The search for life in the Universe

Sergej Plotkin
Institute of Electronics
pl.sr@mail.ru

Annotation. In the paper we propose another way in contacting other civilizations. We give an explanation why Fermi paradox exists ie we explain the Great Silence.
Keywords: Automata; up-and-down; Van Der Pol equation; complexity

1. Introduction

In the book [1] in the article by A. D. Ursul there is given a formula that shows when the contact between Earth and other civilization is possible

$$R_1 / C_1 = R_2 / C_2, \qquad \text{(A)}$$

where R means the result obtained from such contact and C means the costs to establish the contact. The corresponding superscripts denote Earth and other civilization corresponding results and costs.
There is well-known Fermi paradox that is the apparent contradiction between high estimates of the probability of the existence of extraterrestrial civilizations and the lack of evidence for such civilizations.
So, the formula (A) is not fulfilled. If we denote by η the corresponding fraction of the result and cost for Earth and by π for other civilization, then we see that

$$\eta < \pi.$$

If it was otherwise, then obviously the contact would already happened.

Hence we see that η is smaller than needed.
So, one way to contact another civilization is to make η bigger.

As one can see the natural way is to try to reduce the costs by this making η to grow.

In the article mentioned above it is not said exactly what costs mean. In the below discussion we try to formalize the meaning of costs by utilizing the notion of Automaton since such a notion should be common in both civilizations.

Let us study the possibility for a Finite State Automaton (FSA) to find a signal masked by the noise (“white noise”).

2. Main result

Let us recall a definition of FSA. More precisely, a definition of a transducer.

Definition
A finite state transducer is a sextuple $(\Sigma, \Gamma, S, s_0, \delta, \omega)$, where:

- Σ is the input alphabet (a finite non-empty set of symbols).
- Γ is the output alphabet (a finite, non-empty set of symbols).
- S is a finite, non-empty set of states.
- s_0 is the initial state, an element of S.
- δ is the state-transition function: $\delta : S \times \Sigma \to S$.
- ω is the output function $\omega : S \times \Sigma \to \Gamma$.

Let consider the case when $\Sigma = \Gamma = \{0, 1\}$. Other cases can be considered similarly.

Definition
A signal is a sequence $\alpha = (\alpha_1, ..., \alpha_n) \in \{0, 1\}^n$ of $0\ or\ 1$ of length n.

Definition
A signal $\beta = (\beta_1, ..., \beta_n)$ is called masked by the noise if there exists a random vector $\xi = (\xi_1, ..., \xi_n)$ with values in $\{0, 1\}^n$ such that

$$\beta = \alpha \oplus \xi,\ E(\xi_i) = 1/2\ for\ i = 1, ..., n\ and\ for\ all\ n \qquad (1)$$

where $\oplus$ is a boolean addition and E is a expected value of ξ.
For simplicity, we let $\xi_i = \xi\ for\ all\ i$ where ξ is some random variable.

Let there be a transducer $A = (\Sigma, \Gamma, S, s_0, \delta, \omega)$ and let β be its input signal. The transducer output signal is denoted by γ.

Definition
We call a transducer $A = (\Sigma, \Gamma, S, s_0, \delta, \omega)$ capable of recognizing a signal α if

$$D(\gamma_n - \beta_n) \to 0 \qquad (2)$$

when $n \to \infty$, where D is a variance and minus means binary subtraction.

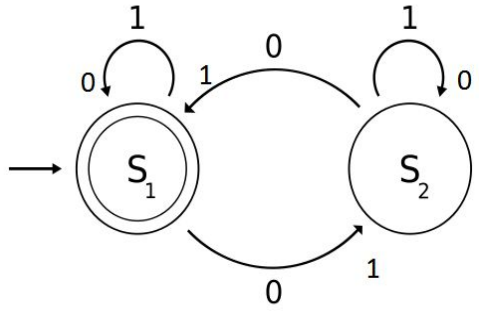
Definition

Up-and-down automaton is an automaton for which the output signal is determined by the rules

if input is 1 and previous input is 0, then the output is 1,
if input is 0 and previous input is 1, then the output is 0,
if input is 0 and previous input is 0, then the output is 1,
if input is 1 and previous input is 1, then the output is 0,
if no previous input, then the output is BLANK. `

As one can see Up-and-down automaton is the automaton that follows the input by taking the previous input and inverting it, and it reminds a well-known method from statistics theory called up-and-down method since it is based on the same principle - “up-and-down” [3].

Example of such up-and-down automaton is given on Pic. 1.



Pic. 1 Example Up-and-down automaton with initial state s_1.

Theorem.

Up-and-down automaton is capable to recognize any signal α.

Proof.

Self-evident. It follows from (2).

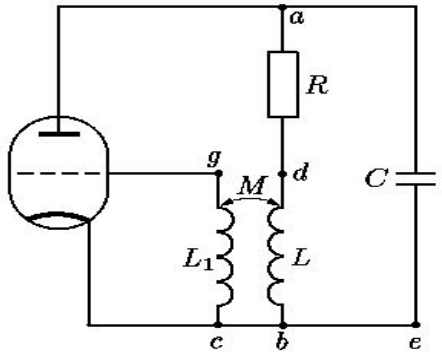
QED

Theorem P.

Any transducer that is not equivalent to Up-and-down automaton is not capable to recognize any signal (there exists a signal α that (2) is not fulfilled).

Proof.

If such transducer exists, then it will be complex enough. The work of such transducers can be described through work of electronic elements and their schemas. It is seen that this transducer will contain enough complex elements such as depicted on the Pic. 2.

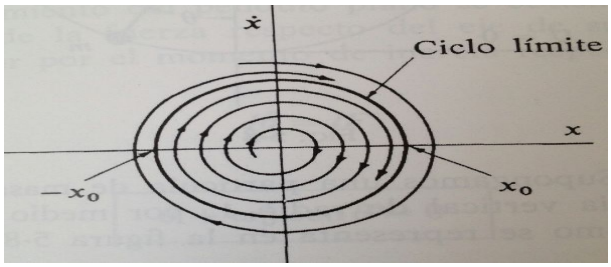


Pic. 2 Typical electronic schema

The signal inside schema depicted on Pic. 2 is described by Van Der Pol equation

$$x'' + \lambda(x^2 - 1)\,x' + x = 0 \qquad (3).$$

See [2] for further discussions. It is known that (3) possess solutions with phase portrait depicted on Pic. 3.



Pic. 3 The Phase portrait of Van Der Pol equation (3)

If $\lambda = 0$, then the solution is harmonic oscillation.
As one can see such output signals can not predict any input signal except of the same type because they are too predictable themselves.
QED

3. Conclusion

The above results can be generalized for the case of “white noise” and bigger input-output alphabets Σ, Γ since any those can be “digitized”.
The results can be generalized to other automata because the result is general and relies only on the theory of differential equations [2].
Such methods give another way in understanding the “reality”.

Theorem P explains why there is the Great Silence: simply people “do not hear” the signals from other civilizations (kind of blind) because their costs or in other words their complexity in everyday life is too high. By this not fulfilling the formula (A).

Theorem P shows that up-and-down method is the only method that can “hear” any signal. It is very simple and requires very low investments. So, it is the only method that gives a possibility to fulfill formula (A).

Using up-and-down method in contacting other civilization one needs take into account the results from [4] which state that up-and-down method with small number of samples is not not able preserve the amplitude of the input signal, ie small number of samples “do not hear” well other civilization. This problem is fixed with bigger number of samples.

References

[1] The problems of a search for extraterrestrial life. Tallinn Simpozium, 1986
[2] Pontrjagin L.S. Ordinary Differential Equations. M.: Fizmatgiz, 1961.
[3] W.J. Dixon, The Up-and-Down method for small samples // Journal of the American Statistical Association Vol. 60, No. 312 (Dec., 1965), pp. 967-978.
[4] K. Krumins, A. Lorencs, V. Plocinsh, A2 Paradox in statistical transform of signals. Automatic Control and Computer Science. 2007. No. 1. P. 3-14.