Rebuttal of the paper "Black-body laws derived from a minimum knowledge of Physics"

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Abstract. Errors in the paper "*Black-body laws derived from a minimum knowledge of Physics*" are described. The paper claims that the density of the thermal current in any number of spatial dimensions is proportional to the temperature to the power of $2(n-1)/(n-2)$, where n represents the number of spatial dimensions. However, it is actually proportional to the temperature to the power of $n+1$. The source of this error is in the claim that the known formula for the fine-structure constant is valid for any number of spatial dimensions, and in the subsequent error that the physical dimensions of Planck's constant become dependent on n.

PACS Numbers: 03.65.Bz Foundations, theory of measurement 04.60 Quantum theory of gravitation 05.90.+m Other topics in statistical physics, thermodynamics, and nonlinear dynamical systems

1. Introduction

Paper [1, eq. 11] uses a dimensional analysis to show the dependence of the density of the thermal current, j_n, versus the temperature, T, in n-dimensional Euclidean space:‡

$$\bullet \quad j_n \propto T^{\frac{2(n-1)}{n-2}}, \tag{1}$$

(In [1], the energy density of the photons, u_n, is used, but it can also be interpreted as j_n.) This formula disagrees with the correct formula which is used in other papers, such as [2, 3, 4, 5]:

$$j_n \propto T^{n+1}. \tag{2}$$

The paper [1] disagrees with the results of the paper [3], which was published before it; therefore it should have been mentioned in the references of [1], instead it was omitted.

In the present paper we will show the errors in [1]. At first sight it seems that the errors are clear, but a detailed inspection of [1] shows that it is necessary to be precise about the source of the errors. The purpose of [1] was also to avoid any theoretical physical background, as much as possible, and to show only what can be achieved with a dimensional analysis. This purpose somewhat reduces possible criticisms of the errors.

Dimensional analyses are interesting and clear, because they can quickly show many properties that otherwise can only be found using long derivations. This rebuttal will explain more about dimensional analyses, and is more transparent than the papers [1, 6].

The motivation for this rebuttal is also that it illuminates various details about multidimensional spaces that can otherwise be overlooked. They are important for understanding the physics. For instance, the formula for the fine-structure-constant is only so simple in three spatial dimensions; the uncertainty principle is a one-dimensional phenomenon. If the power was equal to $2(n-1)/(n-2)$, this would have contradicted the principles of quantum physics, and the rough principles of quantum gravity. Therefore, [1] also behaves like a thought experiment.

It will also be shown that the formula (2) can have a simpler derivation than described in refs. [2, 3, 5].

The paper [1] is cited once, in [4], but the paper [4] is cited 33 times. Although the paper [4] gives the correct formula, (2), it does not mention that the formula (1) is erroneous. It is because such papers mislead people that they need to be rebutted.

In the next section one simple explanation of (2), based on another dimensional analysis, is shown. Arguments that the physical dimensions of Planck's constant in the paper [1] are erroneous, are shown in Section 3. The arguments are also based on dimensional analyses. In Section 4, an analysis is made to show that the formula for the fine-structure constant is erroneously generalized to more dimensions. Another consequence of this fine-structure-constant formula was a different definition of Planck's constant.

‡ The bullet signs will mean formulae from [1], or calculated in accordance with [1].

2. A simple explanation of the Stefan-Boltzmann law with a dimensional analysis

The common calculation gives the result (2), even in statistical non-quantum mechanics, [2], and also gives it in connection with quantum mechanics, [2, 3, 5].

An even simpler insight into why $j_n \propto T^{n+1}$ is evident if we make a dimensional analysis in natural units with Planck's constant $\widetilde{\hbar} = 1$ and the speed of light $\widetilde{c} = 1$. The quantities with a tilde mean that we work in these natural units. This assumption is also very natural, because $\hbar$ and c are only conversion factors between various units.§ It is convenient to introduce $\widetilde{E} = E$, $\widetilde{m} = mc^2$, $\widetilde{q} = qc$, $\widetilde{T} = kT$, $\widetilde{x} = x/(\hbar c)$, $\widetilde{t} = t/\hbar$, etc., where the quantities without tilde are expressed in terms of the conventional units (kg, m, s, or eV, nm, fs) and the quantities with tilde all have the physical dimension $[E]$ to some power, for example: $\widetilde{E}$, $\widetilde{m}$, $\widetilde{q}$, $\widetilde{T} \Rightarrow$ eV; $\widetilde{x}$, $\widetilde{t} \Rightarrow (\mathrm{eV})^{-1}$. Here, E is the energy, m is the mass, q is the momentum, k is Boltzmann's constant, x is the distance, and t is the time. The charge $\widetilde{e}$ and the fine-structure constant (in three spatial dimensions) are dimensionless. Now, it is easy to see the power of temperature in the Stefan-Boltzmann law, where the density of the thermal current j in three spatial dimensions can be expressed using a simple formula:

$$j = \frac{E}{St}, \tag{3}$$

where E is the energy of this thermal current, which is emitted per unit time t and per unit surface S. Then physical dimensions of $\widetilde{j}$ are calculated as follows:

$$\left[\widetilde{j}\right] = \left[\frac{\widetilde{E}}{\widetilde{S}\widetilde{t}}\right] = \frac{[E]}{[E]^{-2}[E]^{-1}} = [E]^4 = (\mathrm{eV})^4, \tag{4}$$

and since the only available quantity is the temperature, with the physical dimension $[E]$, it should appear to the power four. Note that the law cannot depend on any space or time coordinate or on any mass of participating particles in the wall of the cavity containing the black-body radiation. In n dimensions the hypersurface S is of dimension $n-1$, and in j_n the power of the temperature should be $n+1$.

3. The reason for the error associated with Planck's constant

Let us look for the reason why eq. (1) is different from the usual eq. (2). In [1, Page 6] it is evident that the physical dimensions of Planck's constant are defined as follows:

- $$[\hbar_n] = [ML^{n-1}t^{-1}], \tag{5}$$

where $[\hbar_n]$ are the physical dimensions of Planck's constant in any spatial dimensions n, M is the mass, L is the length, and t is the time. At least in our three-dimensional space, it can be considered that $\hbar$ is only a conversion factor, [7], for instance, from

§ Duff, [7], uses totally dimensionless units where he adds that gravitational constant $\widetilde{G} = 1$, but G is not important for the Stefan-Boltzmann law; therefore units in the present paper are not dimensionless.

seconds to $(\text{eV})^{-1}$. Let us assume that we have such units that $\tilde{\hbar} = 1$ and $\tilde{c} = 1$ at $n = 3$. If these units are inserted into (5), we obtain:

- $$[\widetilde{\hbar}_n] = [E]^{3-n}. \tag{6}$$

Thus, $\widetilde{\hbar}_n$ in (6) is not, in general, dimensionless. Consequently, we need a physical body that contains some energy, which means that we need to know the quantity of this energy. This also means that $\hbar_n$ is dependent on the temperature. Physically, this is erroneous, or at least awkwardly.

Furthermore, uncertainty principle introduces Planck's constant almost from fundamentals:

$$\Delta x \Delta q_x \geq \frac{\hbar}{2}. \tag{7}$$

For this, Δx is the uncertainty of the position in the x direction, and Δq_x is the uncertainty of the momentum in this direction. Any direction can be used instead of x. The essence of this inequation is that it only respects one direction, and that the momentum is independent of n. (The same is true for energy.) Because (7) is only in one dimension, it seems that the uncertainty principle is not only linked to three spatial dimensions, thus ineq. (7) should be the same in any number of spatial dimensions. Therefore, if $[\hbar_n]$ is defined as being dependent on n, like in the paper [1], the uncertainty principle in n dimensions should be corrected. The authors should have mentioned this in [1], but they did not.

4. The error in the fine-structure constant

Such physical dimensions of $[\hbar_n]$ were obtained via the fine-structure-constant formula in n dimensions, [1, Page 6],

- $$\alpha_n = \frac{e_n^2}{\hbar_n c}, \tag{8}$$

where e_n is the elementary charge in n dimensions, its physical dimensions are $[M^{1/2}L^{n/2}t^{-1}]$, α_n is dimensionless, and consequently the physical dimensions of $[\hbar_n]$ are as written in (5). This formula was assumed by motivation using Stoney units, [1, Page 5]. It is very simple, but just a dimensional analysis with the assumption of the common physical dimension of $\hbar_n$, $[\hbar_n] = [ML^2t^{-1}]$, gives a different, more complicated, result:

$$\alpha_n = \left(\frac{G_n^{\frac{3-n}{2}}}{\hbar^{n-2} c^{4-n}} \right)^{\frac{2}{n-1}} e_n^2. \tag{9}$$

Eq. (9) is related to [6, eq. 3]. G_n is the gravitational constant in n-dimensional space, its physical dimensions are $[M^{-1}L^n t^{-2}]$. The physical dimensions of e_n are the same as for eq. (8).‖ Thus, with the elementary charge and with some other basic physical

‖ [6, Page 2] contains a lapse, because it states that physical dimensions of e_n are $[ML^n t^{-2}]$, but such physical dimensions are for e_n^2.

constants, a dimensionless number can be composed. Thus, another distinction is that the fine-structure-constant formula in n-dimensional space, (8), was assumed in [1], but in [6] it was calculated.

Together with (9), gravitational coupling constant in n dimensions was calculated:

$$\alpha_{\mathrm{G}n} = \left(\frac{G_n}{\hbar^{n-2}c^{4-n}}\right)^{\frac{2}{n-1}} m_{\mathrm{P}n}^2. \tag{10}$$

Eq. (10) is related to [6, eq. 4].¶ (The index n inside of $m_{\mathrm{P}n}$ means the Planck mass in any dimensions.)1 Thus, in this formula, a combination of powers on $m_{\mathrm{P}n}$, $\hbar$, c, and G_n also gives a dimensionless number.

in natural units, eqs. (9) and (10) together give a more transparent formula:

$$\alpha_n = \widetilde{m}_{\mathrm{P}n}^{n-3}\widetilde{e}_n^2 3.3a \tag{11}$$

We can wonder ourselves about the reason why G_n is present in (9) or that $\widetilde{m}_{\mathrm{P}n}$ is present in (11). Let us write the formula for the electrostatic energy of the charge in n dimensions:

$$V_n \approx \frac{e_n^2}{r^{n-2}}. \tag{12}$$

If we again use a natural system of units, the physical dimension of the charge can be identified as

$$[\widetilde{e}_n]^2 = [E]^{3-n}. \tag{13}$$

This means that the formula (8) can only give a dimensionless value at $n = 3$, but otherwise it is a dimensional value. Therefore, $\widetilde{e}_3$ is dimensionless, but $\widetilde{e}_n$ in general needs something with which together it is dimensionless. This problem of the physical dimensions for electric charge is resolved by the presence of the Planck mass (or energy) in (11). This energy is the only one available in this regime.

At the same time, in [1, Page 5], formulae for the Planck length, l_{P}, the Planck time, t_{P} and the Planck mass m_{P} are used. However, they overlooked that these quantities in n dimensions are different, (10); therefore, they did not describe their values for more dimensions.

Now, $m_{\mathrm{P}n}$ can be incorporated into $\hbar_n$ to make eq. (8) correct; like the temperature can be incorporated into $\hbar_n$ in (6), but this is already a contradiction, i.e. $m_{\mathrm{P}n}$ and the temperature do not represent the same scale of physics, but the reason that they are incorporated is in the same formula (8).

We have made three claims, that (5) and (8) are erroneous, and that (8) is not consistent with (9). Therefore, merging all three claims makes us suspect that they overlooked the fact that the formula (8) is oversimplified, and at the same time, it complicates further calculations.

¶ Although m_{p} in [6, eq. 4] was mentioned as the mass of the proton, it can also be the Planck mass, $m_{\mathrm{P}n}$.

5. Conclusion

We show why the paper [1] is erroneous. At first sight it seems that the errors are clear, but a detailed inspection of the paper shows that it is necessary to be precise at the source of the errors. One fundamental indication of the errors is that the physical dimensions of $\hbar_n$ in [1, Page 5] are dependent on the number of spatial dimensions. The error that was the cause for this one is an oversimplified form of the fine-structure-constant formula, (8), that is independent of the number of spatial dimensions. The third fundamental indication of the error is that rough calculations of quantum gravity disagree with the results of the paper [1]. At the same time, the paper [1] makes us aware of important aspects that the physical dimensions of Planck's constant are independent of the number of spatial dimensions, and that the fine-structure-constant formula is not so simple as in (8). The dimensional analyses in this paper are also more transparent than in [1, 6].

6. Acknowledgment

Thanks to Mitja Rosina for advice and constructive discussions. Thanks to Irena Drevenšek Olenik, Tomaž Mertelj, Alojzij Kodre, and Bojan Božič for opinions and advice. Thanks to the unknown person from an internet forum, who gave me the link to the paper [1].

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