Laplace decomposition and Semigroup decomposition methods to solve Glycolysis system in one dimension

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Abstract

In this article, we formulate two methods to get approximate solution of Glycolysis system. The first is Laplace decomposition methods (is a method combined Lplace transform and Adomian polynomial) and the second is semigroup decomposition method (is a method combined semigroup approach and Adomian polynomial), In both methods the nonlinear terms in Glycolysis system treated with help Adomian polynomial. One example are presented to illustrate the efficiency of the methods, this is done by writing a computer programs with the aid of Maple 13.

keywords: approximate solutions, Laplace decomposition method, semigroup decomposition method.

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1. Introduction

System of partial differential equation have attracted much attention in a verity of applied science because of their wide applicability [5]. Chemical and Biochemical Kinetics has been a rich source to produce a variety of spatial-temporal patterns since the discovery of the oscillating wave in the Belousov-Zhabotinsky reaction in 1950's. These phenomena and observations have been transformed to challenging Mathematical problems through various Mathematical models, especially reaction-diffusion equations [18]. Chemical reactions are modeled by non-linear partial differential equations (PDEs) exhibiting traveling wave solutions. These oscillations occur due to feedback in the system either Chemical feedback (such as autocatalysis) or temperature feedback due to a non-isothermal reaction [10]. In recent published papers [17, 18, 19] by this author, it has been proved that for a class of nonlinear reaction diffusion system in the form

$$\frac{\partial u}{\partial t} = d_1 \triangle u + a_1 u + b_1 v + f(u,v) + g_1(x),$$

$$\frac{\partial v}{\partial t} = d_2 \triangle v + a_2 u + b_2 v - f(u,v) + g_2(x),$$

where the nonlinear reaction term $f(u,v) = u^2 v$ represents the type of cubic autocatalytic Chemical or Biochemical reaction, with homogeneous Dirichlet or Neumann boundary condition on a bounded, locally Lipschitz domain $\Omega \subset \Re^n$, $n \leq 3$, there exist a global attractor in the phase space $L^2(\Omega) \times L^2(\Omega)$, whose Hausdorff and fractal dimensions are finite [20].
This class of reaction-diffusion system includes some significant pattern formation equations arising from modeling of Kinetics of Chemical or Biochemical reactions and from Biological and cellular pattern formation. The following four model equations are typical in this class:
Brusselator model:

$$a_1 = -(b+1), b_1 = 0, a_2 = b, b_2 = 0, f = u^2 v, g_1 = a, g_2 = 0.$$

where a and b are positive constants.

Gray-Scott model:

$$a_1 = -(f+K), b_1 = 0, a_2 = 0, b_2 = -F, f = u^2 v, g_1 = 0, g_2 = F.$$

where F and K are positive constants.

Glycolysis model:

$$a_1 = -1, b_1 = K, a_2 = 0, b_2 = -K, f = u^2 v, g_1 = \delta, g_2 = \rho.$$

where K, ρ and δ are positive constants.

Schnackenberg model:

$$a_1 = -K, b_1 = a_2 = b_2 = 0, f = u^2 v, g_1 = a, g_2 = b.$$

where K, a and b are positive constants

Then one obtains the following system of two nonlinearly coupled reaction-diffusion equations (the Glycolysis model),

$$\begin{aligned} \frac{\partial u}{\partial t} &= d_1 \triangle u - u + Kv + u^2 v + \rho, \quad (t,x) \in (0,\infty) \times \Omega \\ \frac{\partial v}{\partial t} &= d_2 \triangle v - Kv - u^2 v + \delta, \qquad (t,x) \in (0,\infty) \times \Omega \end{aligned} \tag{1.1}$$

with the following initial conditions:

$$\begin{aligned} u(t,x) &= v(t,x) = 0, \qquad t > 0, x \in \partial\Omega \\ u(0,x) &= u_0(x), v(0,x) = v_0, \qquad x \in \Omega \end{aligned} \tag{1.2}$$

where K, ρ and δ are positive constants [10].

2. Laplace Decomposition Method

The Laplace decomposition method (LDM) is a Laplace transform scheme, based on the decomposition method used by many authors for solving differential equation. The numerical technique basically illustrates how the Laplace transform can be used to approximate the solution of the nonlinear differential by manipulating the decomposition method which was first introduced by Adomian [1]. Khuri [4, 5] used this method for solving class of nonlinear ordinary and partial differential equations. Yusufoglu [21] applied this method for the solution of Duffing equation. Elgazery [12]exploit this method to solve Falkner-Skan equation. Khan, *et.al* [9] used this method for solving nonlinear ordinary and partial differential equation. Kumar and Pankaj [8, 7]use this method for solitary wave solutions schrodinger equation and to study solitary wave solutions of coupled nonlinear partial differential equation in 2012 and in 2013 respectively. Therefor Yang and Hou [14] use this method to find an approximate solution of nonlinear fractional differential equation. In 2013 Zafar, *et.al* [22] applied this method for solving Burger's equation. Also Mohamed and Torky [11]exploit this method to numerical solution of nonlinear system of partial differential equations. On the other hand Koroma, *et.al* [6] used modified Laplace decomposition algorithm for solving Blasius boundary Layer equation of the flat plate in a uniform stream. Also in 2013 Yin, *et.al* [15]used modifies Laplace decomposition method for solving Lane-Emden Type differential equations.

Our aim in this section is to apply Laplace decomposition method to solve Glycolysis system in one dimension. The general form of Glycolysis system can be considered in an operator form as follow as:

$$\begin{aligned} Lu + R_1(u,v) - N(u,v) &= h_1, \\ Lv + R_2(u,v) + N(u,v) &= h_2, \end{aligned} \tag{2.1}$$

with initial data:

$$\begin{aligned} u(x,0) &= u_0(x), \\ v(x,0) &= v_0(x). \end{aligned} \tag{2.2}$$

where $L = \frac{\partial}{\partial t}$ is first order differential operator , R_1 and R_2 are remaining linear operator, $N(u,v)$ represents a general non-linear differential operator and h_1 with h_2 are source terms. The methodology consists of applying Laplace transform first on both sides of (2.1), we have:

$$l[Lu] + l[R_1(u,v)] - l[N(u,v)] = l[h_1],$$
$$l[Lv] + l[R_2(u,v)] + l[N(u,v)] = l[h_2],$$

Using the differentiation property of Laplace transform and initial conditions (2.2), we get :

$$sl[u] - u_0(x) + l[R_1(u,v)] - l[N(u,v)] = l[h_1],$$
$$sl[v] - v_0(x) + l[R_2(u,v)] + l[N(u,v)] = l[h_2],$$

Dividing both sides of the above two equations by s (where s Laplace domain function), we get:

$$\begin{aligned} l[u] &= \frac{u_0(x)}{s} - \frac{l[R_1(u,v)]}{s} + \frac{l[N(u,v)]}{s} + \frac{l[h_1]}{s}, \\ l[v] &= \frac{v_0(x)}{s} - \frac{l[R_2(u,v)]}{s} - \frac{l[N(u,v)]}{s} + \frac{l[h_2]}{s}, \end{aligned} \tag{2.3}$$

The second step in Laplace decomposition method is that we represent solution as an infinite series given below:

$$\begin{aligned} u &= \sum_{n=0}^{\infty} u_n, \\ v &= \sum_{n=0}^{\infty} v_n. \end{aligned} \tag{2.4}$$

The nonlinear operator is decompose as:

$$N(u,v) = \sum_{n=0}^{\infty} A_n, \tag{2.5}$$

where A_n are Adomian polynomial [13] of $u_0, u_1, u_2, ..., u_n$ and $v_0, v_1, v_2, ..., v_n$ and it can be calculated by formula given below:

$$A_n = \frac{1}{n!}[\frac{d^n}{d\lambda^n} N(\sum_{i=0}^{\infty} \lambda^i u_i, \sum_{i=0}^{\infty} \lambda^i v_i)]_{\lambda=0}, \tag{2.6}$$

Putting Equations (2.4)-(2.6) in the System (2.3) we will get:

$$\begin{aligned} l[\sum_{n=o}^{\infty} u_n] &= \frac{u_0(x)}{s} - \frac{l[R_1(u,v)]}{s} + \frac{l[\sum_{n=0}^{\infty} A_n]}{s} + \frac{l[h_1]}{s}, \\ l[\sum_{n=o}^{\infty} v_n] &= \frac{v_0(x)}{s} - \frac{l[R_2(u,v)]}{s} - \frac{l[\sum_{n=0}^{\infty} A_n]}{s} + \frac{l[h_2]}{s}, \end{aligned}$$

Or

$$\begin{aligned}\sum_{n=o}^{\infty} l[u_n] &= \frac{u_0(x)}{s} - \frac{l[R_1(u,v)]}{s} + \frac{l[\sum_{n=0}^{\infty} A_n]}{s} + \frac{l[h_1]}{s},\\ \sum_{n=o}^{\infty} l[v_n] &= \frac{v_0(x)}{s} - \frac{l[R_2(u,v)]}{s} - \frac{l[\sum_{n=0}^{\infty} A_n]}{s} + \frac{l[h_2]}{s},\end{aligned} \tag{2.7}$$

On comparing both sides of the system (2.7) we have:

$$\begin{aligned}l[u_0] &= \frac{u_0(x)}{s} + \frac{l[h_1]}{s} = k(x,s),\\ l[v_0] &= \frac{v_0(x)}{s} + \frac{l[h_2]}{s} = p(x,s),\end{aligned} \tag{2.8}$$

$$\begin{aligned}l[u_1] &= -\frac{l[R_1(u_0,v_0)]}{s} + \frac{l[A_0]}{s},\\ l[v_1] &= -\frac{l[R_2(u_0,v_0)]}{s} - \frac{l[A_0]}{s},\end{aligned}$$

$$\begin{aligned}l[u_2] &= -\frac{l[R_1(u_1,v_1)]}{s} + \frac{l[A_1]}{s},\\ l[v_2] &= -\frac{l[R_2(u_1,v_1)]}{s} - \frac{l[A_1]}{s},\end{aligned}$$

In general, the recursive relation is given by:

$$\begin{aligned}l[u_n] &= -\frac{l[R_1(u_{n-1},v_{n-1})]}{s} + \frac{l[A_{n-1}]}{s},\\ l[v_n] &= -\frac{l[R_2(u_{n-1},v_{n-1})]}{s} - \frac{l[A_{n-1}]}{s}, \quad n \geq 1\end{aligned} \tag{2.9}$$

Applying the inverse Laplace transform to Systems (2.8) and (2.9), so our required recursive relation is given below:

$$\begin{aligned}u_0(x,t) &= k(x,t),\\ v_0(x,t) &= p(x,t),\end{aligned} \tag{2.10}$$

and

$$\begin{aligned}u_n &= -l^{-1}[\frac{l[R_1(u_{n-1},v_{n-1})]}{s} + \frac{l[A_{n-1}]}{s}],\\ v_n &= -l^{-1}[\frac{l[R_2(u_{n-1},v_{n-1})]}{s} - \frac{l[A_{n-1}]}{s}], \ \ n \geq 1\end{aligned} \tag{2.11}$$

where k(x,t)and p(x,t) represents the term arising form source terms and prescribe initial conditions. Now first of all we applying Laplace transform of the terms on the right hand side of the system (2.11) then applying inverse Laplace transform we get values of $u_1, v_1, u_2, v_2, u_3, v_3, ..., u_n, v_n$ respectively. Putting these values in (2.4) yields approximate solution of (1.1).

3. semigroup decomposition

Our approach in this section, is reformulating the Glycolysis system (1.1) as a nonlinear integral equations. The semigroup approach is used to recast the System (1.1) as a nonlinear integral equation in time and the integral over nonlinear terms is computed by using Adomian polynomials.

3.1. Semigroup approach

The semigroup method is a well-known analytical tool which may be used to convert partial differential equation to nonlinear integral equation and to calculate estimates associated with the behavior of their solution [3]. The integral solution of the initial value problem

$$u_t = Lu + Nf(u), \tag{3.1}$$

$$u(x, t_0) = u_0(x),$$

is given by

$$u(x,t) = e^{(t-t_0)L}u_0(x) + \int_{t_0}^{t} Nf(u(x,s))ds, \tag{3.2}$$

where L and N are time independent, constant coefficient differential operators. Expressing solution of (3.1) in the form (3.2) allows one to prove existence and uniqueness of the solution, compute estimates of the magnitude of the solution, verify dependence on initial and boundary data, and perform asymptotic analysis of the solution. We used equation (3.2) as a starting point for an efficient numerical algorithm. Based our knowledge, the semi group method has had limited use in numerical calculations. A significant difficulty in designing numerical algorithms based directly on the solution (3.2) is that the operators appearing in (3.1) are not sparse (i.e the matrices representing these operators are dense) [16] .

3.2. Reformulating Semigroup Approach to solve Glycolysis system:

Keiser [3] uses quadrature methods to find integral in Equation (3.2) after recasting partial differentional equation by Semigroup approach as a nonlinear integral equation in time. In this section, for the first time Semigroup approach was used with handling the nonlinear terms Adomian polynomial. To describe our method rewrite Glycolysis system as the form :

$$\begin{aligned} u_t &= L_1(u,v) + Nf(u,v) + h_1, \\ v_t &= L_2(u,v) - Nf(u,v) + h_2, \end{aligned}$$

with the initial conditions (2.2).
So, by semigroup approach, we get

$$\begin{aligned} u(x,t) &= e^{(t-t_0)(L_1(u(x,t),v(x,t))+h_1(x,t))}u_0(x) + \int_{t_0}^{t} e^{(t-s)(L_1(u(x,s),v(x,s))+h_1(x,s))}Nf(u(x,s),v(x,s))ds, \\ v(x,t) &= e^{(t-t_0)(L_2(u(x,t),v(x,t))+h_2(x,t))}v_0(x) - \int_{t_0}^{t} e^{(t-s)(L_2(u(x,s),v(x,s))+h_2(x,s))}Nf(u(x,s),v(x,s))ds, \end{aligned}$$

Next, using Equations (2.4)-(2.6) in the above system, we will get:

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(x,t) &= e^{(t-t_0)(L_1(u(x,t),v(x,t))+h_1(x,t))}u_0(x) + \int_{t_0}^{t} e^{(t-s)(L_1(u(x,s),v(x,s))+h_1(x,s))} \sum_{n=0}^{\infty} A_n ds, \\ \sum_{n=0}^{\infty} v_n(x,t) &= e^{(t-t_0)(L_2(u(x,t),v(x,t))+h_2(x,t))}v_0(x) - \int_{t_0}^{t} e^{(t-s)(L_2(u(x,s),v(x,s))+h_2(x,s))} \sum_{n=0}^{\infty} A_n ds, \end{aligned} \tag{3.3}$$

On comparing both sides of the system (3.3) we have

$$\begin{aligned} u_0(x,t) &= e^{(t-t_0)h_1(x,t)}u_0(x), \\ v_0(x,t) &= e^{(t-t_0)h_2(x,t)}v_0(x), \end{aligned} \tag{3.4}$$

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and

$$\begin{aligned} u_n(x,t) &= e^{(t-t_0)(L_1(u_{n-1}(x,t),v_{n-1}(x,t)))} \\ &+ \int_{t_0}^{t} e^{(t-s)(L_1(u_{n-1}(x,s),v_{n-1}(x,s))+h_1(x,s))} \sum_{n=0}^{\infty} A_{n-1} ds, \\ v_n(x,t) &= e^{(t-t_0)(L_2(u_{n-1}(x,t),v_{n-1}(x,t)))} \\ &- \int_{t_0}^{t} e^{(t-s)(L_2(u_{n-1}(x,s),v_{n-1}(x,s))+h_2(x,s))} \sum_{n=0}^{\infty} A_{n-1} ds, \quad n \geq 1 \end{aligned} \tag{3.5}$$

After finding the values of the above integrals directly, we get values $u_1, v_1, u_2, v_2, u_3, v_3, ..., u_n, v_n$ respectively. Putting these values in (2.4) yields approximate solution of (1.1).

4. Numerical Example

To illustrate efficiency of the presented methods we solve the following example.

Example 4.1:

$$\begin{aligned} \frac{\partial u}{\partial t} &= d_1 \triangle u - u + Kv + u^2 v + \rho, \\ \frac{\partial v}{\partial t} &= d_2 \triangle v - Kv - u^2 v + \delta, \;\; t > 0, \;\; x \in \Omega \end{aligned} \tag{4.1}$$

with the initial conditions

$$\begin{aligned} U(x,0) &= Us + 0.01 sin(\frac{x}{L}), \text{ for } 0 \leq x \leq L \\ V(x,0) &= Vs - 0.12 sin(\frac{x}{L}), \text{ for } 0 \leq x \leq L \end{aligned} \tag{4.2}$$

$U(0,t) = Us, U(L,t) = Us \quad \text{and} \quad V(0,t) = Vs, V(L,t) = Vs.$

We will take

$d_1 = d_2 = 0.01, K = 0.5, \rho = \delta = 0.5, Us = 1, Vs = 0, L = 1.$

Solution:

i) **Using Laplace decomposition method:**

Applying Laplace transform on both sides of (4.1), we get:

$$su(x,s)-u(s,0)=l(0.01\frac{\partial^2 u}{\partial x^2})-u+0.5v+u^2v+0.5),$$
$$sv(x,s)-v(s,0)=l(0.01\frac{\partial^2 v}{\partial x^2}-0.5v-u^2v+0.5),$$

or

$$u(x,s)=\frac{u(s,0)}{s}+l(\frac{0.5}{s})+\frac{l(0.01\frac{\partial^2 u}{\partial x^2}-u+0.5v)}{s}+\frac{u^2v}{s},$$
$$v(x,s)=\frac{v(s,0)}{s}+l(\frac{0.5}{s})+\frac{l(0.01\frac{\partial^2 v}{\partial x^2}-0.5v)}{s}-\frac{u^2v}{s}.$$

The above system, by using initial conditions (4.2) becomes:

$$u(x,s)=\frac{0.01sin(\frac{x}{L})}{s}+l(\frac{0.5}{s})+\frac{l(0.01\frac{\partial^2 u}{\partial x^2}-u+0.5v)}{s}+\frac{u^2v}{s},$$
$$v(x,s)=\frac{1-0.12sin(\frac{x}{L})}{s}+l(\frac{0.5}{s})+\frac{l(0.01\frac{\partial^2 v}{\partial x^2}-0.5v)}{s}-\frac{u^2v}{s}, \tag{4.3}$$

From (4.3), by using Equations (2.10), (2.11) and inverse Laplace transform, we obtain:

$$u_0(x,t)=0.01sin(\pi x)+0.5t,$$
$$v_0(x,t)=1-0.12sin(\pi x)+0.5t, \tag{4.4}$$

and

$$u_n(x,t)=l^{-1}[\frac{l(0.01\frac{\partial^2 u_{n-1}}{\partial x^2}-u_{n-1}+0.5v_{n-1})}{s}]+l^{-1}[\frac{l(\sum_{n=0}^{\infty}A_{n-1})}{s}],$$
$$v_n(x,t)=l^{-1}[\frac{l(0.01\frac{\partial^2 v_{n-1}}{\partial x^2}-0.5v_{n-1})}{s}]-l^{-1}[\frac{l(\sum_{n=0}^{\infty}A_{n-1})}{s}],\ \ n\geq 1 \tag{4.5}$$

where A_n is Adomian polynomials [13] representing the nonlinear terms in the above system. The few components of Adomian polynomials and u_n and v_n, $n\geq 1$ are given as follow:

$$A_0 = \frac{1}{0!}[\frac{d^0}{d\lambda^0}N(\sum_{i=0}^{0}\lambda^i u_i,\sum_{i=0}^{0}\lambda^i v_i)]_{\lambda=0}$$
$$= N(u_0,v_0)=u_0^2v_0, \tag{4.6}$$

$$u_1(x,t) = l^{-1}[\frac{l(0.01\frac{\partial^2 u_0}{\partial x^2} - u_0 + 0.5v_0)}{s}] + l^{-1}[\frac{l(A_0)}{s}],$$
$$v_1(x,t) = l^{-1}[\frac{l(0.01\frac{\partial^2 v_0}{\partial x^2} - 0.5v_0)}{s}] - l^{-1}[\frac{l(A_0)}{s}], \tag{4.7}$$

Hence

$$\begin{aligned} u_1(x,t) &= l^{-1}[\frac{l(0.01\frac{\partial^2(0.01sin(\pi x)+0.5t)}{\partial x^2} - (0.01sin(\pi x)+0.5t) + 0.5(1-0.12sin(\pi x)+0.5t))}{s}] \\ &+ l^{-1}[\frac{l((0.01sin(\pi x)+0.5t)^2(1-0.12sin(\pi x)+0.5t))}{s}] \\ &= -0.03125t^4 - 0.0025sin(\pi x)t^3 + (-0.125 - 0.000075sin(\pi x)^2)t^2 \\ &+ (0.5 - 0.07098696044sin(\pi x) - 0.000001sin(\pi x)^3)t \\ &= U_1, \\ v_1(x,t) &= l^{-1}[\frac{l(0.01\frac{\partial^2(1-0.12sin(\pi x)+0.5t)}{\partial x^2} - 0.5(1-0.12sin(\pi x)+0.5t))}{s}] \\ &- l^{-1}[\frac{l((0.01sin(\pi x)+0.5t)^2(1-0.12sin(\pi x)+0.5t))}{s}] \\ &= 0.03125t^4 + (0.25 - 0.03sin(\pi x))t^3 + (0.0108sin(\pi x)^2 - 0.18sin(\pi x) + 0.625)t^2 \\ &+ (0.5 - 0.2881564747sin(\pi x) - 0.001728sin(\pi x)^3 + 0.0432sin(\pi x)^2)t \\ &= V_1, \end{aligned} \tag{4.8}$$

$$\begin{aligned} A_1 &= \frac{1}{1!}[\frac{d}{d\lambda}N(\sum_{i=0}^{0}\lambda^i u_i, \sum_{i=0}^{0}\lambda^i v_i)]_{\lambda=0} = \frac{d}{d\lambda}[N(u_0+\lambda u_1, v_0+\lambda v_1)]_{\lambda=0} \\ &= \frac{d}{d\lambda}[(u_0+\lambda u_1)^2(v_0+\lambda v_1)]_{\lambda=0} = 2u_0u_1v_0 + u_0^2 v_1, \end{aligned} \tag{4.9}$$

$$u_2(x,t) = l^{-1}[\frac{l(0.01\frac{\partial^2 u_1}{\partial x^2} - u_1 + 0.5v_1)}{s}] + l^{-1}[\frac{l(A_1)}{s}],$$
$$v_2(x,t) = l^{-1}[\frac{l(0.01\frac{\partial^2 v_1}{\partial x^2} - 0.5v_1)}{s}] - l^{-1}[\frac{l(A_1)}{s}], \tag{4.10}$$

Putting u_1, v_1 and A_1 in the above system, we get:

$$\begin{aligned}
u_2(x,t) &= l^{-1}[\frac{l(0.01\frac{\partial^2 U_1}{\partial x^2} - U_1 + 0.5V_1)}{s}] + l^{-1}[\frac{l(2u_0U_1v_0 + u_0^2V_1)}{s}] \\
&= 0.003348214286t^7 + 0.00046875sin(\pi x)t^6 + (0.000028125sin(\pi x)^2 \\
&+ 0.028125)t^5 + (0.01118424011sin(\pi x) + 0.0000009375sin(\pi x)^3 \\
&- 0.0625)t^4 + (0.002552304406sin(\pi x)^2 - 0.035sin(\pi x) + 0.1458333333 \\
&- 0.0000049348022cos(\pi x)^2 + 1.0000000075sin(\pi x)^4)t^3 + (-0.125 \\
&+ 0.000000000000004(-8260643094000 - 74022033cos(\pi x)sin(\pi x) \\
&+ 0.00000000015sin(\pi x)^5 + 0.010725sin(\pi x)^2 - 0.000420703912sin(\pi x)^3)t^2, \\
v_2(x,t) &= l^{-1}[\frac{l(0.01\frac{\partial^2 V_1}{\partial x^2} - 0.5V_1)}{s}] - l^{-1}[\frac{l(2u_0U_1v_0 + u_0^2V_1)}{s}] \\
&= 0.003348214286t^7 + (0.04687500000 - 0.005625000002sin(\pi x))t^6 \\
&+ (-0.06750000002sin(\pi x) + 0.004050000001sin(\pi x)^2 + 0.2593750000)t^5 \\
&+ (0.04050000001sin(\pi x)^2 - 0.001620000001sin(\pi x)^3 \\
&- 0.3082891186sin(\pi x) + 0.7187500000)t^4 + (1.020833333 + 0.0007106115176cos(\pi x)^2 \\
&- 0.012096sin(\pi x)^3 + 0.0003628800001sin(\pi x)^4 + 0.1382681655sin(\pi x)^2 \\
&- 0.6422347122sin(\pi x))t^3 + (0.6250000002 + 0.004263669105cos(\pi x)^2 \\
&- 0.02368035972sin(\pi x)^3 - 0.00000000000016(3197751827cos(\pi x)^2 + 3287347758000)sin(\pi x) \\
&- 0.3732480001e - 4sin(\pi x)^5 + 0.1642726618sin(\pi x)^2 + 0.0015552sin(\pi x)^4)t^2
\end{aligned} \tag{4.11}$$

and so on. Putting the values of $u_1, v_1, u_2, v_2, ...$ in (2.4) yields approximate solution of (1.1), see **Figure 1**.

ii) **Using Semigroup Approach:**

From (1.1), using (3.2), we get:

$$\begin{aligned}
u(x,t) &= e^{(t-0)(0.01\frac{\partial^2 u}{\partial x^2} - u + 0.5v + 0.5)}u_0(x) + \int_{t_0}^{t} e^{(t-s)(0.01\frac{\partial^2 u}{\partial x^2} - u + 0.5v + 0.5)}u^2 v ds, \\
v(x,t) &= e^{(t-0)(0.01\frac{\partial^2 v}{\partial x^2} - 0.5v + 0.5)}v_0(x) - \int_{t_0}^{t} e^{(t-s)(0.01\frac{\partial^2 v}{\partial x^2} - 0.5v + 0.5)}u^2 v ds,
\end{aligned} \tag{4.12}$$

From (3.4) and (3.5) we have

$$u_0(x,t) = e^{0.5t}0.01sin(\pi x),$$
$$v_0(x,t) = e^{0.5t}(1-0.12sin(\pi x)), \tag{4.13}$$

and

$$u_n(x,t) = e^{t(0.01\frac{\partial^2 u_{n-1}}{\partial x^2}-u_{n-1}+0.5v_{n-1})} + \int_{t_0}^{t} e^{t(0.01\frac{\partial^2 u_{n-1}}{\partial x^2}-u_{n-1}+0.5v_{n-1}+0.5)}A_{n-1}ds,$$
$$v_n(x,t) = e^{t(0.01\frac{\partial^2 v_{n-1}}{\partial x^2}-0.5v_{n-1})} - \int_{t_0}^{t} e^{t(0.01\frac{\partial^2 v_{n-1}}{\partial x^2}-0.5v_{n-1}+0.5)}A_{n-1}ds, \;\; n \geq 1. \tag{4.14}$$

The few components of u_n and v_n, $n \geq 1$ are given as follow:

For $n = 1$, putting A_0 in (4.14), we have:

$$u_1(x,t) = e^{t(0.01\frac{\partial^2 u_0}{\partial x^2}-u_0+0.5v_0)} + \int_{t_0}^{t} e^{t(0.01\frac{\partial^2 u_0}{\partial x^2}-u_0+0.5v_0+0.5)}u_0^2 v_0 ds,$$
$$= e^{t(-0.0001e^{0.5t}sin(\pi x)\pi^2-0.01e^{0.5t}sin(\pi x)+0.5e^{0.5t}(1-0.12*sin(\pi x))-0.000001(e^{0.5t})^3 sin(\pi x)^3)} - 0.5$$
$$+ \frac{e^t sin(\pi x)^3(e^{-0.07098696044e^{0.5t}tsin(\pi x)+0.5e^{0.5t}t-0.000001e^{1.5t}tsin(\pi x)^3}-1)}{70986.96044sin(\pi x) - 5000000000 + e^t sin(\pi x)^3}$$
$$= U_1$$
$$v_1(x,t) = e^{t(0.01\frac{\partial^2 v_0}{\partial x^2}-0.5v_0)} - \int_{t_0}^{t} e^{t(0.01\frac{\partial^2 v_0}{\partial x^2}-0.5v_0+0.5)}u_0^2 v_0 ds,$$
$$= e^{t(0.0012e^{0.5t}sin(\pi x)\pi^2-0.5e^{0.5t}(1-0.12sin(\pi x))+(e^{0.5t})^3(1-0.12sin(\pi x))^3-0.5}$$
$$+ \frac{g_1}{g_2} = V_1, \tag{4.15}$$

where

$$g_1 = 4e^t(-15625 + 5625sin(\pi x) - 675sin(\pi x)^2 + 27sin(\pi x)^3)$$
$$.e^{0.07184352528e^{0.5t}tsin(\pi x)-0.5e^{0.5t}t+e^{1.5t}t-0.36te^{1.5t}sin(\pi x)+0.0432e^{1.5t}tsin(\pi x)^2}$$
$$.e^{-0.001728e^{1.5t}tsin(3.141592654\pi x)^3)-1},$$

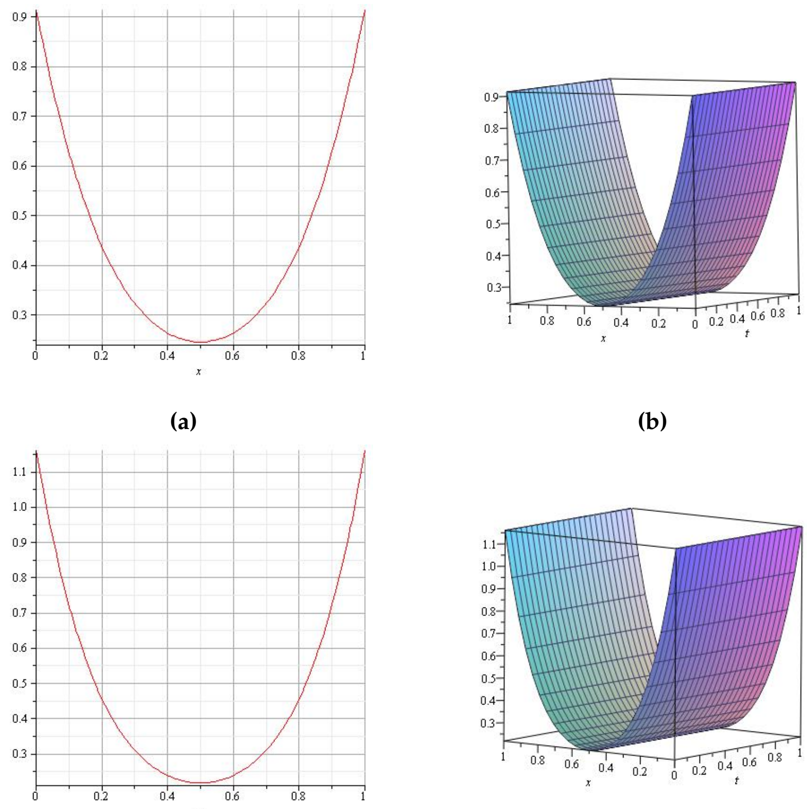
and

$$g_2 = -4490.220330sin(\pi x) + 31250 - 62500e^t + 22500e^t sin(\pi x)$$
$$- 2700e^t sin(\pi x)^2 + 108e^t sin(\pi x)^3.$$

For $n = 2$, putting A_1, U_1 and V_1 in (4.14), we obtain:

$$\begin{aligned} u_2(x,t) &= e^{t(0.01\frac{\partial^2 U_1}{\partial x^2} - U_1 + 0.5V_1)} + \int_{t_0}^{t} e^{t(0.01\frac{\partial^2 U_1}{\partial x^2} - U_1 + 0.5V_1 + 0.5)}(2u_0U_1v_0 + u_0^2V_1)ds, \\ v_2(x,t) &= e^{t(0.01\frac{\partial^2 V_1}{\partial x^2} - 0.5V_1)} - \int_{t_0}^{t} e^{t(0.01\frac{\partial^2 V_1}{\partial x^2} - 0.5V_1 + 0.5)}(2u_0U_1v_0 + u_0^2V_1)ds. \end{aligned} \tag{4.16}$$

and so on. Putting the values of $u_1, v_1, u_2, v_2, ...$ in (2.4) yields approximate solution of (1.1), see **Figure 2**.



(a) (b)

(c) (d)

Figure 1: *plots of results for solution of Example **(4.1)**, by means of Laplace Decomposition Method, with n=5.*

(a) *u(x,t) when, 0<x<1 and t=0.5.* ***(b)*** *u(x,t) when, 0<x<1 and 0<t<1.*

(c) *v(x,t) when, 0<x<1 and t=0.5.* ***(d)*** *v(x,t) when, 0<x<1 and 0<t<1.*

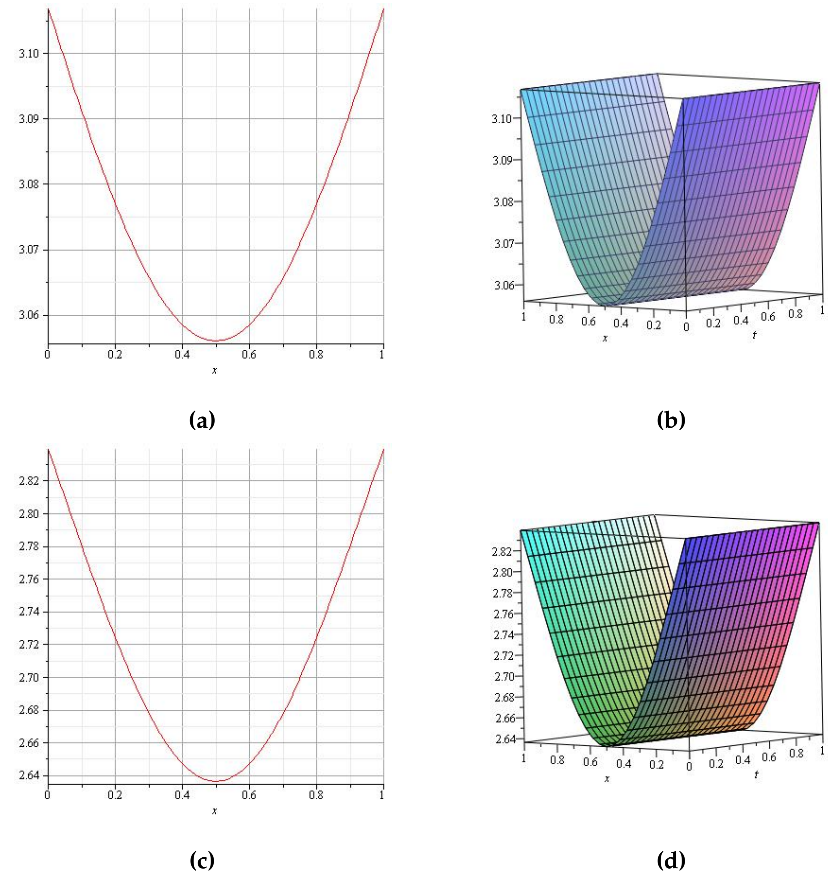


Figure 2: *plots of results for solution of Example* ***(4.1)****, by means of Simegroup decomposition approach, with n=2.*
(a) *u(x,t) when, 0<x<1 and t=0.5.* ***(b)*** *u(x,t) when, 0<x<1 and 0<t<1.*
(c) *v(x,t) when, 0<x<1 and t=0.5.* ***(d)*** *v(x,t) when, 0<x<1 and 0<t<1.*

5. **Conclusion:**

In this paper, we have successfully reformulate Laplace decomposition method to obtain approximate solution of Glycolysis system (1.1). We, also described a new method by combining semigroup approach and Adomian polynomial to obtain approximate solution of (1.1). We believe that the methods that we've used can also give exact solutions but since the exact solution of this system is unknown, we frankly cannot decide. The result that we have obtain are very close to the result that reference [10] has obtained. And that these methods are very efficient for solving Glycolysis system(1.1).

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