A^X+B^X=C^X$ by maximal common factor of the three terms, then you will get a set of solution of positive integers without a common prime factor. It is obvious that this conclusion is in contradiction with proven the Beal's conjecture as the true, thus we have proved Fermat's Last Theorem as easy as the pie extra.