Modern Physics Letters A
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GALACTIC ENTROPY IN EXTENDED KALUZA-KLEIN COSMOLOGY

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We use a Kaluza-Klein model with variable cosmological and gravitational terms to discuss the nature of galactic entropy function. For this purpose, we assume a universe filled with dark fluid and consider five-dimensional field equations using the Gamma law equation. We mainly discuss the validity of the first and generalized second laws of galactic thermodynamics for viable Kaluza-Klein models.

Keywords: Kaluza-Klein theory; thermodynamics; entropy; dark energy.

PACS Nos.: 04.50.+h; 04.70.Dy; 65.40.Gr; 95.35.+d

1. Introduction

It is commonly believed that our Universe is homogeneous, flat and isotropic over large scale and the dark contents put our Universe into the accelerated expansion phase [1–5]. Planck-2013 observational evidences [1] indicated that, in our universe, the matter is dominated by 68.3 percent dark energy, 26.8 percent dark matter and 4.9 percent other cosmic ordinary matters.

Many cosmologists believe that extra dimensions are very convenient to investigate the dark part of universe. The Kaluza-Klein theory is one of the most interesting multi-dimensional theories. Kaluza [6] and Klein [7] defined an extra dimension in Einstein's theory of general relativity to unify the electromagnetism and gravity and into one theory. There are many suggestions to be a candidate for the explanation of galactic dark energy effect, but the dark nature is still completely unknown [8]. Tachyon [9], K-essence [10], quintom [11], phantom [12], quintessence [13], intergalactic gas models [14,15], modified gravity [16,17] and reconstruction in modified gravity [18] are possible candidates for the dark universe. The simplest candidate to identify the galactic dark effect is the cosmological constant, as a matter of fact it is the earliest theoretical instrument, but it gives problems such as cosmic-coincidence puzzle and fine-tuning [19]. Currently, the cosmological parameter is equal to $10^{-55}cm^{-2}$, but in particle physics the value is 10^{120} times grater than this factor [20, 21]. We know this difference as the fine-tuning problem [21]. On the other hand, the cosmic-coincidence puzzle raises due to the comparison of the galactic dark matter and dark energy [20]. Li et al. [22], including a survey of some theoretical models, gave a very useful introduction about the dark universe. In the present paper, we take the five-dimensional (5D) Kaluza-Klein model in which dark matter interacts with dark energy and investigate the nature of universal entropy function.

The scheme of present work is the following. In the second section, we assume the 5D Friedmann-Robertson-Walker spacetime (FRW) as the representation of the Universe and introduce the selected galactic dark effect scenario. Also, we formulate the corresponding extended Friedmann and continuity equations. In the third section of the paper, we analyze of the validity of the first and generalized second laws of galactic thermodynamics and examine these laws for viable models. In the fourth section, we give final discussions.

2. Dark effect scenario

We focus on the extended 5D Kaluza-Klein cosmology given as follows. Here, we consider the following Kaluza-Klein space-time model [23]

$$ds^2 = dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1-kr^2)d\psi^2], \tag{1}$$

where $a(t)$ denotes the scale factor while k shows the curvature parameter (the corresponding values of curvature parameter 0, -1 and $+1$ describes the flat, closed and open universes, respectively). We consider the Universe to be filled with dark energy and dark matter. Additionally, energy-momentum tensor of a dark fluid in five dimensions is described by

$$T_{\mu\nu} = (\rho + P)U_\mu U_\nu - g_{\mu\nu}P, \qquad (\mu, \nu = 0, 1, 2, 3, 5), \tag{2}$$

where P, ρ and U_μ shows the pressure, the energy density and 5-velocity vector, respectively. Besides, note that $\rho = \rho_m + \rho_e$ and $P = P_m + P_e$ where the subscripts m and e indicates dark matter and dark energy, respectively. Here, we also have $U_\mu U^\mu = 1$.

The Einstein's field equations with time varying cosmological and gravitational parameters are defined as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(t)T_{\mu\nu} + \Lambda(t)g_{\mu\nu}, \tag{3}$$

where $R_{\mu\nu}$, $g_{\mu\nu}$, R, $G(t)$ and $\Lambda(t)$ are the Ricci tensor, the metric tensor, the curvature scalar, the time-varying gravitational term and the time-varying cosmological term, respectively. Making use of equations (2) and (3), we get

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(t)\left[(\rho + P)U_\mu U_\nu - g_{\mu\nu}P\right] + \Lambda(t)g_{\mu\nu}. \tag{4}$$

This field equation with the space-time model (1) gives the following independent equations

$$H^2 + \frac{k}{a^2} = \frac{4\pi G(t)}{3}\rho + \frac{\Lambda(t)}{6}, \tag{5}$$

$$2H^2 + \dot{H} + \frac{k}{a^2} = -\frac{8\pi G(t)}{3}P + \frac{\Lambda(t)}{3}, \tag{6}$$

where $H = \frac{\dot{a}}{a}$ defines the well-known Hubble parameter. Furthermore, the continuity equation yields [24]

$$\dot{\rho} + 4H(\rho + P) = -\rho\frac{\dot{G}}{G} - \frac{1}{8\pi}\frac{\dot{\Lambda}}{G}. \tag{7}$$

Here, we see that the varying nature of scalars G and Λ causes nonconservative case and the equivalence principle gives

$$\dot{G} = -\frac{\dot{\Lambda}}{8\pi\rho}, \tag{8}$$

thence, the conservation law holds and we have

$$\dot{\rho} + 4H(\rho + P) = 0. \tag{9}$$

Using the Gamma law equation

$$P = (\gamma - 1)\rho, \tag{10}$$

we can rewrite the continuity relation in another nice form

$$\dot{\rho} + 4H\gamma\rho = 0, \tag{11}$$

where γ defines the state parameter. The value of Gamma-law-equation parameter γ describes three different phases of the dark energy such as vacuum ($\gamma = 0$), phantom ($\gamma < 0$) and quintessence ($\gamma > 0$).

In this dark energy scenario, making use of the following fractional densities:

$$\Omega_m = \frac{4\pi G\rho_m}{3H^2}, \qquad \Omega_e = \frac{4\pi G\rho_e}{3H^2}, \qquad \Omega_\Lambda = \frac{\Lambda}{6H^2}, \qquad \Omega_k = -\frac{k}{H^2a^2}, \tag{12}$$

one can rewrite the Friedmann equation (5) in a simple form

$$\sum_{i=1}^{4}\Omega_i = 1, \tag{13}$$

where $i = e, m, \Lambda, k$. Since, we assume mixture of the galactic dark energy and dark matter, the conservation equation should be decomposed into

$$\dot{\rho_m} + 4H\gamma_m\rho_m = \Sigma, \tag{14}$$

$$\dot{\rho_e} + 4H\gamma_e\rho_e = -\Sigma, \tag{15}$$

where Σ is the energy exchange term and describes the interaction between the dark energy and the dark matter. To be in general, we assume the following form of exchange term [25]:

$$\Sigma = 3\xi H(\rho_e - \rho_m). \tag{16}$$

The model is the best suitable one with the current observations. It is obvious that signs of ξ which is a coupling parameter for the interaction indicate the directions of energy transitions. The positive value of ξ corresponds to an energy flow from the dark energy sector into the dark matter sector, and negativity stands for the reverse process. The case $\xi = 0$ describes the non-interacting model, and in some conditions ξ is chosen [26] in the range $[0, 1]$. Cosmic Microwave Background observational data indicate that the coupling parameter is a small positive constant of the order of the unity ($\xi^2 < 0.025$) [27, 28] and the negative coupling parameter cases are avoided due to the violation of laws of gravitational thermodynamics. Recently, Abell Cluster A586 observations have given some evidences showing a transition from the dark energy sector into the dark matter one [29,30]. This mysterious nature may appear effectively as a self-conserved dark energy (with a non-trivial state equation imitating phantom or quintessence) as in the ΛXCDM scenario [31–34]. Nevertheless, the significance of above interaction has not been clearly explained [11], but it is consistent with the gravitational thermodynamics using the arguments [25] both for the early as well as the late time [35].

One can also develop the equivalent effective uncoupled model

$$\dot{\rho_m} + 4H\gamma_m^{eff}\rho_m = 0, \tag{17}$$

$$\dot{\rho_e} + 4H\gamma_e^{eff}\rho_e = 0, \tag{18}$$

where the effective state parameters are defined as

$$\gamma_m^{eff} = \gamma_m - \frac{\Sigma}{4H\rho_m}, \qquad \gamma_e^{eff} = \gamma_e + \frac{\Sigma}{4H\rho_e}. \tag{19}$$

3. Nature of the entropy

Here, we explore the validity of gravitational thermodynamics laws on the dynamical apparent horizon to investigate the nature of galactic entropy function.

Now, we start with the Gibb's law [36]

$$T_h dS_I = PdV + dE_I. \tag{20}$$

Thence, one can write

$$dS_I = \frac{PdV + dE_I}{T_h}, \tag{21}$$

where P, S_I, E_I and T_h denote the pressure, internal entropy, internal energy and temperature of the system, respectively. The derivation with respect to the cosmic time yields

$$\dot{S}_I = \frac{P\dot{V} + \dot{E}_I}{T_h}, \tag{22}$$

and we can write

$$\dot{S}_m = \frac{p_m\dot{V} + \dot{E}_m}{T}, \qquad \dot{S}_e = \frac{p_e\dot{V} + \dot{E}_e}{T}. \tag{23}$$

The system is assumed to be in equilibrium which implies that all components of the system have the same temperature [37]. The internal energy in the system is defined as

$$E_I = \rho V. \tag{24}$$

The extra-dimensional volume written in the above relation is given as

$$V = \frac{1}{2}\pi^2 r_h^4, \tag{25}$$

where

$$r_h = \left[H^2 + \frac{k}{a^2}\right]^{-\frac{1}{2}} \tag{26}$$

is the dynamical apparent horizon in the FRW universe [38, 39].

3.1. *The first law of thermodynamics*

The first law of gravitational thermodynamics on the dynamical apparent horizon is

$$-dE_I = T_h dS_h, \tag{27}$$

It is known that all the fluids in the Universe acquire the same temperature after establishing of the equilibrium [40], otherwise the energy transfer may deform this geometry [31, 36]. The temperature of dynamical apparent horizon T_h is related to its radius r_h [31, 41, 42], i.e. $T_h = (2\pi r_h)^{-1}$. The measure of energy crossing on the horizon is given by [43]

$$-dE_I = 2\pi^2 r_h^4 H T_{\mu\nu} U^\mu U^\nu dt = 2\pi^2 r_h^4 H(\rho + P)dt. \tag{28}$$

Using Friedmann equations (5) and (6), we find

$$\rho + P = \frac{3}{8\pi G(t)}\left[\frac{k}{a^2} - \dot{H}\right]. \tag{29}$$

Inserting above result into equation (28) yields

$$-dE_I = \frac{3\pi H}{4G(t)} \left(H^2 + \frac{k}{a^2} \right)^{-2} \left[\frac{k}{a^2} - \dot{H} \right] dt. \tag{30}$$

In black hole thermodynamics, it is known that the temperature is proportional to the surface gravity and the entropy depends on the area of the horizon [44, 45]. Bardeen et al. [46] showed that, for a black hole, the entropy, temperature and mass satisfy the first law of thermodynamics. This nature motivates physicists to find a connection between the black hole thermodynamics and general relativistic field equations. In 1973, Bekenstein focused on the aforementioned problem and obtained a relation between the black hole thermodynamics and event horizon [37]. Later, in 2006, Wang et al. [47] obtained very interesting conclusions for the first and generalized second laws of universal thermodynamics and found that the Universe should be non-static and the usual relations thermodynamical quantities on the event horizon may be more complicated than in the static universe.

The entropy of dynamical apparent horizon is [48] $S_h = \frac{A_3}{4G}$. After considering the definition of A_3 (the area of 4-sphere), the entropy takes the following form

$$S_h = \frac{\pi^2 r_h^3}{2G(t)}. \tag{31}$$

Hence, using equations (26) and (31), we obtain

$$T_h dS_h = \frac{3\pi H}{4G} \left(H^2 + \frac{k}{a^2} \right)^{-2} \left[\frac{k}{a^2} - \dot{H} \right] dt - \frac{\pi \dot{G}}{4G^2} \left(H^2 + \frac{k}{a^2} \right)^{-1} dt. \tag{32}$$

Equations (30) and (32) lead to following result

$$T_h dS_h + T_h d\breve{S}_{G\Lambda} = -dE_I, \tag{33}$$

where

$$T_h d\breve{S}_{G\Lambda} = \frac{\pi \dot{G}}{4G^2} \left(H^2 + \frac{k}{a^2} \right)^{-1} dt. \tag{34}$$

We get an extra term having time dependence in the left-hand-side of relation (33), so that this extra term cannot be vanish during the certain stage of spacetime evolution. One may conclude that $-dE_I \neq T_h dS_h$. Thus, the first law of gravitational thermodynamics does not hold in the selected Kaluza-Klein model. All of the matter fields feel the same Hawking temperature and see the same horizon, but some gravitational degrees of freedom may feel a different spacetime metric, Hawking temperature and horizon. Recently, Dubovski and Sibiryakov [49] have studied the spontaneous breaking of Lorentz invariance on thermodynamics of a black hole, and found a similar conclusion. The additional entropy term, i.e. $T_h d\breve{S}_{G\Lambda}$, may be considered as produced entropy term which is developed due to the non-equilibrium framework in the extended Kaluza-Klein cosmology.

Furthermore, if we assume that $G = constant$, the additional entropy term will be diminished and the first law of gravitational thermodynamics holds for all time.

The results obtained in this limiting case are exactly the same as those obtained in Ref. [50] given with eqns. (34) and (35). Additionally, the selected model and our results can be reduced to those ones that obtained in Ref. [20] for a flat Kaluza-Klein universe. In literature, Sharif and Saleem focused on the first law of galactic thermodynamics in the flat Kaluza-Klein geometry [20], and they get $-dE_I = \frac{1}{\pi}T_h dS_h$ which means the first law of thermodynamics is not valid in the Kaluza-Klein universe with a flat FRW model. But, this conclusion is not right. Taking $\Lambda = 0$ and $G = 1$ in equations (30) and (32) gives the model chosen in Ref. [20] and it is found that the first law is recovered exactly, i.e. $-dE_I = T_h dS_h$.

3.2. *The generalized second law of thermodynamics*

To investigate the generalized second law, we consider the derivative of total entropy. Using the Gibb's law, we write

$$T_h dS_m = P_m dV + dE_m, \tag{35}$$

$$T_h dS_e = P_e dV + dE_e. \tag{36}$$

Using energy relations $E_m = \rho_m V$ and $E_e = \rho_e V$ we get

$$T_h dS_m = (\rho_m + P_m)(\dot{V} - 4HV), \tag{37}$$

$$T_h dS_e = (\rho_e + P_e)(\dot{V} - 4HV). \tag{38}$$

Adding of these results gives us

$$T_h d(S_m + S_e) = (\rho + P)(\dot{V} - 4HV), \tag{39}$$

where $\rho + P = \rho_m + P_m + \rho_e + P_e$. Inserting the definition of volume given in equation (25) and the relation (29) into this equation yields

$$T_h d(S_m + S_e) = \frac{3\pi r_h^3}{4G}(\dot{r}_h - Hr_h)\left[\frac{k}{a^2} - \dot{H}\right] dt. \tag{40}$$

For the horizon entropy, we found the following auxiliary relations

$$\dot{r}_h = Hr_h^3\left[\frac{k}{a^2} - \dot{H}\right], \tag{41}$$

$$\dot{r}_h - Hr_h = -Hr_h^3(\dot{H} + H^2). \tag{42}$$

Thence, using these auxiliary relations, it is found that

$$T_h d(S_m + S_e) = \frac{3\pi H r_h^6}{4G}(\dot{H} + H^2)\left[\dot{H} - \frac{k}{a^2}\right] dt. \tag{43}$$

On the other hand, for the horizon entropy, it can be written by using equation (32) that

$$T_h dS_h = \frac{3\pi H r_h^4}{4G}\left[\frac{k}{a^2} - \dot{H}\right] dt - \frac{\pi \dot{G} r_h^2}{4G^2} dt. \tag{44}$$

Thus, it is calculated that

$$T_h(\dot{S}_m + \dot{S}_e + \dot{S}_h) = \frac{3\pi \dot{r}_h r_h^3}{4G}\left[\frac{k}{a^2} - \dot{H}\right] dt - \frac{\pi \dot{G} r_h^2}{4G^2} dt. \tag{45}$$

Note that assuming $\Lambda = constant$ and $G = constant$ reduces this result into the one obtained in Ref. [50] while taking $\Lambda = 0$ and $G = 1$ transforms our model to the one considered in Ref. [20] and gives the corrected forms of the results obtained in that study. According to the second law of universal thermodynamics, the entropy function is always increasing. In other words, the derivative of any entropy function of thermodynamical system can never be decreased, i.e. $\dot{S} \geq 0$. Equation (45) shows that the validity of the second law depends on the selected model. For instance, assuming $\Lambda = 0$ and $G = constant$ (Einstein's theory of general relativity without cosmological constant in 5D) gives

$$T_h \dot{S}_t = \frac{3\pi H r_h^6}{4G}\left(\frac{k}{a^2} - \dot{H}\right)^2 \geq 0, \tag{46}$$

which indicates that the generalized second law of universal thermodynamics is always satisfied. Furthermore, using the relation (29) equation (46) can be rewritten in another form:

$$T_h \dot{S}_t = \alpha(t) 8G\pi^2 H r_h^5 (\rho + P)^2 \geq 0, \tag{47}$$

where $\alpha(t) = \frac{2\pi r_h}{3}$. In the general relativity, the corresponding relation in 4D was obtained [51, 52] as

$$T_h \dot{S}_t = 8G\pi^2 H r_h^5 (\rho + P)^2 \geq 0, \tag{48}$$

which is similar to our result. Next, $T_h \dot{S}_t$ may tend to infinity in another specific case. This outlandish nature transforms all the energy in usable form into an unusable one, the entropy will act very mysterious and have maximum value. This interesting condition is the heat death scenario of a system which is known also as one of the predicted fates of the Universe. At this level, all of the free energy will be derogated while the motion of life cannot sustain any more. On the other hand, for very high expansion rate, the Hubble parameter may tend to infinity and we may get $r_h \approx \frac{1}{H} \to 0$. Therefore this condition yields $T_h \dot{S}_t = 0$ which describes the reversible adiabatic expansion of our Universe.

3.3. *Entropy for viable models and graphical discussions*

To discuss evolutionary nature of the galactic entropy function we need to know the time evolution of the cosmological parameter Λ, the scale factor a, the gravitational parameter G and the Hubble parameter H. In what follows, we want to examine the validity of the galactic thermodynamics laws for some viable models. On this purpose, we start with two special solutions obtained in literature and investigate the nature of universal entropy in 5D to discuss the corresponding effect of extra dimension. We further consider a spatially flat spacetime, i.e. $k = 0$, which is compatible with the recent cosmological data [53, 54].

- Model 1: Assuming $\Lambda \sim \left(\frac{\dot{a}}{a}\right)^2$ yields

$$\Lambda(t) = \frac{\epsilon}{t^2}, \qquad a(t) = t^{3/\gamma(6-\epsilon)}, \qquad G(t) = \frac{9t^{2\epsilon/(6-\epsilon)}}{8\pi b\gamma^2(6-\epsilon)}, \tag{49}$$

where b is a positive parameter and ϵ is a constant [24]. The corresponding Hubble parameter and its time-derivative are obtained as

$$H = \frac{3}{\gamma(6-\epsilon)t}, \qquad \dot{H} = \frac{-3}{\gamma(6-\epsilon)t^2}. \tag{50}$$

We also calculate the deceleration parameter as

$$q = -1 - \frac{\dot{H}}{H^2} = -1 - \frac{\gamma(6-\epsilon)}{3}. \tag{51}$$

Here, due to the fact that our Universe is in the phase of accelerated expansion, i.e. $q < 0$, from equation (51) we need to have $\gamma(6-\epsilon) > -3$.

The variations of the cosmological parameter, the scale factor, the gravitational parameter and the Hubble parameter versus the cosmic time t are plotted in Figures 1 and 2. Figure 1 shows that the cosmological parameter, the scale factor, the gravitational parameter and the Hubble parameter, respectively, increases, decreases, decreases and increases in the phantom divide. For the quintessence phase, Figure 2 indicates that the cosmological parameter, the scale factor, the gravitational parameter and the Hubble parameter, respectively, increases, increases, decreases and decreases the during history of the Universe.

For the first law of universal thermodynamics the model yields

$$\begin{aligned} T_h\dot{\check{S}}_{G\Lambda} &= \frac{\pi\dot{G}}{4G^2H^2} \\ &= \frac{4b\epsilon\pi^2}{81}\gamma^4(6-\epsilon)^2 t^{1-\frac{2\epsilon}{6-\epsilon}}. \end{aligned} \tag{52}$$

With the help of the numerical results obtained for the cosmological parameter, the scale factor, the gravitational parameter and the Hubble parameter presented in Figures 1 and 2, the variation of the additional entropy term $T_h\dot{\check{S}}_{G\Lambda}$ versus time is plotted in Figure 3. We get the same results both for the phantom and quintessence divides. Figure 3 shows that the additional entropy term does not vanish which indicates the violation of the first law and sustains the conclusion obtained by Dubovski and Sibiryakov [49].

Besides, for the second law, it is found that

$$\begin{aligned} T_h\dot{S}_t &= \frac{\pi}{4GH^2}\left[\frac{3\dot{H}^2}{H^3} - \frac{\dot{G}}{G}\right] \\ &= \frac{2b\pi^2}{81}\gamma^4(6-\epsilon)^2\left[\gamma(6-\epsilon)^2 - 2\epsilon\right] t^{1-\frac{2\epsilon}{6-\epsilon}}. \end{aligned} \tag{53}$$

In Figure 4, we plot the evolutionary behavior of the generalized second law both for the phantom and quintessence eras. Figure 4 shows that the second law of galactic thermodynamics is satisfied throughout the history of the Universe.

- Model 2: Considering $\Lambda \sim \frac{\ddot{a}}{a}$ we have [24]

$$\Lambda(t) = \frac{2\beta}{t^2}, \qquad a(t) = \frac{3(1-2\gamma)}{3-\beta\gamma}t^2, \qquad G(t) = \frac{\beta}{16\pi b\gamma}\left[\frac{3(1-2\gamma)}{3-\beta\gamma}\right]^{4\gamma} t^{8\gamma}, \tag{54}$$

where β is a free parameter. Hence, we obtain the following values for the Hubble parameter and its time-derivative:

$$H = \frac{2}{t}, \qquad \dot{H} = -\frac{2}{t^2}, \tag{55}$$

and the deceleration parameter is $q = -\frac{1}{2}$. The time evolution of the above parameters can be obtained by numerical analyzing. The numerical results found for Λ, a, G and H are plotted in Figure 5 (the phantom phase) and Figure 6 (the quintessence phase). The Figure 5 shows that Λ, a, G and H, respectively, increases, increases, decreases and decreases during the history of the Universe. On the other hand, Figure 6 indicates that the evolutionary behaviors of above parameters depend on the choice of auxiliary parameters in the quintessence era.

The results of Λ, a, G and H illustrated in Figures 5 and 6 help us to find the variation of the entropy function for the selected model. So, we get the following relation for the first law of galactic thermodynamics:

$$T_h \dot{\check{S}}_{G\Lambda} = \frac{8b\pi^2}{\beta}\gamma^2 \left[\frac{3-\beta\gamma}{3(1-2\gamma)}\right]^{4\gamma} t^{1-8\gamma}. \tag{56}$$

Here the validity of the first law depends on the condition $\beta\gamma = 3$. The cases $\beta < 0$ and $\beta > 0$ correspond the phantom ($\gamma < 0$) and quintessence ($\gamma > 0$) phases of our Universe, respectively. Hence, the first law is satisfied in both phases for the selected cosmological model under this condition. Additionally, in Figure 7, we plot the time-evolution of $T_h \dot{\check{S}}_{G\Lambda}$. The Figure 7 shows that this term decreases for the phantom divide while increases for the quintessence phase during the history of the Universe. Both of these indications mean the first law is violated and agree with the Dubovski and Sibiryakov results [49].

Furthermore, for the generalized second law, we have

$$T_h \dot{S}_t = \frac{b\pi^2\gamma}{\beta}\left(\frac{3}{2} - 8\gamma\right)\left[\frac{3-\beta\gamma}{3(1-2\gamma)}\right]^{4\gamma} t^{1-8\gamma}. \tag{57}$$

This relation does not give us any idea whether it increases or decreases. But, assuming $\beta\gamma = 3$ provides $T_h \dot{S}_t = 0$ which shows that the generalized second law holds. Using the numerical results illustrated in Figures 5 and 6, we plot $T_h \dot{S}_t$ versus the cosmic time in Figure 8. This figure indicates that the generalized second law is satisfied for this model throughout the history of the Universe.

3.4. *Conclusions*

In the present study, the type of universe is assumed as the five dimensional Kaluza-Klein model and it has been considered that the Universe is in the thermal equilibrium state and filled with dark energy and dark matter. The validity of thermodynamics laws on the dynamical apparent horizon with the Hawking temperature have been investigated to discuss the five dimensional nature of galactic entropy function. On this purpose, the variation of entropy function for each dark fluid contents and for the dynamical apparent horizon itself have been evaluated separately. The first and generalized second laws have been turned out to be independent of the selected dark energy model fifth dimension. According to these investigations, we have discussed some special conditions: (i) the effect of spontaneous breaking of Lorentz invariance and the violence of the first law, (ii) the reversible adiabatic expansion of our Universe, (iii) the heat death scenario for the Universe. We have also discussed our results graphically. The results have been showed that the first laws of thermodynamics transforms another form, i.e. $T_h dS_h + T_h d\breve{S}_{G\Lambda} = -dE_I$, which includes an additional entropy term $T_h d\breve{S}_{G\Lambda}$. This result sustains Dubovski and Sibiryakov's conclusions and agrees with their investigations [49]. In addition to this, we have seen that the second law of galactic thermodynamics is satisfied for the extended Kaluza-Klein gravity model. We also want to mention here that our results are consistent with the general relativity [51, 52] and previous studies published in literature [20, 50].

Acknowledgement

We would like to thank the referee for giving such constructive comments and important suggestions which substantially helped improving the quality of the paper. This study was supported by the Research Fund of Mersin University in Turkey with project number: 2015-AP4-1231.

References

1. Planck Collaboration (P.A.R. Ade et al.), arXiv e-Print: 1303.5072 (2013).
2. Supernovae Search Team Collaboration (A. G. Riess et al.), Astron. J. **116** (1998) 1009.
3. Boomerang Collaboration (P. de Bernardis et al.), Nature **404** (2000) 955.
4. Supernovae Cosmology Project Collaboration (S. Perlmutter et al.), Astrophys. J. **517** (1999) 565.
5. Supernovae Cosmology Project Collaboration (R. A. Knop et al.), Astrophys. J. **598** (2003) 102.
6. T. Kaluza, Sits. Press. Akad. Wiss. Math. Phys. K **1** (1921) 895.
7. O. Klein, Zeits. Phys. **37** (1926) 895.
8. S. Chakraborty and A. Biswas, Astrophys. Space Sci. **343** (2013) 791.
9. A. Sen, Lect. Notes Phys **653** (2004) 141.
10. T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D **62** (2000) 023511.
11. B. Feng, X.L. Wang and X.M. Zhang, Phys. Lett. B **607** (2005) 35.
12. R.R. Caldwell, Phys. Lett. B **545** (2002) 204.

13. E.J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15** (2006) 1753.
14. A. Yu. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B **511** (2001) 265.
15. J. Christensen-Dalsgard, "Lecture Notes on Stellar Structure and Evolution" (Aarhus Univ. Press, Aarhus, 2004) p.13.
16. B. Boisseau, G. Esposito-Farese, D. Polarski and A. A. Starobinsky, Phys. Rev. Lett. **85** (2000) 2236.
17. S. Capozziello, Int. J. Mod. Phys. D **11** (2002) 483.
18. V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **15** (2006) 2105.
19. V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9** (2000) 373.
20. M. Sharif and R. Saleem, Mod. Phys. Lett. A **28** (2013) 1350072.
21. F. Darabi, arXiv: hep-th/1107.3307
22. M. Li, X.D. Li, S. Wang and X. Zhang, JCAP **06** (2009) 036.
23. C. Ozel, H. Kayhan and G.S. Khadekar, Adv. Stud. Theor. Phys. **4** (2010) 117.
24. R.K. Dubey, A. Mitra and B.K. Singh, Int. J. Contemp. Math. Sci. **7** (2012) 2041.
25. C.-Y. Sun and R.-H. Yue, Phys. Rev. D **85** (2012), 043010
26. H. Zhang and Z.H. Zhu, Phys. Rev. D **73** (2006) 043518.
27. K. Ichiki and Y.Y. Keum, J. Cosmol. Astropart. Phys. **6** (2008) 5.
28. L. Amendola, G.C. Campos and R. Rosenfeld, Phys. Rev. D **75** (2007) 083506.
29. O. Bertolami, F. Gil Pedro and M. Le Delliou, Phys. Lett. B **654** (2007) 165.
30. M. Jamil and M.A. Rashid, Eur. Phys. J. C **58** (2008) 111.
31. M. Jamil, E.N. Saridakis, M.R. Setare, Phys. Rev. D **81** (2010) 023007.
32. J. Sola and H. Stefancic, Phys. Lett. B **624** (2005) 147.
33. I.L. Shapiro and J. Sola, Phys. Lett. B **682** (2009) 105.
34. J. Grande, J. Sola and H. Stefancic, JCAP **011** (2006) 0608.
35. M. Sharif and F. Khanum, Gen. Rel. Gravit. **43** (2011) 2885.
36. G. Izquierdo and D. Pavon, Phys. Lett. B **633** (2006) 420.
37. J. D. Bekenstein, Phys. Rev. D **7** (1973) 2333.
38. E. Poisson and W. Israel, Phys. Rev. D **41** (1990) 1796.
39. Y.G. Gong and A. Wang, Phys. Rev. Lett. **99** (2007) 211301.
40. M. R. Setare, JCAP **0701** (2007) 023.
41. T. Jacobson, Phys. Rev. Lett. **75** (1995) 1260.
42. A. Paranjape, S. Sarkar and T. Padmanabhan, Phys. Rev. D **74** (2006) 104015.
43. R. Bousso, Phys. Rev. D **71** (2005) 064024.
44. N.A. Bahcall, J.P. Ostriker, S. Perlmutter and P.J. Steinhardt, Science **284** (1999) 1481.
45. A.G. Riess et al., Astron.J. **116** (1998) 1009.
46. J.M. Bardeen, B. Carter and S.W. Hawking, Commun. Math. Phys. **31** (1973) 161.
47. B. Wang, Y. Gong and E. Abdalla, Phys. Lett. B **624** (2005) 141.
48. R.G. Cai and S.P. Kim, JHEP **0502** (2005) 050.
49. S. Dubovsky and S. Sibiryakov, Phys. Lett. B **638** (2006) 509.
50. M. Salti, O. Aydogdu and H. Yanar, Mod. Phys. Lett. A **30** (2015) 1550109.
51. K. Karami, JCAP **01** (2010) 015.
52. K. Karami, M.S. Khaledian and N. Abdollahi, EPL **98** (2012) 30010.
53. WMAP Collaboration (E. Komatsu et al.), Astrophys. J. Suppl. **192** (2011) 18.
54. WMAP Collaboration (G. Hinshaw et al.), Astrophys. J. Suppl. **208** (2013) 19.

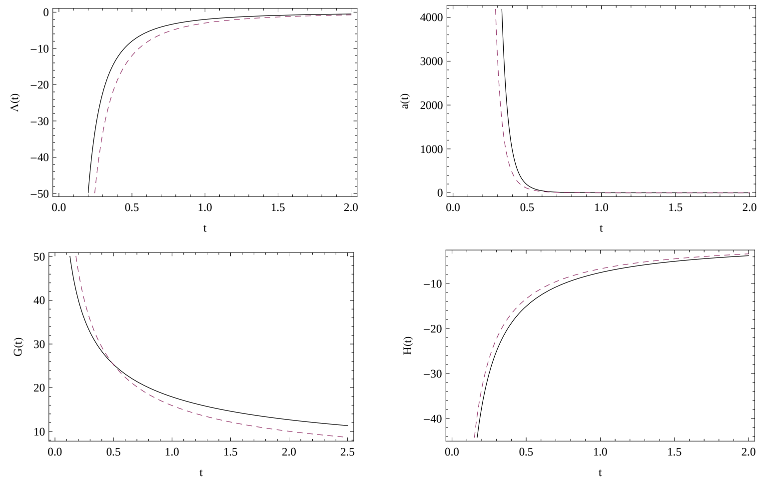

Fig. 1. The variation of the cosmological parameter Λ, the scale factor a, the gravitational parameter G and the Hubble parameter H versus the cosmic time t in the phantom sector of the Universe for Model 1. Auxiliary parameters are $b = 1$, $\gamma = -0.05$, $\epsilon = -2$ (solid line) and $\epsilon = -3$ (dashed line).

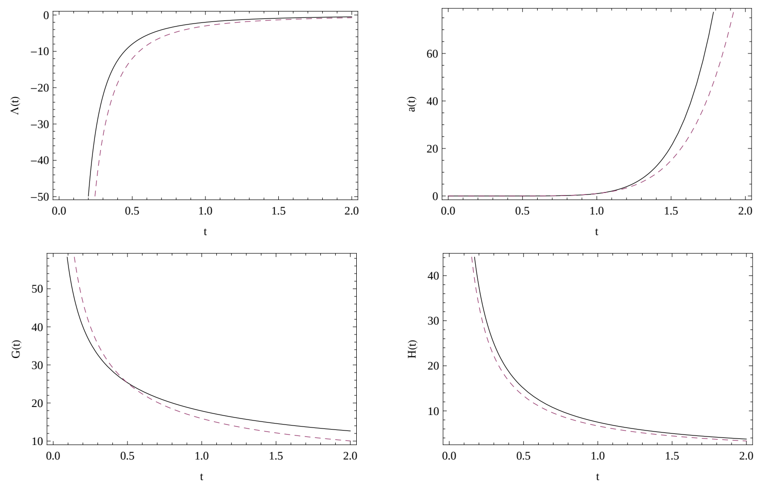

Fig. 2. The variation of the cosmological parameter Λ, the scale factor a, the gravitational parameter G and the Hubble parameter H versus the cosmic time t in the quintessence sector of the Universe for Model 1. Auxiliary parameters are $b = 1$, $\gamma = 0.05$, $\epsilon = -2$ (solid line) and $\epsilon = -3$ (dashed line).

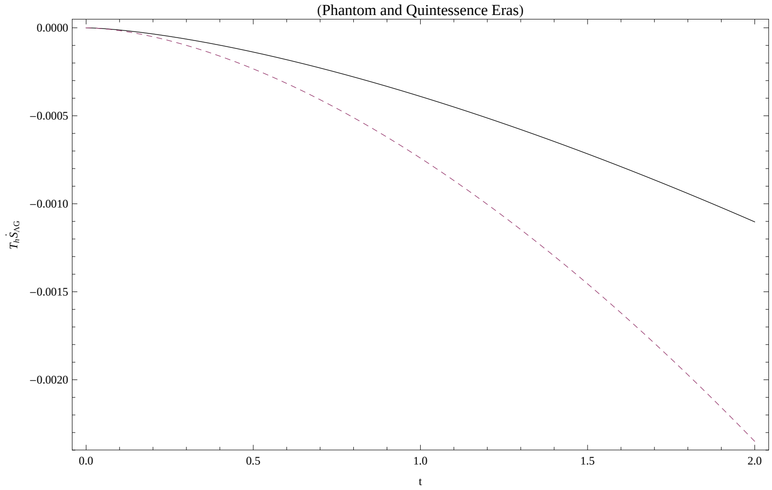

Fig. 3. The variation of $T_A\dot{S}_{\Lambda G}$ versus the cosmic time t for Model 1. Auxiliary parameters are $b = 1$, $\gamma = -0.05$ (taking $\gamma = 0.05$ does not change the results), $\epsilon = -2$ (solid line) and $\epsilon = -3$ (dashed line).

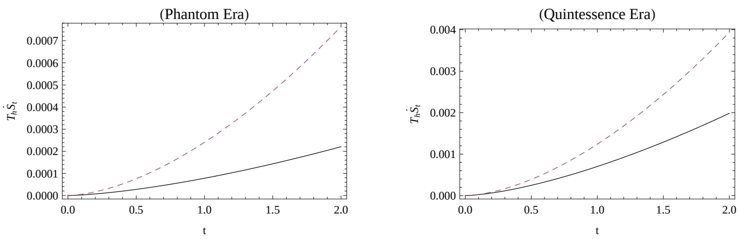

Fig. 4. The variation of $T_A\dot{S}_t$ versus the cosmic time t for Model 1. Auxiliary parameters are $b = 1$, $\gamma = -0.05$, $\epsilon = -2$ (solid line) and $\epsilon = -3$ (dashed line) for the phantom era and $b = 1$, $\gamma = 0.05$, $\epsilon = -2$ (solid line) and $\epsilon = -3$ (dashed line) for the quintessence era.

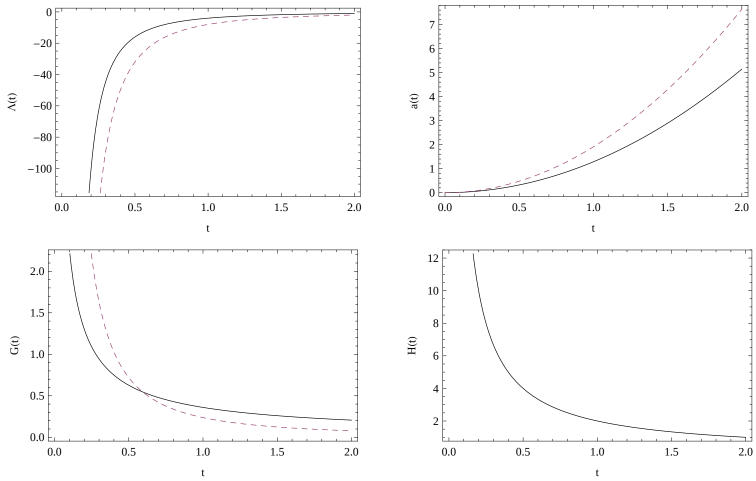

Fig. 5. The variation of the cosmological parameter Λ, the scale factor a, the gravitational parameter G and the Hubble parameter H versus the cosmic time t in the phantom sector of the Universe for Model 2. Auxiliary parameters are $b = 1$, $\gamma = -0.1$ and $\beta = 2$ for the solid line and $b = 1$, $\gamma = -0.2$ and $\beta = -4$ for the dashed line.

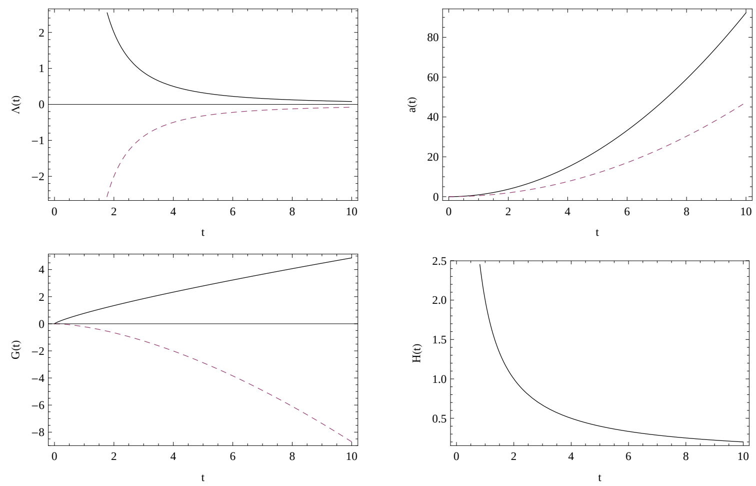

Fig. 6. The variation of the cosmological parameter Λ, the scale factor a, the gravitational parameter G and the Hubble parameter H versus the cosmic time t in the quintessence sector of the Universe for Model 2. Auxiliary parameters are $b = 1$, $\gamma = 0.1$ and $\beta = 4$ for the solid line and $b = 1$, $\gamma = 0.2$ and $\beta = -4$ for the dashed line.

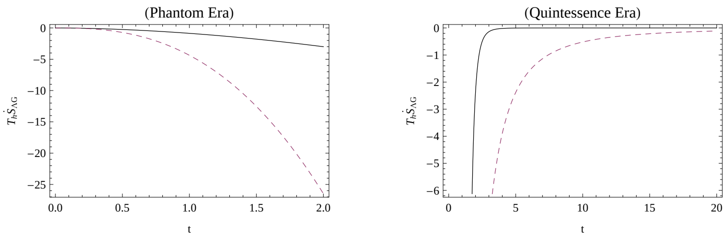

Fig. 7. The variation of $T_A\dot{S}_{\Lambda G}$ versus the cosmic time t for Model 2. Auxiliary parameters are $b = 1$, $\beta = -1$, $\gamma = -0.1$ (solid line), $\gamma = -0.2$ (dashed line) for the phantom era and $b = 1$, $\beta = -2, \gamma = 1$ (solid line) and $b = 1$, $\beta = -4, \gamma = 0.4$ (dashed line) for the quintessence era.

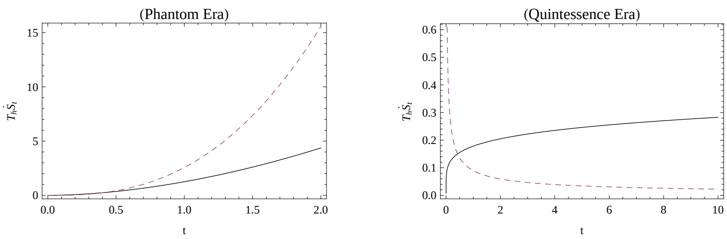

Fig. 8. The variation of $T_A\dot{S}_t$ versus the cosmic time t for Model 2. Auxiliary parameters are $b = 1$, $\beta = -2, \gamma = -0.1$ (solid line) and $b = 1$, $\beta = -4, \gamma = -0.2$ (dashed line) for the phantom era and $b = 1$, $\beta = 4, \gamma = 0.1$ (solid line) and $b = 1$, $\beta = -4, \gamma = 0.2$ (dashed line) for the quintessence era.