Another bold conjecture on Fermat pseudoprimes

Abstract. In my previous paper “Bold conjecture on Fermat pseudoprimes” I stated that there exist a method to place almost any Fermat pseudoprime to base two (Poulet number) in an infinite subsequence of such numbers, defined by a quadratic polynomial, as a further term or as a starting term of such a sequence. In this paper I conjecture that there is yet another way to place a Poulet number in such a sequence defined by a polynomial, this time not necessarily quadratic.

Conjecture:

If we express the prime factors of a Poulet number, not divisible by 3, P = d1*d2*...di, where d1, d2, ..., di are its prime factors, as P = (2n + 1)*(m1*n + 1)*...*(mi*n + 1), then there exist an infinity of Poulet numbers of this form.

As example, the first Poulet number, 341, is equal to 11*31 and we have 11 = 2*5 + 1 (so 2*n + 1) and 31 = 6*5 + 1 (so 6*n + 1); the conjecture states that there exist an infinity of Poulet numbers of the form (2*n + 1)*(6*n + 1).

Note that not any Poulet number not divisible by 3 (though the most of them) can be written the way described above; as example, the 2-Poulet number 6601 = 7*23*41 (7 is equal to 2*3 + 1, but 23 isn’t equal to m*3 + 1 neither 41).

Examples:
(for few from the first Poulet numbers not divisible by 3)

: 341 = 11*31 is the starting term, obtained for n = 5, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1) which has the following terms: 645, 2465, 2821, 4033 (...) obtained for n = 7, 14, 15, 18 (...);

: 1105 = 5*13*17 is the starting term, obtained for n = 2, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1)*(8*n + 1) which has the following terms: 13981, 137149, 278545, 493885 (...) obtained for n = 5, 11, 14, 17 (...);

: 1387 = 19*73 is the second term, obtained for n = 9, in the sequence of Poulet numbers (2*n + 1)*(8*n + 1) which has as the first term, obtained for n = 8, the Poulet number 1105, and as the following terms: 2047, 3277, 6601 (...) obtained for n = 11, 14, 20 (...);

: 1729 = 7*13*19 is the starting term, obtained for n = 3, in the sequence of Poulet numbers (2*n + 1)*(4*n + 1)*(6*n + 1) which has the following terms: 18705, 172081, 294409 (...) obtained for n = 7, 15, 18 (...);

: 2047 = 23*89 is the third term, obtained for n = 11, in the sequence of Poulet numbers (2*n + 1)*(8*n + 1) which has as the first and second terms, obtained for n = 8 and n = 9, the Poulet numbers 1105 and 1387;

: 2465 = 5*17*29 is the starting term, obtained for n = 2, in the sequence of Poulet numbers (2*n + 1)*(8*n + 1)*(14*n + 1) which has the following terms: 176149 (...) obtained for n = 9 (...);

: 2701 = 37*73 is the second term, obtained for n = 18, in the sequence of Poulet numbers (2*n + 1)*(4*n + 1) which has as the first term, obtained for n = 8, the Poulet number 561;

: 2821 = 7*13*31 is the starting term, obtained for n = 3, in the sequence of Poulet numbers (2*n + 1)*(4*n + 1)*(10*n + 1) which has the following terms: 63973, 285541, 488881 (...) obtained for n = 9, 15, 18 (...);

: 4033 = 37*109 is the fifth term, obtained for n = 18, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1) which has as the previous terms, obtained for n = 5, 7, 14, 15 (...) the Poulet numbers 341, 645, 2465, 2821 (...);

: 4369 = 17*257 is the starting term, obtained for n = 8, in the sequence of Poulet numbers (2*n + 1)*(32*n + 1);

: 4681 = 31*151 is the third term, obtained for n = 15, in the sequence of Poulet numbers (2*n + 1)*(10*n + 1) which has as the first and second terms, obtained for n = 5 and n = 9, the Poulet numbers 561 and 1729;

: 5461 = 43*127 is a term, obtained for n = 21, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1);

: 7957 = 73*109 is a term, obtained for n = 36, in the sequence of Poulet numbers (2*n + 1)*(3*n + 1);

: 8321 = 53*157 is a term, obtained for n = 26, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1);

: 8911 = 7*19*67 is the first term, obtained for n = 3, in the sequence of Poulet numbers (2*n + 1)*(6*n + 1)*(22*n + 1) which has the following terms: 63973 (...) obtained for n = 6 (...).