Could be explained the origin of dark matter and dark energy through the introduction of a virtual proper time ?

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Abstract

If we introduce a virtual proper time in the space-time metric, then any physical field is complemented by its own virtual field. This virtual field has an energy-momentum and a massive in the presence of field sources. In this article we consider the above phenomenon for the electromagnetic (Maxwell`s) field whose own virtual field is scalar-electric. This virtual scalar-electric field is massive in the presence of electric charges and currents. In the case of gravitational field its massive virtual field has an energy-momentum and manifests itself in gravitational interactions. Such massive virtual field could explain the origin of dark matter and dark energy.

Introduction

In Minkowski space with the metric $ds^2 = c^2dt^2 - d\mathbf{x}^2$ the proper time t is determined by the equality $ds^2 = c^2dt^2$. For a moving particle the proper time t is measured by the clock which move with this particle and at rest relative to it [1]. Therefore, in a moving inertial reference system (further, i.r.s.) the proper time t is not observable and not measured directly by the clock of time t.

We consider the system of two expressions ds^2 given above for the invariant interval s. Thereby, we pass from Minkowski space to the four-dimensional space-time with the double metric. In this four-dimensional bimetric space-time all variables depend not only on the physical coordinates t, x, y, z, but are also dependent on virtual proper time t.

Now moving i.r.s. includes the clock of virtual proper time t that is separated from the clock of physical time t . The virtual proper time t is not observable time in a moving i.r.s. We accept by definition that the clock of time t is synchronized with the clock of time t in i.r.s. at rest.

With inclusion of a virtual proper time in the metric of Minkowski space the physical electromagnetic (Maxwell's) field is complemented by its virtual scalar-electric field. In the plane scalar-electromagnetic wave the physical electromagnetic wave has a transverse polarization and the virtual scalar-electric wave has a longitudinal polarization. The virtual scalar-electric field is massive in the presence of electric charges and currents. Then the massless photons of electromagnetic field with spin 1 and two projections ± 1 are becoming the massive photons of scalar-electromagnetic field with spin 1 and three projections $0, \pm 1$. This result is physically equivalent to what we have in the case of spontaneous breaking of the gauge U(1) - symmetry for Abelian vector field [2,3,4]. The massive scalar-electromagnetic field may also explain the origin of the electron self-energy.

We use the following abbreviations:

the i.r.s. - the inertial reference system ,

the t-clock - the clock of time t ,

SEM - scalar-electromagnetic,

SE - scalar-electric.

We assume that the indices

i, j, k take on the values 1, 2, 3;

α, β, γ take on the values 0, 1, 2, 3;

μ, ν, λ take on the values 0, 1, 2, 3, 5.

I. 4-dimensional bimetric pseudo-euclidean space-time $V_{4|5}$

1. 4-dimensional biometric pseudo-euclidean space of 5-vectors $V_{4|5}$

1) Space V_4

V_4 - 4-dimensional pseudo-euclidean linear space consisting of 4-vectors $x^{\alpha} = (x^0, x^i)$

with the metric $(ds^2)_{V4} = (dx^0)^2 - (dx^i)^2$.

2) Space V_1

V_1 - 1-dimensional linear space consisting of 1-vectors (scalars) x^5 with the metric

$(ds^2)_{V1} = (dx^5)^2$.

3) Space V_5

V_5 - 5-dimensional pseudo-euclidean linear space consisting of 5-vectors

$x^{\mu} = (x^{\alpha}, x^5) = (x^0, x^i, x^5)$ with the metric $(ds^2)_{V5} = (dx^0)^2 - (dx^i)^2 + (dx^5)^2$.

4) Space $V_{4|5}$

A) $V_{4|5}$ - 4-dimensional linear space consisting of 5-vectors $x^{\mu} = (x^{\alpha}, x^5) \in V_5$ such

that $x^{\alpha} x_{\alpha} = (x^5)^2$ and that later we will call 4|5–vectors.

B) $V_{4|5}$ - pseudo-euclidean space with the double metric (bimetric) which is the system

$$(ds^2)_{V4|5} = (dx^0)^2 - (dx^i)^2 = (ds^2)_{V4} ,$$

$$(ds^2)_{V4|5} = (dx^5)^2 = (ds^2)_{V1} ,$$

or

$$(ds^2)_{V4|5} = \frac{1}{2}\left[(dx^0)^2 - (dx^i)^2 + (dx^5)^2\right] = \frac{1}{2}(ds^2)_{V5} ,$$

$$(ds^2)_{V4|5} = (dx^5)^2 = (ds^2)_{V1} .$$

It is the latter form of the double metric we call the canonical form of the metric in $V_{4|5}$ since V_5 includes the space $V_{4|5}$.

Definition

For the 4|5–vector $x^\mu = \left(x^\alpha,\, x^5\right)$ the 4–vector x^α is called *the base part* and is denoted $x^\alpha = x^\mu{}_{\text{base}}$, the 1–vector (scalar) x^5 is called *the own part* and is denoted $x^5 = x^\mu{}_{\text{own}}$. Thus, the 4|5–vector $x^\mu = \left(x^\alpha,\, x^5\right) = \left(x^\mu{}_{\text{base}},\, x^\mu{}_{\text{own}}\right)$.

2. 4-dimensional bimetric pseudo-euclidean space-time $V_{4|5}$

1) The double metric in $V_{4|5}$

Let the 4-vector $x^\alpha = \left(x^0,\, x^i\right) = \left(ct,\, \mathbf{x}\right) \in V_4$, where V_4 - 4-dimensional basic space-time (Minkowski space) with the metric $ds^2 = dx^\alpha dx_\alpha = c^2 dt^2 - d\mathbf{x}^2$. At each point $A\,(t, \mathbf{x})$ we deal only with the physical (observable) coordinates t and $\mathbf{x}$.

Let the 1-vector (scalar) $x^5 = c\tau \in V_1$, where τ is the proper time. That is, the metric in V_1: $ds^2 = c^2 d\tau^2$. Then the 5-vector $x^\mu = \left(x^\alpha,\, x^5\right) = \left(ct,\, \mathbf{x},\, c\tau\right) \in V_5$, where $V_5 = V_4 \oplus V_1$ - 5-dimensional space-time with the metric

$$\left(ds^2\right)_{V5} = 2\, ds^2 = dx^\mu dx_\mu = c^2 dt^2 - d\mathbf{x}^2 + c^2 d\tau^2 .$$

Definition

4-dimensional bimetric pseudo-euclidean space-time $V_{4|5}$ is the linear space consisting of 4|5-vectors x^μ, for which

a) the double metric in the projective

$$ds^2 = dx^\alpha dx_\alpha = c^2 dt^2 - d\mathbf{x}^2 ,$$

$$ds^2 = dx^5 dx_5 = c^2 d\tau^2 \;;$$

b) the double metric in the canonical form

$$2ds^2 = dx^{\mu}dx_{\mu} = c^2dt^2 - d\mathbf{x}^2 + c^2dt^2 ,$$

$$ds^2 = dx^5dx_5 = c^2dt^2 .$$

2) Inertial reference system in the space-time $V_{4|5}$

In each moving i.r.s. there is the clock of virtual proper time t that is separated from the clock of physical time t. The rate and direction of time coincide for the t - and t-clocks in each i.r.s. where the t-clock at rest.

Corollaries

α) Since the space-time $V_{4|5}$ is four-dimensional, then the virtual proper time t is not observable in a moving i.r.s..

β) $s = |x^5| = \pm ct$. Here and elsewhere the sign $\pm$ corresponds to the forward / backward direction of the virtual proper time t.

γ) If $\Delta t \neq 0$, then the interval Δs is always timelike, that is, $\Delta s^2 > 0$.

δ) The event in $V_{4|5}$ is defined by the point $A(t, \mathbf{x}, t)$. Thus, in the moving i.r.s. not all components of $A(t, \mathbf{x}, t)$ corresponding to the event are physical (observable).

3) Transformation group in $V_{4|5}$

In the space-time $V_{4|5}$ isomorphic to Minkowski space V_4 as a continuous transformation group of components x^{α} of the 4|5–vector $x^{\mu} = (x^{\alpha}, x^5)$ we examine the Poincare group or, in a special case, the 6-parametric Lorentz group. The component $x^5 = ct$ is Lorentz invariant.

Remarks

α) The last statement about transformation of components x^α remains valid also for any 4|5–vector $a^\mu = (a^\alpha, a^5)$ such that $a^\alpha a_\alpha = a_5{}^2$. Here, the base part $a^\mu{}_{\text{base}} = a^\alpha$, the own part $a^\mu{}_{\text{own}} = a^5$.

β) For any 4-vector a^α there is the couple 4|5-vectors $a^\mu{}_\pm = (a^\alpha, \pm a^5)$ such that $a^\alpha a_\alpha = (\pm a_5)^2$ and conversely. On physical reasons, the signs of a^0 and a^5 must be coincident. Thus, there is a one-to-one correspondence between a^α and $a^\mu = (a^\alpha, a^5)$.

3. Invariant systems for 4|5–vectors in the space-time $\mathrm{V}_{4|5}$

Respect to transformations of the Lorentz group we have the invariant expressions written below in the form of systems:

1) the 4|5–vector $x^\mu = (x^\alpha, x^5) = (ct, \mathbf{x}, ct)$

$x^\alpha x_\alpha = \text{inv}$,

$x^5 x_5 = \text{inv}$, i.e. $x^\mu x_\mu = 2x_5{}^2$;

2) the 4|5–vector of velocity u^μ (the 4|5-velocity)

$$u^\mu = \frac{dx^\mu}{ds} = (u^\alpha, u^5) = \pm\left(\frac{dt}{dt}, \mathbf{u}, 1\right) = \pm\left(\frac{1}{\varepsilon}, \frac{\mathbf{v}}{c\,\varepsilon}, 1\right).$$ Here,

$$ds = |dx^5| = \pm c\,dt = \pm c\,\varepsilon\,dt, \quad \varepsilon = \sqrt{1-(\mathbf{v}/c)^2}, \quad \mathbf{v} = \frac{d\mathbf{x}}{dt}.$$

Then $u^\alpha u_\alpha = 1$, $u^5 u_5 = 1$, i.e. $u^\mu u_\mu = 2$.

Corollary

If $a^5 \neq 0$, then $a^\mu = a^5 u^\mu$ is the 4|5–vector $a^\mu = (a^\alpha, a^5)$, such that $a^\alpha a_\alpha = a_5{}^2$ or $a^\mu a_\mu = 2a_5{}^2$.

3) the momentum 4|5–vector (the 4|5–momentum) of a massive point particle

$$\pm\, p^{\mu} = mcu^{\mu} = \pm\left(p^{\alpha},\, p^{5}\right) = \pm\left(\frac{mc}{\varepsilon},\, \frac{m\mathbf{v}}{\varepsilon},\, mc\right),$$

$$p^{\alpha}p_{\alpha} = m^2c^2, \quad p^5p_5 = m^2c^2, \quad \text{i.e.} \quad p^{\mu}p_{\mu} = 2m^2c^2.$$

Here, m is the virtual mass of a moving particle. By value m coincides with the physical mass m_0 of a particle at rest.

4) the energy-momentum $\pm\, p^{\mu}c = mc^2u^{\mu} = \pm\,(E, \mathbf{p}c, \mathrm{E}\,)$.

Here, $E = \frac{1}{\varepsilon}mc^2$, $\mathbf{p} = \frac{m\mathbf{v}}{\varepsilon}$, $\mathrm{E} = mc^2$, respectively: the physical energy, 3-momentum, the virtual self-energy of a moving particle. By value the virtual self-energy $\mathrm{E} = mc^2$ coincides with the physical self-energy $E_0 = m_0c^2$ of a particle at rest.

$$E^2 - \mathbf{p}^2\mathrm{c}^2 = m^2c^4,$$

$$\mathrm{E}^2 = m^2c^4, \qquad \text{i.e.} \quad E^2 - \mathbf{p}^2\mathrm{c}^2 + \mathrm{E}^2 = 2m^2c^4.$$

Remark

The 5-acceleration $w^{\mu} = \dfrac{du^{\mu}}{ds} = \left(w^{\alpha},\, w^{5}\right)$, $w^5 = 0$, and the 5-force

$f^{\mu} = \dfrac{dp^{\mu}}{ds} = \left(f^{\alpha},\, f^{5}\right)$, $f^{5} = 0$, are not 4|5-vectors, since, in general case,

$w^{\alpha}w_{\alpha} \neq 0$ and $f^{\alpha}f_{\alpha} \neq 0$.

4. The mass current 4|5-vector and the energy-momentum 4|5-tensor of a point particle in the space-time $\mathrm{V}_{4|5}$

The mass current 4|5-vector of a moving point particle

$$j^{\mu}_{m} = \rho_{m}cu^{\mu} = mc\int \delta^{4|5}\left(x^{\lambda} - x^{\lambda}(\vartheta)\right)\varepsilon(\vartheta)\,u^{\mu}(\vartheta)\,d\vartheta\,, \quad \text{where}$$

$\delta^{4|5}\left(x^{\lambda}-x^{\lambda}(\vartheta)\right) = \delta^{4}\left(x^{\alpha}-x^{\alpha}(\vartheta)\right)\,\delta\left(x^{5}-x^{5}(\vartheta)\right)$, $\int \delta^{4|5}\left(x^{\lambda}-x^{\lambda}(\vartheta)\right) dx^{\alpha} = 1$,

$\rho_m = m\varepsilon\,\delta\left(\mathbf{x}-\mathbf{x}(t)\right)\delta\left(t - t\,(t)\right)$ is the physical mass density.

Let the energy-momentum 4|5–tensor of a massive particle $T_m^{\mu\nu} = j_m^{\mu} c u^{\nu}$.

Trace of $T_m^{\mu\nu}$: $T_{\mu}^{\mu} = 2T_{\alpha}^{\alpha} = 2\rho_m c^2$. $T_m^{\mu 5} = \left(T_m^{\alpha 5}, T_m^{55}\right)$, where $T_m^{55} = -L_m = \rho_m c^2$.

$T_m^{\mu 5} = j_m^{\mu} c u^5 = j_m^{\mu} c$, for which the conservation equation $\partial_{\mu} T_m^{\mu 5} = 0$ and $\partial_5 T_m^{\mu 5} = 0$.

The value $T_m^{\mu 0} = j_m^{\mu} c u^0 = T_m^{\mu 5} u^0$ or $T_m^{\mu 0} = \dfrac{c}{\varepsilon}\, j_m^{\mu} = \not{\rho}_m c^2 u^{\mu}$ is not the 4|5-vector and is usually called the momentum density of a particle. The moment 4|5-vector of a particle $P^{\mu 0} = \dfrac{1}{c}\int T_m^{\mu 0} d^3 x = m c u^{\mu} = p^{\mu}$ is called the 4|5-momentum.

For the symmetric energy-momentum 4|5–tensor of a particle $T_m^{\mu\nu} = \left(T_{\text{base}}^{\mu\nu}, T_{\text{own}}^{\mu\nu}\right)$ the base part $T_{\text{base}}^{\mu\nu} = T_m^{\alpha\beta} = j_m^{\alpha} c u^{\beta}$ is the symmetric 4–tensor,

the own part $T_{\text{own}}^{\mu\nu} = \left(T_m^{\mu 5}, T_m^{5\mu}\right)$ are two equal 4|5-vectors.

By analogy with the 4|5-vector x^{μ} for the 4|5-vector $T_m^{\mu 5} = \left(T_m^{\alpha 5}, T_m^{55}\right)$ there are the invariant equalities: $T_{\alpha 5} T^{\alpha 5} = T_{55}{}^{2}$, $T_{\mu 5} T^{\mu 5} = 2T_{55}{}^{2}$.

Respectively, for the 4|5-tensor $T_m^{\mu\nu}$ there are the invariant equalities:

$T_{\alpha\beta} T^{\alpha\beta} = T_{\gamma 5} T^{\gamma 5}$, $\alpha < \beta$,

$T_{\alpha\nu} T^{\alpha\nu} = 2T_{\gamma 5} T^{\gamma 5}$, $\alpha < \nu$.

5. The charge current 4|5-vector of a massive point particle

The charge current 4|5-vector of a particle with the charge physical e and virtual mass m

$j_e^{\mu} = \rho_e c u^{\mu}$ or $j_e^{\mu} = \dfrac{e}{m}\, j_m^{\mu} = \dfrac{e}{mc}\, T_m^{\mu 5}$, where the virtual charge density $\rho_e = \dfrac{e}{m}\,\rho_m$.

If we assume the positive direction of time t and t, then the charge current 4|5-vector of a particle $j^{\mu} = \rho c u^{\mu} = \left(j^{\alpha},\, j^{5}\right) = \left(\not\rho c,\, \mathbf{j},\, \rho c\right)$, where ρ is the virtual charge density, $\not\rho = \dfrac{\rho}{\varepsilon}$ is the physical charge density, $\mathbf{j} = \rho c\,\mathbf{u} = \not\rho\,\mathbf{v}$. Here, $j_{\alpha}\, j^{\alpha} = j_{5}^{2}$ or $\not\rho^{2} c^{2} - \mathbf{j}^{2} = \rho^{2} c^{2}$.

The equation of current continuity $\partial_{\mu}\, j^{\mu} = \partial_{\alpha}\, j^{\alpha} = 0$. That is, $\partial_{5}\, j^{5} = \dfrac{\partial \rho}{\partial t} = 0$.

Then the physical charge $Q = \int \not\rho\, dV = \int \rho\, d\mathrm{V}$, $dV = \varepsilon\, d\mathrm{V} = d^{3}x$.

II. The scalar-electromagnetic field in the space-time $\mathrm{V}_{4|5}$

1. 5-potential of the SEM-field

Let the scalar-electromagnetic potential $A^{\mu}\left(x^{\nu}\right) = \left(A^{\alpha},\, A^{5}\right) = (\varphi,\, \mathbf{A},\, \phi)$ is the 5-vector $x^{\lambda} \in \mathrm{V}_{5}$, $\mathrm{V}_{5} = \mathrm{V}_{4} \oplus \mathrm{V}_{1}$, but not the 4|5-vector $x^{\nu} \in \mathrm{V}_{4|5}$. That is, the scalar potential ϕ is a virtual invariant, but the inequality $A^{\alpha} A_{\alpha} \neq A_{5}^{2}$ or $\varphi^{2} - \mathbf{A}^{2} \neq \phi^{2}$ takes place respect to the transformations of the Lorentz group (of boosts and spatial rotations). In the case of a massive SEM-field with sources the 4-potential $A^{\alpha}\left(x^{\nu}\right)$ clearly depends on the virtual proper time t, i.e. $\partial_{5} A^{\alpha} \neq 0$. Thus, the massive SEM-field with the 5-potential $A^{\mu}\left(x^{\nu}\right)$ is considered in $\mathrm{V}_{4|5}$, where the 4|5-vector $x^{\nu} = \left(x^{\alpha},\, x^{5}\right)$.

In the case of a massless SEM-field without sources the 5-potential A^{μ} does not depend on the virtual proper time t, that is, $\partial_{5} A^{\alpha} = 0$. Thus, the massless SEM-field is considered actually in V_{4} and is given by the 5-potential $A^{\mu}\left(x^{\alpha}\right)$, $x^{\alpha} \in \mathrm{V}_{4}$. The theory of a massless SEM-field is invariant respect to gauge transformations of the potential A_{μ}:

$A_{\mu} \rightarrow A_{\mu}{}' = A_{\mu} - \partial_{\mu} f$, where $f:\ \partial_{\mu}\partial^{\mu} f = 0$.

In what follows we will use mainly the Heaviside-Lorentz system of units, where

$e^2 = 4\pi\alpha$, $\mathbf{h} = c = 1$.

2. 5-tensor of the SEM-field strengths

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (\mathbf{E},\mathbf{H},\mathrm{C},-\mathbf{Э}) = \begin{pmatrix} 0 & E_x & E_y & E_z & C \\ -E_x & 0 & -H_z & H_y & -Э_x \\ -E_y & H_z & 0 & -H_x & -Э_y \\ -E_z & -H_y & H_x & 0 & -Э_z \\ -C & Э_x & Э_y & Э_z & 0 \end{pmatrix}, \text{ where}$$

the physical electric field $\mathbf{E} = -\,\mathrm{grad}\,\varphi - \frac{\partial \mathbf{A}}{\partial t}$, the physical magnetic field $\mathbf{H} = \mathrm{rot}\,\mathbf{A}$,

the virtual electric field $\mathbf{Э} = -\,\mathrm{grad}\,\phi - \frac{\partial \mathbf{A}}{\partial t}$, the virtual scalar field $\mathrm{C} = \frac{\partial \phi}{\partial t} - \frac{\partial \varphi}{\partial t}$.

For the antisymmetric 5-tensor $F_{\mu\nu} = \left(F_{\mu\nu}^{\text{base}}, F_{\mu\nu}^{\text{own}}\right)$ the base part $F_{\mu\nu}^{\text{base}} = F_{\alpha\beta} = (\mathbf{E},\mathbf{H})$ is the antisymmetric 4-tensor of the physical EM-field , the own part $F_{\mu\nu}^{\text{own}} = \left(F_{\mu 5}, F_{5\mu}\right)$, where $F_{\mu 5} = (\mathrm{C}, -\mathbf{Э}, 0) = -F_{5\mu}$, are two opposite 5-vectors of the virtual SE-field which is not observable directly in a moving i.r.s.

By analogy with the 4|5-vector A^μ, for the 4|5-tensor $F_{\mu\nu}$ we have the inequalities:

$F_{\alpha\beta}F^{\alpha\beta} \neq F_{\gamma 5}F^{\gamma 5}$, $\alpha < \beta$, i.e. $\mathbf{H}^2 - \mathbf{E}^2 \neq \mathrm{C}^2 - \mathbf{Э}^2$,

$F_{\alpha\nu}F^{\alpha\nu} \neq 2F_{\gamma 5}F^{\gamma 5}$, $\alpha < \nu$.

In the general case, $F_{\gamma 5}F^{\gamma 5} \neq F_{55}{}^2$, i.e. $\mathrm{C}^2 - \mathbf{Э}^2 \neq 0$. The equality takes place in the special case for a plane SEM-wave.

3. Transformation of the virtual SE-field strengths

The physical EM- and virtual SE- fields transform independently under the Lorentz group. As a result of boosts the virtual SE-field transforms as the 4-vector $F^{\alpha 5} = (\mathrm{C}, \mathbf{Э})$:

$$\mathrm{C}' = \frac{1}{\varepsilon}(\mathrm{C} + \mathbf{V}\mathbf{Э}), \quad \mathbf{Э}' = \mathbf{Э} + \frac{\mathbf{V}}{\varepsilon}\left(\frac{\mathbf{V}\mathbf{Э}}{\varepsilon + 1} + \mathrm{C}\right), \quad \text{where} \quad \varepsilon = \sqrt{1 - V^2}, \quad V = |\mathbf{V}|.$$

4. The first union of the SEM-field equations

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0\,, \quad \text{i.e.}$$

$$\text{rot}\,\mathbf{E} = -\frac{\partial\,\mathbf{H}}{\partial t}\,, \qquad \text{div}\,\mathbf{H} = 0\,,$$

$$\text{rot}\,\mathbf{Э} = -\frac{\partial\,\mathbf{H}}{\partial t}\,, \qquad \text{grad}\,\text{C} = \frac{\partial\,\mathbf{E}}{\partial t} - \frac{\partial\,\mathbf{Э}}{\partial t}\,.$$

5. The charge current 5-vector

$j^{\,\mu} = \left(j^{\,\alpha}, j^{\,5} \right) = (\not\rho,\, \mathbf{j},\, \rho)$, where $j^{\,\alpha}$ is physical, $j^{\,5} = \rho$ is virtual, but $j^{\,\mu} \neq \rho u^{\,\mu}$.

Thus, $j^{\,\mu}$ is not the 4|5-vector. From this , $j_\alpha\, j^{\,\alpha} \neq j_5^{\,2}$ or $\not\rho^{\,2} - \mathbf{j}^{\,2} \neq \rho^{\,2}$.

6. Lagrangians of the massive SEM-field with sources

The full Lagrangian $\text{L} = \text{L}_{\text{f}} + \text{L}_{\text{int}}$, where

$$\text{L}_{\text{f}} = \text{L}_{\text{SEM}} = -\frac{1}{4} F_{\mu\nu}F^{\,\mu\nu} + \frac{1}{2}\mathcal{M}^2 A_\mu A^\mu = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2 + \mathbf{Э}^2 - \text{C}^2\right) + \frac{1}{2}\mathcal{M}^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right)\,,$$

$\mathcal{M}$ this is the virtual mass of a quantum SE-field, $\text{L}_{\text{int}} = -A_\mu\, j^{\,\mu}$.

Since $\text{L}_{\text{f}} = \text{L}^{\text{f}}_{\text{own}} + \text{L}^{\text{f}}_{\text{base}}$, i.e. $\text{L}_{\text{SEM}} = \text{L}_{\text{SE}} + \text{L}_{\text{EM}}$, then the own part of Lagrangian L :

$$\text{L}_{\text{own}} = \text{L}^{\text{f}}_{\text{own}} + \text{L}^{\text{int}}_{\text{own}} = -\frac{1}{2} F_{\mu 5}F^{\,\mu 5} + \frac{1}{2}\mathcal{M}^2 A_\mu A^\mu - A_5\, j^{\,5} =$$

$$= \frac{1}{2}\left(\mathbf{Э}^2 - \text{C}^2\right) + \frac{1}{2}\mathcal{M}^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right) - \rho\phi\,,$$ the base part of Lagrangian L :

$$\text{L}_{\text{base}} = \text{L}^{\text{f}}_{\text{base}} + \text{L}^{\text{int}}_{\text{base}} = -\frac{1}{4} F_{\alpha\beta}F^{\,\alpha\beta} - A_\alpha\, j^{\,\alpha} = \frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2\right) - (\not\rho\,\varphi - \mathbf{j}\mathbf{A})\,.$$

Obviously, Lagrangian $\text{L}^{\text{f}}_{\text{base}} = \text{L}_{\text{EM}}$ is gauge invariant, but $\text{L}^{\text{f}}_{\text{own}} = \text{L}_{\text{SE}}$ is not.

7. The second union of equations of the massive SEM-field with sources. Elimination of the infrared divergences

Proca equations for the massive SEM-field with sources are obtained from the Lagrangians L and L_{own} as the system

$\partial_\nu F^{\nu\mu} + м^2 A^\mu = j^\mu$,

$\partial_5 F^{5\alpha} + м^2 A^\alpha = 0$. From this, the equations:

$$\mathrm{div}\,\mathbf{E} + м^2\varphi = \not{\rho} + \frac{\partial C}{\partial t}\,, \quad \mathrm{rot}\,\mathbf{H} + м^2\mathbf{A} = \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{Э}}{\partial t}\,,$$

$\frac{\partial C}{\partial t} = м^2\varphi$, $\frac{\partial \mathbf{Э}}{\partial t} = м^2\mathbf{A}$. In the difference of these equations we obtain :

$\partial_\alpha F^{\alpha\beta} = j^\beta$, that is, $\mathrm{div}\,\mathbf{E} = \not{\rho}$, $\mathrm{rot}\,\mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{E}}{\partial t}$. Also, from the system we obtain

the equation: $\partial_\alpha F^{\alpha 5} + м^2 A^5 = j^5$, that is, $\mathrm{div}\,\mathbf{Э} + м^2\phi = \rho - \frac{\partial C}{\partial t}$.

Corollary

As follows from the equations of the massive SEM-field, the virtual SE-field as the unobservable own part of SEM-field varies in time t and, therefore, is massive in the presence of field sources . The small virtual mass $м$ of quantum SE-field protects from the infrared catastrophe in QED [5]. The physical EM-field as the observable base part of SEM-field is massless and long-range.

8. Wave equations for a massive SEM-field with sources

Using the Stückelberg Lagrangians with the interaction term

$$\mathrm{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2}\left(\partial_\mu A^\mu\right)^2 + \frac{1}{2} м^2 A_\mu A^\mu - A_\mu j^\mu\,,$$

$$\mathrm{L}_{\mathrm{own}} = -\frac{1}{2} F_{\mu 5} F^{\mu 5} - \frac{1}{2}\left(\partial_5 A^5\right)^2 + \frac{1}{2} м^2 A_\mu A^\mu - A_5 j^5\,,$$

we obtain the system of SEM-field equations

$\partial_\nu F^{\nu\mu} + м^2 A^\mu = j^\mu$,

$\partial_5 F^{5\alpha} + м^2 A^\alpha = 0$, with the condition $\partial_\mu A^\mu = 0$.

That is equivalent to the system of wave equations for the 5-potential A^{μ}

$$\left(\mathrm{I} - м^2\right) A^{\mu} = -j^{\mu}\ , \quad \text{where} \quad \mathrm{I} = -\partial_{\nu}\partial^{\nu},$$

$$\left(\partial_5^{\,2} + м^2\right) A^{\alpha} = 0\ , \quad \text{with the condition} \quad \partial_5 A^5 = 0 .$$

Since $\partial_{\nu}\partial^{\nu} = \partial_{\gamma}\partial^{\gamma} + \partial_5^{\,2}$, then from the system we get the equations :

$$\mathbf{W} A^{\alpha} = -j^{\alpha}, \ \left(\mathbf{W} - м^2\right) A^5 = -\ j^5, \quad \text{where } \mathbf{W} = -\partial_{\gamma}\partial^{\gamma} .$$

Then the system of wave equations for the SEM-field strengths

$$\left(\mathrm{I} - м^2\right) F^{\mu\nu} = -J^{\mu\nu}, \quad \text{where} \quad J^{\mu\nu} = \partial^{\mu} j^{\nu} - \partial^{\nu} j^{\mu},$$

$$\left(\partial_5^{\,2} + м^2\right) F^{\alpha\beta} = 0 .$$

From here, the wave equations for EM-field strengths :

$$\mathbf{W} F^{\alpha\beta} = -J^{\alpha\beta}, \ \left(\partial_5^{\,2} + м^2\right) F^{\alpha\beta} = 0 .$$

The wave equations for SE-field strengths : $\left(\mathrm{I} - м^2\right) F^{\alpha 5} = -J^{\alpha 5}$.

That is, for the strengths Э and C of the virtual massive SE-field:

$$\left(\mathrm{I} - м^2\right) \text{Э} = \operatorname{grad}\rho + \frac{\partial \mathbf{j}}{\partial t}\ , \quad \left(\mathrm{I} - м^2\right) \mathrm{C} = \frac{\partial \not{p}}{\partial t} - \frac{\partial \rho}{\partial t} .$$

9. The equation of current continuity. Conserved charges

From the SEM-field equations it follows the equation of current continuity $\partial_{\mu} j^{\mu} = 0$

together with the condition $\partial_5 j^5 = 0$. Therefore, in $\mathrm{V}_{4|5}$ the physical charge

$Q_0 = \int j^0\left(x^{\lambda}\right) d^3x = \int \not{p}\ d^3x$, the virtual charge $Q_5 = \int j^5\left(x^{\lambda}\right) d^3x = \int \rho\ d^3x$, where

$|Q_0| > |Q_5|$, since $\not{p} = \dfrac{\rho}{\varepsilon}$, are conserved in time: $\dfrac{d}{dt} Q_0 = 0$, $\dfrac{d}{dt} Q_5 = 0$.

10. The canonical energy-momentum tensor of the massive SEM-field

From the full Lagrangian L_{f} of the massive SEM-field we can obtain the energy-

momentum tensor $T^{\mu\nu} = (T^{\mu\nu}_{\text{base}}\ ,\ T^{\mu\nu}_{\text{own}})$, or in the matrix form

$$T^{\mu\nu} = \begin{pmatrix} T^{\alpha\beta} & T^{\alpha 5} \\ T^{5\alpha} & T^{55} \end{pmatrix} = \begin{pmatrix} \mathrm{w} & \mathbf{g} & \mathrm{v} \\ \mathbf{S} & -\hat{\sigma} & \mathbf{h} \\ \mathrm{v} & \mathbf{R} & \mathrm{u} \end{pmatrix}.$$

All equalities below are given with an accuracy to terms that disappear upon integration over d^3x in $\mathrm{V}_{4|5}$.

The base part of the energy-momentum tensor $T^{\mu\nu}_{\text{base}} = T^{\alpha\beta}$ has the physical components: the energy density ($c = 1$)

$$\mathrm{w} = T^{00} = \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{H}^2 - \mathbf{Э}^2 - \mathrm{C}^2\right) - \frac{1}{2}\,м^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right) = T^{00}_{\text{EM}} - T^{00}_{\text{SE}}\ ,$$

$\mathbf{g}$ - the momentum density 3-vector, $c\mathbf{g} = \left\{T^{0i}\right\} = [\mathbf{E}\mathbf{H}] - \mathrm{C}\mathbf{Э} = \left\{T^{0i}\right\}_{\text{EM}} - \left\{T^{0i}\right\}_{\text{SE}}$,

$\mathbf{S}$ - the energy flux density 3-vector (the Poynting vector) , $\frac{1}{c}\mathbf{S} = \left\{T^{i0}\right\} = [\mathbf{E}\mathbf{H}] - \mathrm{C}\mathbf{Э}$,

the stress 3-tensor $-\hat{\sigma} = \left\{T^{ij}\right\}$.

The own part of the energy-momentum tensor $T^{\mu\nu}_{\text{own}} = \left(T^{\mu 5}, T^{5\mu}\right)$ has the components:

$$\mathrm{u} = T^{55} = -\frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2 - \mathbf{Э}^2 + \mathrm{C}^2\right) + \frac{1}{2}\,м^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right) = \mathrm{L}_{\text{SE}} - \mathrm{L}_{\text{EM}}\ ,$$

virtual: $\mathrm{v} = T^{05} = \mathbf{Э}\mathbf{E}$, $c\mathbf{h} = \left\{T^{i5}\right\} = [\mathbf{Э}\mathbf{H}] + \mathrm{C}\mathbf{E}$, $\frac{1}{c}\mathbf{R} = \left\{T^{5i}\right\} = [\mathbf{Э}\mathbf{H}] + \mathrm{C}\mathbf{E}$.

Trace of the energy-momentum tensor of the massive SEM-field

$$T^{\mu}_{\mu} = T^{\alpha}_{\alpha} + T^{5}_{5} = -\frac{1}{2}\left(\mathbf{E}^2 - \mathbf{H}^2 + \mathbf{Э}^2 - \mathrm{C}^2\right) - \frac{1}{2}\,м^2\left(\varphi^2 - \mathbf{A}^2 + \phi^2\right) = -\mathrm{L}_{\text{SEM}}\ .$$

Corollary

The virtual SE-field brings in the negative contribution to the energy-momentum of the SEM-field. Thus, for hydrogen atom in an external field the virtual SE-field shifts the energy levels and this leads to two additional amendments.

11. Equations of the virtual SE-waves. The plane SEM-wave

We can obtain the equations of the SEM-waves from the equations for a massive SEM-field, when $j^{\mu} = \mathcal{M} = 0$. In particular, the equations of the virtual SE-waves:

$$\mathrm{div}\,\mathbf{Э} = -\frac{\partial \mathrm{C}}{\partial t}\ , \quad \mathrm{rot}\,\mathbf{Э} = \mathbf{0}\ , \quad \mathrm{grad}\,\mathrm{C} = -\frac{\partial \mathbf{Э}}{\partial t}\ .$$

Here below the dot above denotes the differentiation with respect to time t.

Let the propagation direction of the plane SEM-wave $\mathbf{n} // Ox$. We can find i.r.s. in which $A^0 = \varphi = \mathrm{const} \neq 0$. Then $\mathbf{E} = -\dot{\mathbf{A}}$, $\mathbf{H} = \mathrm{rot}\,\mathbf{A}$.

Thus, $\mathbf{E} = [\mathbf{H}\mathbf{n}]$, $\mathbf{H} = [\mathbf{n}\mathbf{E}]$, $|\mathbf{E}| = |\mathbf{H}|$, $\mathbf{n}\mathbf{E} = 0$, $\mathbf{n}\mathbf{H} = 0$, $\mathbf{E}\mathbf{H} = 0$.

That is, the physical EM-waves are transverse. Further, $\mathbf{Э} = -\,\mathrm{grad}\,\phi = \mathbf{n}\dot{\phi}$, $\mathrm{C} = \dot{\phi}$.

Thus, $\mathbf{Э} = \mathbf{n}\mathrm{C}$, $\mathrm{C} = \mathbf{n}\mathbf{Э}$, $\mathbf{Э} // \mathbf{n}$, $|\mathrm{C}| = |\mathbf{Э}|$, $\mathbf{Э}\mathbf{E} = 0$, $\mathbf{Э}\mathbf{H} = 0$.

That is, the virtual SE-waves are longitudinal.

Since $\mathbf{H}^2 = \mathbf{E}^2$, $\mathrm{C}^2 = \mathbf{Э}^2$, then $T_{55} = 0$ and the energy density of the plane SEM-wave $T_{00} = \mathbf{E}^2 - \mathbf{Э}^2$. The Poynting vector $\mathbf{S} = [\mathbf{E}\mathbf{H}] - \mathrm{C}\mathbf{Э} = \mathbf{S}_{\mathrm{EM}} - \mathbf{S}_{\mathrm{SE}} = \mathbf{n}\left(\mathbf{E}^2 - \mathbf{Э}^2\right) = \mathbf{n}T_{00}$.

That is, $T_{00} = \mathbf{n}\mathbf{S}$.

12. The equation of motion of the charged point particle in an external massive SEM-field

$f_{\mathrm{i}}^{\mu} = j_{\nu} F^{\mu\nu} - \mathcal{M}^2 \partial^{\mu}\left(A_{\nu}A^{\nu}\right)$, where j^{ν} - the charge current 5-vector, f_{i}^{μ} - the 5-force acting on the particle with the physical charge e and virtual mass m.

This point particle moves in an external massive SEM-field in the forward direction of time t and time t. Other hand, $f_{\mathrm{i}}^{\mu} = \frac{d}{ds} T_m^{\mu 5} = \rho_m \frac{du^{\mu}}{ds}$, where ρ_m - the physical mass density of a particle (see I.4) and $T_m^{\mu 5}$ - the momentum density 4|5-vector of a particle.

Hence, $f_{\mathrm{i}}^{0} = \mathbf{j}\mathbf{E} + \rho\mathrm{C} - \mathcal{M}^2 \partial^0\left(A_{\nu}A^{\nu}\right)$, $\mathbf{f}_{\mathrm{i}} = \rho\mathbf{E} - \rho\mathbf{Э} + [\mathbf{j}\,\mathbf{H}] - \mathcal{M}^2\,\vec{\nabla}\left(A_{\nu}A^{\nu}\right)$,

$f_{\mathrm{i}}^{5} = \mathbf{j}\,\mathbf{Э} - \not{\partial}\mathrm{C} - м^{2}\partial^{5}\left(A_{\nu}A^{\nu}\right)$. However, $f_{\mathrm{i}}^{5} = \rho_{m}\dfrac{du^{5}}{ds} = 0$. Therefore,

$0 = j_{\nu}F^{5\nu} - м^{2}\partial^{5}\left(A_{\nu}A^{\nu}\right)$. We can see that $\partial_{5}\left(A_{\nu}A^{\nu}\right) = 0$. Then, $j_{\nu}F^{5\nu} = \mathbf{j}\,\mathbf{Э} - \not{\partial}\mathrm{C} = 0$,

i.e. $\not{\partial}\mathrm{C} = \mathbf{j}\,\mathbf{Э}$ or $\mathrm{C} = \mathbf{v}\mathbf{Э}$.

On the other hand, the 5-force acting on a moving charge from an external massive SEM-field with the energy-momentum tensor $T^{\mu\nu}$, is equal $f_{\mathrm{f}}^{\mu} = \partial_{\nu}T^{\mu\nu}$. From the equality $f_{\mathrm{f}}^{\mu} = f_{\mathrm{i}}^{\mu}$ follows that $f_{\mathrm{f}}^{5} = f_{\mathrm{i}}^{5} = 0$. That is, $\partial_{\nu}T^{5\nu} = 0$.

13. The field origin of the electron virtual mass

The consideration of only the physical EM-field cannot explain the origin of mass, self-energy and momentum of an electron. The stability of an electron cannot be achieved only through physical electromagnetic forces [6,7]. It should also take into account the virtual massive SE-field.

In the space-time $\mathrm{V}_{4|5}$ the virtual mass m of a moving electron (see I.3) has the origin of a massive SEM-field and is explained by the presence of the virtual self-SEM-field of an electron. The latter corresponds to the nonzero base part of the energy-momentum 5–vector of the massive SEM-field, i.e., $T^{\mu 5}_{\mathrm{base}} = T^{\alpha 5} \neq 0$.

Since the momentum density of the virtual self-SEM-field of an electron

$\mathbf{h} = \dfrac{1}{c}\left\{T^{i5}\right\} = \dfrac{1}{c}\left([\mathbf{Э}\mathbf{H}] + \mathrm{C}\mathbf{E}\right)$, where $c = 1$, then the 3-momentum

$$\mathbf{И} = \int \mathbf{h}\, d^{3}x = \frac{1}{c}\int\left([\mathbf{Э}\mathbf{H}] + \mathrm{C}\mathbf{E}\right) d^{3}x\,.$$

In i.r.s., where the electron at rest, $\mathrm{C}' = 0$, $\mathbf{H}' = \mathbf{0}$. From the transformation of the SEM-field strengths (see II.3) it follows that $\mathrm{C} = \dfrac{1}{c}\,\mathbf{V}\mathbf{Э}$, $\mathbf{H} = \dfrac{1}{c}[\mathbf{V}\mathbf{E}]$.

Then, $\mathbf{И} = \frac{1}{c^2}\,\mathbf{V}\int \mathbf{ЭE}\; d^3x = m\mathbf{V}$, where the virtual mass of an electron

$m = \frac{1}{c^2}\int \mathbf{ЭE}\; d^3x$, $T^{05} = \mathbf{ЭE}$. Thus, the virtual self-energy $mc^2 = \int \mathbf{ЭE}\; d^3x$.

Also, we have proved the equality $\frac{1}{c}\int T^{\alpha 5}\, d^3x = \int j_m^{\alpha}\; d^3x$, where j_m^{α} - the virtual mass current 4-vector (see I.4).

Remark

By value the virtual self-energy of a moving electron $\mathrm{E} = mc^2 = \int \mathbf{ЭE}\; d^3x$ coincides with the physical self-energy of an electron at rest $E_0 = m_0 c^2 = \int \mathbf{E}^2\; d^3x$, i.e. with the energy of the electrostatic field [8,9].

References

1. Landau L. D., Lifshitz E. M.: Course of Theoretical Physics, Pergamon, Oxford, 1975, Vol. 2: The Classical Theory of Fields, Ch.1.
2. Chang T.-P., Li L.-F.: Gauge Theory of Elementary Particle Physics, Oxford, 2000, Ch.8.
3. Ryder L. H.: Quantum Field Theory, Cambridge University Press, Cambridge, 1985,Ch.8.
4. Kane G.: Modern Elementary Particle Physics, Addison Wesley, 1987, Ch.8.
5. Itzykson C., Zuber J-B.: Quantum Field Theory , McGraw Hill, New York, 1980, Ch.3.
6. Feynman R. P., Leighton R. B., Sands M.: The Feynman Lectures on Physics, Addison Wesley, 1964 , Vol. 2, Ch.28.
7. Eddington A.S.: The Mathematical Theory of Relativity, Cambridge University Press, Cambridge, 1924, Ch.6.
8. Jackson J.D.: Classical Electrodynamics, Wiley, New York, 1962, Ch.17.
9. Tamm I.E.: Fundamentals of the Theory of Electricity, Mir Publish, Moscow, 1979, Ch.1.