# A Statistical Model of Spacetime, Black Holes and Matter

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**Abstract**

I propose first a simple model for quantum black holes based on a harmonic oscillator describing the black hole horizon covered by Planck length sized squares carrying soft hair. Secondly, I discuss a more involved statistical model with the partition function sum taken over black hole stretched horizon constituents which are black holes themselves. Attempting a unified quantum structure for spacetime, black holes and matter, I apply the statistical model picture also to matter particles using a composite model for quarks and leptons.

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# 1 Introduction

Thermodynamical properties, like entropy and temperature, of black holes have been established about four decades ago [1, 2]. More recently thermodynamics has been considered as the major agent behind general relativity by Padmanabhan [3, 4]. Thermogravity suggests that the role of the metric tensor is secondary thus providing the basis for models of the type considered in this note. Thermodynamical concepts have been applied to local Rindler frames [5, 6] leading to observer dependent phenomena. In [7] Mäkelä has applied the acceleration frame considerations to a statistical model of stretched horizon black holes by calculating the partition function of the system.

I describe first a warm-up model for the structure of quantum black holes. The black hole horizon is a spherical membrane covered with $l_{\mathrm{Pl}}^2$ size squares each of which can be in k states. The membrane dynamics is represented by a two dimensional harmonic oscillator.

Secondly, I redefine the partition function as a sum over black hole stretched horizon constituents following [7]. This model has a first order phase transition and it gives as predictions Hawking radiation and Bekenstein-Hawking entropy formula. The author of [7] regards the stretched horizon black holes as atoms of spacetime. I propose here a unified statistical picture for spacetime, black holes and matter in terms of the stretched horizon model and a composite model for quarks and leptons [8].

This note is organized as follows. After the Introduction a simple oscillator model for black holes is outlined in section 2. In section 3 I present the main points of the stretched horizon black hole model. Consistent quantization of spacetime and matter is discussed in section 4 in connection with a model for composite quarks and leptons. Finally in section 5 I give a brief discussion of results and conclusions. An appendix is provided for main results of the stretched horizon model phase transition. The presentation is very concise throughout.

# 2 Membrane Model of Horizon

As the first scenario for quantum black holes I assume the standard picture of a hole as a spherical horizon covered with squares of size $\gtrapprox l_{\mathrm{Pl}}^2$. The minimal horizon radius is of the order of $l_{\mathrm{Pl}}$. All physics takes place on the surface of the sphere, and tentatively, none inside. Suppose there are $n$ squares on the horizon and each square can be in $k$ soft hair states [9]. Then the total number of states is $k^n$. This gives for entropy S of the sphere the well known result

$$S = k_B \; log \; k^n = k_B \; n \; ln \; k \propto \frac{A}{l_{\mathrm{Pl}}^2} \tag{2.1}$$

where $k_B$ is the Boltzmann constant and A is the area of the horizon.

The vibrations of an oscillator can be calculated in normal way with certain frequency restrictions due to the grid. The geometry is a two dimensional sphere

$$H = \frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega^2\vec{x}^2 \tag{2.2}$$

The energy eigenvalues for a square are given $E_n = \hbar\omega(n+1)$ where $n = 0,\, 1,\, 2,\, \dots$ .

The partition function Q is (to be used in section 3)

$$Z = \Sigma_n g_n exp(E_n/kT) \tag{2.3}$$

where $g_n$ (= k in (2.1)) is the degeneracy of the n'th state and $E_n$ its energy and $T$ the temperature.

# 3 Model of Stretched Horizon

I consider a micro black hole dressed by a (virtual reality [10]) stretched horizon, which is a membrane hovering about a Planck length outside the event horizon and which is both physical and hot. A treatment of the stretched horizon has been done by Mäkelä in [7] which paper I cite below. He assumes that the stretched horizon consists of finite number of discrete constituents. Candidate constituents may be something geometrical, like a Planck scale string or area, but not volume, scalar fields or, very likely, graviton configurations. In [7] the area model is chosen, each constituent contributing to the stretched horizon an area of a non-negative integer times a constant

$$A = b\, l_{\rm Pl}^2\, (n_1 + n_2 + ... + n_N) \tag{3.1}$$

where $N$ is the number of constituents, the $n_i$ define constituent area quantum states and $b$ is a number of the order unity that can be determined later. Equation (3.1) is not, however, a typical quantum form because its zero point area is zero. For the constituents themselves one assumes simply black holes of size $l_{\rm Pl}$. Therefore in this model each stationary quantum state of a black hole is determined by the quantum numbers $n_1, n_2, ..., n_N$ of its stretched horizon. As in section 2, no physics is tentatively associated inside the horizon.

For the partition function we need the energy levels of the system. The energy of a black hole from the point of view of an observer on its stretched horizon is called Brown-York energy [11]

$$E = \frac{ac^2}{8\pi G} A \tag{3.2}$$

where $a$ is the constant proper acceleration of an observer on the stretched horizon and $A$ is the area of the horizon. For our system in consideration this becomes from (3.1)

$$E_n = nb\frac{\hbar a}{8\pi c} \tag{3.3}$$

where $n = n_1 + n_2 + ... + n_N$. The number of microscopic states associated with energy $E_n$ is the number of ways of writing a given positive integer $n$ as a sum of exactly $N$ positive integers, whith $N \leq n$. This is given by the binomial coefficient

$$\Omega_N(n) = \left( \begin{array}{c} n-1 \\ N-1 \end{array} \right) \tag{3.4}$$

It gives the degeneracy function $g(E_n)$ for the partition function 2.3 which is calculated into a simple form [7]

$$Z(\beta) = F(\beta)(1 - F(\beta)^{N+1}) \tag{3.5}$$

where

$$F(\beta) = \frac{1}{2^{\beta T_C} - 2} \tag{3.6}$$

and the temperature

$$T_C = \frac{b\, \hbar\, a}{4(\ln 2)\pi k_B c} \tag{3.7}$$

is called the critical temperature of the hole. For the purposese of the next section it is sufficient that that partition function is convergent.

How do we interprete the stretched horizon model? I assume it applies both to black holes and atoms of spacetime. The next question is are matter particles pointlike in this spacetime or do particles have some kind of substructure? In a unified model both spacetime and matter particles have the same type of substructure as we see in the next section.

# 4 Unified Model of Spacetime and Matter

Statistical methods of section 3 for spacetime offer a possibility to study the matter sector from a novel point of view to build a consistent, unified statistical picture of both spacetime and matter. This goal would imply some internal structure at scale of the order of $l_{\text{Pl}}$ for particles. Such a model has been proposed in [8] (though there never was experimental support for it).

The basic idea in [8] is that the quarks and lepton are made of maxons with charge 0 or $\frac{1}{3}$ and 'color' (i, j, k) as permutation index as follows

$$\begin{aligned} u_k &= \epsilon_{ijk} m_i^+ m_j^+ m^0 \\ \bar{d}_k &= m_k^+ m^0 m^0 \\ e^- &= \epsilon_{ijk} m_i^- m_j^- m_k^- \\ \nu &= \epsilon_{ijk} m_i^0 m_j^0 m_k^0 \end{aligned} \tag{4.1}$$

The construction (4.1) on maxon level is matter-antimatter symmetric and 'color' singlet, which is desirable for early cosmology.

The maxons in this note are black holes with stretched horizons. Their energy scale is at the Planck scale (and (4.1) would be superheavy particles). To get the standard model particles the large mass difference has to be explained. It is accomplished from (3.1) by setting $N = 1, n_1 = 0$, which leads by (3.2) to zero mass remnant of the hole. This maxon, without stretched horizon, may interact with the Higgs field and gain mass from it. I assume that the quarks and leptons are bound states of maxons with the Higgs, or other scalar, mediating the binding. The question of existence of free single maxons is postponed to another study. It may be a new particle or it can also be assumed that some kind of confinement is operating.

The gauge bosons and the Higgs would be elementary (but their composite nature is not ruled out). The three generations would be due to a gravitational mechanism of the stretched horizon or a new symmetry. Missing at the moment are calculational methods for the bound states. Numerical methods can usually be developed at some level of accuracy.

# 5 Discussion and Conclusions

If a non-inertial observer perceives a horizon, he will attribute to it the Davies-Unruh temperature, see (3.7)

$$T = \frac{\hbar}{k_B c} \frac{a}{2\pi} \tag{5.1}$$

where $a$ is the acceleration of the observer. This result makes the notion of temperature and all of thermodynamics observer dependent phenomena. This problem was taken into account in the local Rindler frame considerations in the earlier sections.

It has turned out that horizons have profound importance in gravity both on thermodynamical and statistical levels. There are interesting questions of heat as inertial effect in a quantum equivalence principle and static observer's virtual reality in [10].

In the UV black holes cannot be probed deeper than $l_{\text{Pl}}$. With increasing energy the hole begins to grow approaching the classical regime. This model is therefore consistent with the concept of self-completeness [12].

The regime of real quantum gravity is limited to the vicinity of mini black holes and very early universe. Otherwise classical theory is accurate.

A model of decay and radiation of black holes has been proposed in [13, 14]. The lightest black hole state $E_{n=0}$, the remnant or gravon, is expected to decay into standard model particles. Otherwise black holes radiate by the Hawking mechanism and by a classical no-hair theorem based mechanism producing non-thermal particles, dominantly light leptons.

There are at present a number competing theoretical schemes for quantum gravity like string theory, loop quantum gravity, causal dynamical triangulation, and others. The model of section 3 goes deep into the structure of the physical universe and can be considered a promising candidate. In that scenario the horizon properties of black holes and local Rindler frames are the origin of gravity, and general relativity is its IR limit. In this note, these ideas have been applied to the matter sector using a preon model in section 4.

The results of this simple model can be considered encouraging for the development of a more realistic model. A more involved model for the stretched horizon should include gravitons and is expected to produce a smoother transition for the system in getting down from Planck scale to standard model particle mass scale. Finally, this note should be considered a program definition for future work with a number of model results as guidelines.

# Appendix

# A The Phase Transition

I give below a brief sketch of the stretched horizon model first order phase transition though not directly needed for the main section 4 of the present note. All properties of the model are carefully derived in [7].

The average energy at temperature $T = 1/\beta$ can be calculated from the partition function (3.5)

$$E(\beta) = -\frac{\partial}{\partial\beta} \ln Z(\beta) \tag{A.1}$$

of the black hole which yields

$$E(\beta) = \left[\frac{2^{\beta T_C}}{2^{\beta T_C} - 2} - \frac{(N+1)2^{\beta T_C}}{(2^{\beta T_C} - 1)^{N+2} - 2^{\beta T_C} + 1}\right] T_C \ln 2 \tag{A.2}$$

The average energy per constituent is for large $N$

$$\bar{E}(\beta) = \bar{E}_1(\beta) + \bar{E}_2(\beta) \tag{A.3}$$

where

$$\begin{aligned} \bar{E}_1(\beta) &= \frac{1}{N}\frac{2^{\beta T_C}}{2^{\beta T_C} - 2} T_C \ln 2 \\ \bar{E}_2(\beta) &= -\frac{2^{\beta T_C}}{(2^{\beta T_C} - 1)^{N+2} - 2^{\beta T_C} + 1} T_C \ln 2 \end{aligned} \tag{A.4}$$

where $(N+1)/N \approx 1$ has been used.

At $T = T_C$ the average energy per a constituent of the stretched horizon is, in SI units,

$$\bar{E} = k_B T_C \ln 2 \tag{A.5}$$

and its rate of change

$$\frac{d\bar{E}}{dT}|_{T=T_C} = \frac{1}{6}k_B(\ln 2)^2 N + \mathcal{O}(1) \tag{A.6}$$

where $\mathcal{O}(1)$ denotes the terms which are of the order $N^0$, or less. At large $N$ the hole undergoes a phase transition at $T = T_C$. When $T < T_C$, $\bar{E}$ is about zero. When $T = T_C$, the function $\bar{E}(T)$ becomes practically vertical jumping by the latent heat $\bar{L} = 2k_B T_C \ln 2$. $\bar{E}(T)$ depends on $T$ approximately linearly when $T \gg T_C$.

$T_C$ may be written in terms of the Schwarzschild mass $M$ and the radial coordinate $r$ of an observer on the stretched horizon as

$$T_C = \frac{b}{8\pi \ln 2}\left(1 - \frac{2M}{r}\right)^{-1/2}\frac{M}{r^2} \tag{A.7}$$

An observer just outside of the event horizon, where $r \approx 2M$, measures a temperature

$$T_C = \frac{\alpha}{32\pi \ln 2}\left(1 - \frac{2M}{r}\right)^{-1/2}\frac{1}{M} \tag{A.8}$$

for the black hole. Due to the non-zero temperature of the hole, thermal Hawking radiation comes out of it, and the Tolman relation [15] implies that an observer at asymptotic infinity measures a temperature for the radiation

$$T_\infty = \frac{\alpha}{32\pi \ln 2}\frac{1}{M} \tag{A.9}$$

With some more effort one can obtain the Bekenstein-Hawking entropy law for the Schwarzschild black hole

$$S(A) = \frac{1}{4}\frac{k_B c^3}{\hbar G}A \tag{A.10}$$

When $T = T_C$, the energy of the hole from the point of view of an observer on its stretched horizon is exactly

$$E = (N+2)k_B T_C \ln 2 \tag{A.11}$$

It is interesting that, up to an unimportant numerical factor $2\ln 2$, this expression for energy is the same as the one used as a starting point in the scenario for an entropic theory of gravity [16].

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