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Semi-classical exchange of unphysical photons between two widely separated quantum black holes

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We consider the possibility that electromagnetism is caused by the exchange of unphysical L=0 photons between fundamental particles with properties that resemble quantum black holes. This force is infinite if the known far-field cross section is assumed. However, the divergence is generated at low energy where the photon wavelengths are large and the far-field limit fails. An estimate of the near-field correction removes the infinity and leads to an estimate of the inverse fine structure constant of α^{-1}~139. A speculative scaling of a term controlling the near-field correction by $1/(1+\alpha)$ gives the result α^{-1} =137.038. A suggested speculative link to the higher-order QED corrections to the anomalous magnetic moment of the electron gives α^{-1}=137.036 and a calculated elementary charge $q=1.602177\times10^{-19}$ C. These calculations suggest elementary particles are quantum black holes.

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One of the greatest mysteries of physics is the value of the fine structure constant [1]. Despite many attempts, there is still no accepted theory to explain its value. In the present letter we consider the possibility that electromagnetism and its strength are generated by the semi-classical exchange of unphysical photons between quantum black holes. It is known that L=0 photons cannot be emitted to infinity by a single Schwarzschild black hole. However, we consider the possibility that low energy L=0 photons can be exchanged between two quantum black holes via quantum means. If the far-field L=0 photon black-hole interaction cross section is assumed, then their exchange leads to an infinite radiation pressure. This unphysical result is generated in the limit as the photon energy, ε, goes to zero where the wavelength, λ, becomes larger than the distance between the two quantum black holes, d, where the far-field limit is not valid. A simple estimate of the near-field correction removes the divergence, and causes the exchange to be dominated by photons with $\lambda\sim2\pi d$. This force is independent of the masses of both black holes and is near that between two charged leptons.

One of the simplest methods for estimating the emission rate of massless particles from an object at temperature T is to use transition-state theory [2] and we write the decay width for emission from a spherical object as

$$\Gamma=\frac{1}{2\pi}\int_0^\infty \sum_{\text{helicity}}\sum_{L=0}^{\infty}(2L+1)T_L(\varepsilon)\exp(-\varepsilon/T)d\varepsilon \ , \qquad (1)$$

where the $T_L(\varepsilon)$ are the angular momentum and energy dependent transmission coefficients. In the limit of a large classical object, the maximum orbital angular momentum that can be taken away by a massless particle is $L_{\max}\hbar=r\varepsilon/c$, where r is the radius. Substituting in $T_L(\varepsilon)=1$ for $L\le L_{\max}$ in the limit of a large object, assuming two states of helicity (polarization), and making the semi-classical replacement of $\exp(-\varepsilon/T)$ with $1/(\exp(\varepsilon/T)-1)$ for the emission of real photons gives the result

$$\Gamma_\gamma=\frac{4\pi r^2}{4\pi^2\hbar^2c^2}\int_0^\infty\frac{\varepsilon^2}{\exp(\varepsilon/T)-1}d\varepsilon\,. \qquad (2)$$

The corresponding emission power is

$$P=\frac{4\pi r^2}{4\pi^2\hbar^3c^2}\int_0^\infty\frac{\varepsilon^3}{\exp(\varepsilon/T)-1}d\varepsilon=4\sigma_{\text{a}}\sigma_{\text{SB}}T^4=A\sigma_{\text{SB}}T^4\ , \qquad (3)$$

where σ_a and A are the classical absorption cross section and the surface area of the black body, and $\sigma_{\text{SB}}=\pi^2/(60\hbar^3c^2)$ is the Stefan-Boltzmann constant. In the limit of low gravity, Eq. (3) gives the power of Hawking radiation from a large black hole with radius r_S and temperature $T_{\text{bh}}=\hbar c/(4\pi r_s)$ [3-5].

The emission and absorption of photons by a single Schwarzschild black hole are controlled by the energy dependent absorption cross sections [6,7]

$$\sigma_a=\sum_{L=1}^{\infty}\frac{(2L+1)\pi r_S^2}{4(M\omega)^2}T_L(M\omega)=\sum_{L=1}^{\infty}(2L+1)\pi\bar{\lambda}^2T_L(\varepsilon), \qquad (4)$$

where M is the black-hole mass and ω is the angular frequency. The conversion between the $M\omega$ used by Crispino *et al*. and the photon energy is $M\omega=\varepsilon/(8\pi T_{\text{bh}})$. The L=0 photon emission from black holes is, in part, known to be unphysical because its forced inclusion into the absorption cross section would cause the emission power to be infinite. However, for a single quantum black hole with a fixed mass, this unphysical emission would be completely suppressed by energy conservation. We temporarily ignore this important fact and use transition-state theory to write the semi-classical emission of unphysical L=0 photon power emitted to infinity from an infinitely small black hole (A) with infinite temperature as

$$P_{A\to\infty} = \frac{1}{\pi\hbar}\int_0^\infty \varepsilon\, d\varepsilon. \quad (5)$$

The violation of conservation of energy by the emission from a single quantum black hole can be rectified by the absorption of the photon by a neighboring quantum black hole on a time scale less than ~ħ/(2ε) given by the time-energy uncertainty principle. The photon energies involved in the exchange are therefore

$$T_{\rm ex} \sim \frac{\hbar c}{2d}. \quad (6)$$

For widely spaced quantum black holes with $d>>r_S$ of either of the black holes, the constraints on the exchanging photon energies imposed by the time-energy uncertainty principle rule out all but the unphysical L=0 exchanges. Assuming that Eq. (6) defines an effective exchange temperature, we speculate that the photon power being exchanged from quantum black hole A to quantum black hole B can be given as

$$P_{A\to B} = \frac{1}{\pi\hbar}\int_0^\infty \frac{\varepsilon T_{L=0}(\varepsilon)}{(\exp(\varepsilon/T_{\rm ex})-1)}\frac{\sigma_{L=0}(\varepsilon)}{4\pi d^2}d\varepsilon. \quad (7)$$

Despite the fact that for emission to infinity $T_{L=0}(\varepsilon)$=1, we leave it in Eq. (7) because, as discussed later, the transmission coefficients need to be modified for the case of exchange between objects when the λ become comparable to, or larger than d. Given Eq. (7), in the limit of a pair of widely spaced small quantum black holes, the force generated by the two-way exchange of unphysical photons between them is not dependent on the size and/or temperature of either black hole and is given by

$$F = \frac{2}{\pi\hbar c}\int_0^\infty \frac{\varepsilon T_{L=0}(\varepsilon)}{(\exp(\varepsilon/T_{\rm ex})-1)}\frac{\sigma_{L=0}(\varepsilon)}{4\pi d^2}d\varepsilon. \quad (8)$$

Substituting $\sigma_{L=0}(\varepsilon) = \pi(\hbar c/\varepsilon)^2\cdot T_{L=0}(\varepsilon)$ [see Eq. (4)] into Eq. (8) gives

$$F = \frac{\hbar c}{2\pi d^2}\int_0^\infty \frac{(T_{L=0}(\varepsilon))^2}{\varepsilon(\exp(\varepsilon/T_{\rm ex})-1)}d\varepsilon. \quad (9)$$

If this exchange force is assumed to be the origin of electromagnetism, then the fine structure constant can be expressed as

$$\alpha = \frac{1}{2\pi}\int_0^\infty \frac{(T_{L=0}(\varepsilon))^2}{\varepsilon(\exp(\varepsilon/T_{\rm ex})-1)}d\varepsilon. \quad (10)$$

Using the standard value of $T_{L=0}(\varepsilon)$=1 gives an infinite fine structure constant. However, the origin of the divergence is the lowest energy photons where $\lambda>d$ and the transmission coefficients need to be modified to lower values to correct for near-field effects.

The L=0 absorption cross section is $\pi(\hbar c/\varepsilon)^2$ in the far-field limit where the photons approach from infinity as plane waves. This means that, in a semi-classical sense, two black holes exchanging photons with $\lambda<<d$, do so as though they are classical spherical objects with a radius $r=\hbar c/\varepsilon$. This is the expected relevant length scale for near-field effects. This translates to a relevant energy scale of $\varepsilon^*=\hbar c/d$. The two semi-classical interaction spheres will start to overlap as the photon energy drops through $\varepsilon = 2\varepsilon^*$. Therefore, from simple overlap arguments one might expect the near-field corrections to be small but starting to grow rapidly as ε decreases through $2\varepsilon^*$. For $\varepsilon < \varepsilon^*$ both black holes are inside the semi-classical interaction sphere of the other, and near-field corrections should be large with a significant reduction in the effective interaction cross section, relative to the far-field value, by more than a factor of two.

Without detailed theoretical calculations, intuitive guesses of the functional form of the near-field corrections should be viewed with skepticism. Nonetheless, we make an intuitive guess here. We assume that around each quantum black hole there is a distribution of photon interaction sites and that the effective density of these sites can be represented by a function $\psi(r/\lambda)$. This is to ensure that the interaction cross section scales with λ^2. For photon exchanges between a pair of black holes with $d>>\lambda$ there is no overlap of the interaction sites from different black holes, and thus no near-field corrections. However, when the λ are comparable to or larger than d, the interaction sites from the different black holes overlap. We assume this overlap controls the increase of the near-field corrections with decreasing ε as the λ grow larger than d. Guided by the functional form of the harmonic oscillator wave function and an intuitive feeling that the interaction cross section should be flat at the lowest ε (at least before the addition of QED corrections) we assume

$$\psi(r/\lambda) \propto \exp(\frac{-r^2}{2\bar{\lambda}^2}). \quad (11)$$

We further speculate that only emission sites along the line joining the two black holes control the near-field corrections and write the change in the L=0 effective interaction radius as

$$\frac{r_{\rm n}}{r_{\rm f}} = \frac{2}{\bar{\lambda}\sqrt{2\pi}}\int_0^d \exp(\frac{-z^2}{2\bar{\lambda}^2})dz. \quad (12)$$

where the n and f subscripts are for near and far field. The corresponding near-field modified transmission coefficient is

$$T_{L=0}(\varepsilon,d) = \left\{\frac{2\varepsilon}{\sqrt{2\pi}}\int_0^{d/(\hbar c)} \exp(\frac{-z^2\varepsilon^2}{2})dz\right\}^2. \quad (13)$$

The corresponding effective interaction cross sections are displayed in Fig. 1.

Substituting Eq. (13) into Eq. (10), and switching the energy into units of $T_{\rm ex}$ gives

$$\alpha = \frac{1}{2\pi}\int_0^\infty \frac{\mathrm{erf}^4(\varepsilon/(2^{3/2}))}{\varepsilon(\exp(\varepsilon)-1)}d\varepsilon. \quad (14)$$

The corresponding calculated value is α^{-1}=138.9099, with a relative difference from the known value of ~2α. We speculate that QED effects will slightly decrease the near-field corrections from the expectations of simple geometrical considerations. To lowest order, the absorption process can be represented by a Feynman diagram with a single vertex. The next higher order diagrams contain three vertices and add a fuzziness to the absorption location. However, this fuzziness does not affect the far-field cross section of $\pi(\hbar c/\varepsilon)^2$, but perhaps can affect the near-field correction. To apply a simplistic estimate of a QED-based near-field correction to Eq. (14) that will not change the limiting values of $T_{L=0}(\varepsilon\rightarrow 0)=0$ and $T_{L=0}(\varepsilon\rightarrow\infty)=1$, and without changing the length scale embedded in the argument of the erf function, we suggest a change in the power of 4 in Eq. (14) to $4/(1+\alpha)$. This change gives a calculated value of $\alpha^{-1}=137.0378$, which differs by ~1 part in 10^5 from the known value of 137.035999 [8-11]. The elementary charge corresponding to the calculated value of $\alpha^{-1}=137.0378$ is $q=(\alpha\hbar c4\pi\varepsilon_o)^{1/2}=1.60217\times10^{-19}$ C. This good match to experiment is possibly fortuitous, but demonstrates that corrections to Eq. (14) of the order of α can lead to apparent excellent matches to experimental data. The photon exchange power spectrum for the $\alpha^{-1}=137.0378$ calculation discussed above is displayed in Fig. 2. The calculated spectrum of the exchanging photons is close to Planckian, with a temperature of $\sim 2\varepsilon^*/7$. The photons responsible for the exchange force have a mean energy of $\sim\varepsilon^*$ and reduced wavelength of $\sim d$.

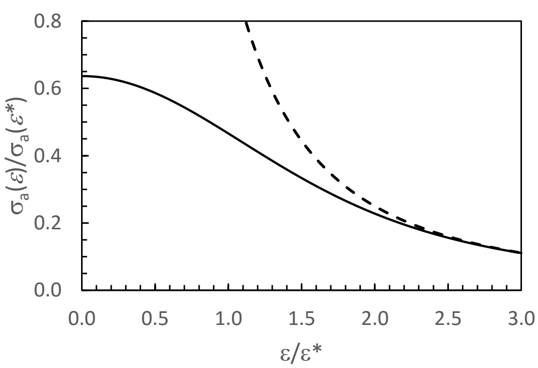

Fig. 1. Effective interaction cross sections obtained using the near-field corrected transmission coefficients represented by Eq. (13) (solid curve). The dashed curve displays the far-field result.

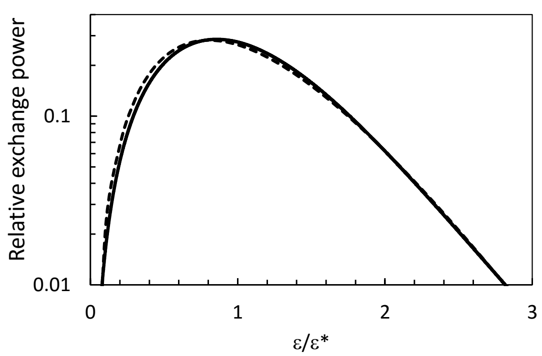

Fig. 2. Power spectrum for the exchanging photons involved in the $\alpha^{-1}=137.0378$ calculation discussed in the text (solid curve). The dashed curve displays a Planckian fit to the calculated power spectrum with a temperature $2\varepsilon^*/7$.

The speculative QED correction of order α feels like the well-known QED correction to the magnetic moment of the electron [12-13]. Based on the complexity of the higher-order corrections to the magnetic moment of an electron [14-19], the higher-order corrections for the exchange scenario presented here will not follow a simple pattern, and obtaining them directly via calculation would be a monumental task.

The Feynman diagrams for the higher-order corrections that generate the fuzziness in the interaction locations (discussed above) look like the diagrams for the anomalous magnetic moment of the electron, but with an extra photon line that connects to the other exchange partner a distance d away. It is therefore plausible that the QED corrections needed for the exchange scenario presented here are related to the QED calculation of the anomalous magnetic moment of the electron. The electron-only QED calculation of $(g-2)/2$ can be written as [19]

$$\frac{g-2}{2}=\sum_{n=1}^{\infty}A_1^{(2n)}(\frac{\alpha}{\pi})^n. \qquad (15)$$

The first three $A_1^{(2n)}$ are known precisely and are $A_1^{(2)}=0.5$, $A_1^{(4)}=-0.328478965579\ldots$, and $A_1^{(6)}=1.1812456587\ldots$ The $(g-2)/2$ can be loosely thought of as a ratio of two length scales. These are related to the relative path lengths in space-time where the electron location is fuzzy due to a surrounding cloud of virtual photons and electron-positron pairs, and the corresponding length scale for when the electron is naked. The exchange problem is related to effective interaction surface areas, and thus has a change in symmetry relative to that associated with the anomalous electron magnetic moment. Perhaps, for the exchange problem, we need a ratio of a QED corrected fuzzy effective surface area to the corresponding sharp surface area. Inspired by this suggestion we speculate that the $A_1^{(2n)}$ in Eq. (15) are related to dimensionless length scales. For our near-field correction problem, we convert these length scales into effective surface areas of spheres and express the fine structure constant as

$$\alpha=\frac{1}{2\pi}\int_0^{\infty}\frac{\mathrm{erf}^{\,4/(1+\eta(\alpha))}(\varepsilon/(2^{3/2}))}{\varepsilon(\exp(\varepsilon)-1)}\,d\varepsilon. \qquad (16)$$

with the near-field correction term

$$\eta(\alpha)=\sum_{n=1}^{\infty}4\pi(A_1^{(2n)})^2(\frac{\alpha}{\pi})^n. \qquad (17)$$

Eq. (17) leads to a calculated inverse fine structure constant of $\alpha^{-1}=137.035891$, which differs by 1 in the 7th significant digit from the known value. The corresponding calculated fundamental unit of charge is $q=1.602177\times10^{-19}$ C. The $n=4$ and 5 terms [19] only influence the 10th and beyond significant digits. The good match to experiment obtained here is possibly fortuitous, but demonstrates that plausible high-order QED corrections to Eq. (14) can lead to an improved match to the experimental data.

Using the speculative assumption of Eq. (7), a semi-classical estimate of the repulsive force generated by the exchange of unphysical photons between a pair of widely spaced quantum black holes can be obtained. If only the far-field estimate of the photon black-hole interaction cross section (and corresponding transmission coefficients) is used, then the calculated exchange force is infinite. Simple estimates of the near-field corrections obtained by intuitive overlap arguments remove the divergence, lead to an exchange force that is inversely proportional to the square of the separation distance, that is independent of the properties of the black holes (mass, size and temperature), and give an estimate of α^{-1}~139. The insensitivity to several black-hole properties suggests that the objects do not necessarily have to be black holes. However, the objects are required to have a propensity to emit L=0 photons via Eq. (5) that is completely suppressed due to energy conservation for emission to infinity, while the exchange of low-energy photons between like objects is allowed via the time-energy uncertainty principle. The nature of the photon exchange calculated here has a QED feel, with the photons involved being unphysical (perhaps we could label them virtual) and with the dominant exchange photons having $\lambda \sim 2\pi d$. In a semi-classical sense this means the energy, path, and direction of an individual exchange is not definable. It would thus not be surprising in a more detailed quantum mechanical calculation that the details of the semi-classical exchange suggested here are lost, and the only surviving property is a single photon-particle coupling constant.

Although the semi-classical arguments proposed here can only generate a repulsive force between like objects, an attractive force between oppositely charged objects can be generated by assuming the opposite charge is associated with a hole in a Fermi-sea of negative energy objects. Magnetism is not discussed here but falls out via standard Lorentz transformations between inertial frames. Possible reasons for the fractional charges of quarks are not discussed. Only a static configuration of a pair of objects is considered here. It would be interesting to consider the possibility of the generation of "real" photons in an extension of the presented scenario to a dynamical case.

Invoking the speculative QED modification represented by Eq. (17) gives the value α^{-1}=137.035891. The corresponding effective charge for quantum black holes is $q=1.6021773\times10^{-19}$ C, which differs from the known elementary charge $e=1.60217662\times10^{-19}$ C [11] by 7 in the 8th significant digit. The list of assumptions needed to obtain this result are: an effective exchange temperature of $T_{ex}=\hbar c/(2d)$; near-field corrections controlled by a harmonic oscillator wave function with a length scale of $\lambda/2\pi$; and QED corrections to the near-field correction that are analogous to the QED calculation of the electron's $(g-2)/2$ but with the $A_1^{(2n)}$ terms replaced by $4\pi\cdot(A_1^{(2n)})^2$. As discussed previously, the match to experiment may be fortuitous, and quantum field theory calculations are needed to confirm or negate the speculations presented here. In particular, the nature of near-field corrections and the possibility of second- and higher-order QED corrections should be studied.

If the presented speculations are confirmed by quantum field theory calculations (not semi-classical arguments) the implications are too numerous to be discussed here. However, an important one is that the strength of electromagnetism would be controlled by simple geometrical factors and QED corrections, and α would be a mathematical constant like π and e, and not a physical one (at least in flat spacetime). This would have significant consequences for ideas related to possible time dependencies of α, the anthropic principle, string theories, and multiverse scenarios.

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