A new method for solving fuzzy linear fractional programs with Triangular Fuzzy numbers

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Abstract

Several methods currently exist for solving fuzzy linear and non-linear programming problems. In this paper an efficient method for FLFP has been proposed, in order to obtain the fuzzy optimal solution. This proposed method based on crisp linear programming and has a simple structure. It is easy to apply the proposed method compare to exiting method for solving FLFP problems in real life situations. To show the efficiency of our proposed method a numerical example has been illustrated with a practical problem.

Keywords : Fuzzy linear fractional programming (FLFP) problem, Triangular fuzzy numbers, Ranking function.

1. Introduction

Bellman and Zadeh [8] proposed the concept of decision making in fuzzy environment.The fuzzy linear programming problem is an attractive topics for researchers adopt this concept for solving [2, 9–11] problem.Many researchers [12, 13] adopted this method for solving fuzzy optimal solutions. Dehghan et al.[9] proposed a fuzzy linear programming approach for finding the exact solution of fuzzy linear system of equations. Lotfi et al. [10] proposed a method to obtain the approximate solution of fully fuzzy linear programming problems. In [3], a new model introduced the formulation of fuzzy linear programming(FLP) problem into crisp linear programming(CLP) problems with the using of symmetric trapezoidal fuzzy number. Fuzzy linear fractional programming(FLFP) problems deals with problems in which objective function is a ratio of two fuzzy linear functions. Maximizing the efficiency of an economic system leads to optimization problems whose objective function is a fuzzy ratio. Fuzzy linear fractional problems may be found in different fields such

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as data development analysis, tax programming, risk and portfolio theory, logistic and location theory [5–7, 14, 15]. Also, fuzzy linear fractional programming is used to achieve the highest ratio of outcome to cost, the ratio representing the highest efficiency.

Charnes and Cooper [1] proposed several methods for solving linear fractional program by transforming it to an equivalent linear program. Bitran and Novaes [19] considered updated objective functions method to solve linear fractional program by solving a sequence of linear programs whereas Dinkelbach [16] used parametric approach to solve a linear fractional programming problems. Later on, several authors extended the approach by Dinkelbach [16] to solve fractional programming problems, *e.g.*, generalized fractional programming problems [17**?**] and the minimum spanning tree with sum of ratios problems [**?**]. Almogy and Levin [18] extended the parametric approach of Dinkelbach [16] to solve sum of ratios problems. Falk and Palocsay [**?**] showed that the approach in [18] does not always lead to appropriate solutions and they extended the parametric approach of Dinkelbach to solve sum of ratios, product of ratios and product of linear functions in [**?**]. Tammer *et al.* [**?**] considered Dinkelbach approach to solve multiobjective linear fractional programming problems by estimating the parameters. However, their approach does not necessarily guarantee an efficient solution. Gomes *et al.* [**?**] focused on multiobjective linear programming problem having weights established some optimality conditions.

During recent years, complexity of problems arising in different fields prompted researchers to develop efficient algorithms to solve fuzzy linear fractional programs. Nidhal Dheyab [4] suggested an finding the optimal solution for fractional programming problems with fuzzy numbers.

In the present paper, we introduce a new type of fuzzy arithmetic for triangular numbers in which the coefficients of the objective function and the values of the right hand side were represented by triangular fuzzy numbers but variables are real numbers. Here, A new method is proposed to convert FLFP problem into crisp linear fractional programming (CLFP) problem. We used transformation technique to convert CLFP problem into CLP problem. A new method is proposed for finding optimal solutions with converting them to crisp linear programming problems. A new method is proposed for finding the fuzzy optimal solution of FLFP problems with inequality constraints. To illustrate the proposed method, a real life numerical examples are solved and the obtained results are discussed. In Section 2, we first give some necessary notations and definitions of fuzzy set theory for triangular fuzzy number. The formulation of FLFP problems and transform to FLP problem is given in Section 3. Results of this paper are presented in Section 4. Finally, conclusions are discussed in last section.

2. Notations and Definitions

Definition 2.1[11] Let X denote a universal set.Then a fuzzy subset $\widetilde{A}$ of $X$is defined by its membership function $\mu_{\widetilde{A}} : X \to [0, 1]$; which assigned a real number $\mu_{\widetilde{A}}(X)$ in the interval [0,1], to each element $x \in X$,where the value of $\mu_{\widetilde{A}}(X)$ at x shows the grade of membership of x in $\widetilde{A}$. A fuzzy subset $\widetilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\widetilde{A}}(X)$ and is often written $\widetilde{A}$=$(x, \mu_{\widetilde{A}}(X)) : x \in X$ is called a fuzzy set.

Definition 2.2[11] A fuzzy number $\widetilde{A}$ =(a, b, c) is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\widetilde{A}}(X) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b, \\ \frac{(x-c)}{(b-c)}, & b \leq x \leq c, \\ 0, & \text{else.} \end{cases}$$

Definition 2.3[2] A triangular fuzzy number (a, b, c) is said to be non-negative fuzzy number iff $a \geq 0$.
Definition 2.4[2] Two triangular fuzzy number $\widetilde{A}$=(a, b, c) and $\widetilde{B}$=$(, d, e, f)$ are said to be equal iff $a = d, b = e, c = f$.
Definition 2.5[2] A ranking is a function $R : F(R) \to R$, where F(R) is a set of fuzzy number defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\widetilde{A}$=(a, b, c) is a triangular fuzzy number then $R(\widetilde{A})$=$\frac{a+2b+c}{4}$.
Definition 2.6[11] Let $\widetilde{A}$=(a, b, c),$\widetilde{B}$=(d, e, f) be two triangular fuzzy number then:
i.$\widetilde{A} \oplus \widetilde{B}$=$(a, b, c) \oplus (d, e, f)$=$(a + d, b + e, c + f)$,
ii.$\widetilde{A} \ominus \widetilde{B}$=$(a, b, c) \ominus (d, e, f)$=$(a - f, b - e, c - d)$,
iii.Let $\widetilde{A}$=(a, b, c) be any triangular fuzzy number and $\widetilde{B}$ =(d, e, f) be a non negative triangular fuzzy number then

$$\widetilde{A} \otimes \widetilde{B} = \begin{cases} \text{(ad,be,cf)}, & a \geq 0 \\ \text{(af,be,cf)}, & a < 0, c \geq 0 \\ \text{(af,be,cd)}, & c < 0 \end{cases}$$

3. Fuzzy linear fractional programming problem

We proposed a new model for solving FLFP problems which the coefficients of the objective functions and the values of the right hand sides were represented by triangular fuzzy numbers but the left hand sides variables are real numbers.This problem can be consider the following FLFP problems.

$Max\ \widetilde{z}$= $\frac{\widetilde{c}^t\ x \oplus\ \widetilde{\alpha}}{\widetilde{d}^t\ x \oplus\ \widetilde{\beta}}$
$s.t \qquad A \otimes x \leq \widetilde{b}$
$\qquad\quad x \geq\ 0.$

where A=$(A_1, A_2, ..., A_n)$ is an m by n matrix; $\widetilde{b}$ is a column vector with m-components; $\tilde{\alpha}$ and $\tilde{\beta}$ are the scalars.Here all the parameters $\widetilde{c}$, $\widetilde{d}$, $\widetilde{x}$ are triangular fuzzy number.

First, convert fuzzy linear fractional programming problem into the following CLFP problem using ranking function of triangular fuzzy number.

$Max\ z =\ \frac{c^t\ x+\ \alpha}{d^t\ x+\ \beta}$
$s.t \qquad A \times x \leq b$
$\qquad\quad x \geq\ 0.$

Now convert the above CLFP problem in to CLP problem by using Charnes-Cooper transformation method:
$Max\ z = c^t\ y +\ \alpha\ t$

$s.t \quad Ay - b\ t \leq 0$
$d^t y + \beta\ t = 1$
$y \geq 0, t \geq 0$

In the next section we find the solution with the help of crisp linear programming(CLP) problem.

4. Proposed method to find fuzzy optimal solution of FLFP problem

In this section, we find that the fuzzy solution of the FLFP problem with the help of crisp linear programming(CLFP) problem. We use ranking function of each fuzzy number in the fuzzy problem under consideration, which leads to an equivalent CLP problem and it solved with a standard method.Using all arithmetic operation are performed on fuzzy numbers while in our proposed method, all arithmetic operations are also used by crisp numbers.

The steps of the proposed method are as follows:

Step 1: $Max\ \widetilde{z} = \frac{\widetilde{c}^t\ x \oplus\ \tilde{\alpha}}{\widetilde{d}^t\ x \oplus\ \tilde{\beta}}$

$s.t \quad A \otimes x \leq \widetilde{b},$
$x \geq\ 0.$

x is non-negative fuzzy number.

Step 2: Convert fuzzy linear fractional programming problem into the following CLFP problem using ranking function of triangular fuzzy number.

$Max\ z = \frac{c^t\ x +\ \alpha}{d^t\ x +\ \beta}$

$s.t \quad A \times x \leq b$
$x \geq\ 0.$

x is non-negative fuzzy number.

Step 3: Convert CLFP problems in to CLP problems by using Charnes-Cooper transformation method

$Max\ z = c^t\ y +\ \alpha\ t$

$s.t \quad Ay - b\ t \leq 0$
$d^t y + \beta\ t = 1$
$y \geq 0, t \geq 0$

x is non-negative fuzzy number.

Step 4: Find the optimal solution y in step 3.

Step 5: Find the optimal solution x using the value y in step 3.

Step 6: Find the fuzzy optimal value.

Next, we solve an example by our methods.

4. Example (Real life Application)

In this section, we applied our proposed method using a simple example and a real life problem.Also a mathematical programming solver Lingo will be used to solve the following mathematical problems.

Example 4.1.

In a tea stall in India the owner produces two types of Tea i.e. Type A is Red tea and Type B is Milk tea. Each Tea of the Type A requires twice as much time as the Type B. If all Teas are of the type B only, the owner produce a total (12,20,28) Teas a day. The market limits daily sales of the Type A and type B to (4,7,10) and (8,10,12) Teas. Assuming that the profits per teas are

Rs (1,2,3) for Type A and Rs (0,1,2) for Type B. However, the cost for each teas of Type A is Rs (0,1,2) and type B is Rs (1,2,3). Consider that a fixed cost of around (0,1,2) is added. Determine the number of teas to be produced of each type so as to maximize the profit.

Solution

In this case, let x_1 and x_2 to be the units of Type A and Type B to be produced. Then the above problem can be formulated as:

$max\ \ \widetilde{z} = \frac{(1,2,3)x_1 \oplus (0,1,2)x_2}{(0,1,2)x_1 \oplus (1,2,3)x_2 \oplus (0,1,2)}$

$s.t \quad 2x_1 + x_2 \leq\ (12, 20, 28),$

$x_1 \leq\ (4, 7, 10),$

$x_2 \leq\ (8, 10, 12),$

$x_1, x_2 \geq\ 0.$

We first substitute the ranking function of each fuzzy number for corresponding fuzzy problem, which leads to an equivalent CLFP problem:

$max\ z = \frac{2x_1 + x_2}{x_1 + 2x_2 + 1}$

$s.t\ \ 2x_1 + x_2 \leq 20,$

$x_1 \leq 7,$

$x_2 \leq 10,$

$x_1, x_2 \geq\ 0.$

Now we transform this CLFP problem into CLP problem by using charnes cooper transformation method:

$max\ z =\ 2y_1 + y_2$

$s.t\ \ \ y_1 + 2y_2 + t =\ 1,$

$2y_1 + y_2 - 20t \leq\ 0,$

$y_1 - 7t \leq\ 0,$

$y_2 - 10t \leq\ 0,$

$y_1, y_2, t \geq\ 0.$

we construct the standard form of crisp linear programming problem:

$max\ z =\ 2y_1 + y_2$

$s.t\ \ \ y_1 + 2y_2 + t =\ 1,$

$2y_1 + y_2 - 20t + y_4 = 0,$

$y_1 - 7t + y_5 =\ 0,$

$y_2 - 10t + y_6 =\ 0,$

$y_1, y_2,\ \ y_4,\ \ y_5,\ \ y_6,\ \ t \geq\ 0.$

This problem is CLP problem and solved by simplex method.

We find $y_1 = 0.875$, $y_2 = 0$ and $t = 0.125$.

Thus using step 4, we have $x_1 = \frac{y_1}{t} = 7, x_2 = \frac{y_2}{t} = 0.$

In step 5, using these values we solves the objective function value, $\widetilde{z} = 1.75.$

Thus x_1 and x_2 fuzzy numbers approximate very closely the numbers 6.8 and 0.3 respectively. Also the triangular fuzzy number Z approximate very closely the real number $\frac{13.9}{8.4}$ which represents the optimal value of the problem.

Here, our optimal value of objective functions approximates very well close to the original result, which is exactly the optimal value of the problem which we proposed with.

5.Conclusions

In the fuzzy linear fractional programming (FLFP) problem proposed in this study, the coefficients of the objective function and the value of right hand side are triangular fuzzy number and rest of the part is real number with inequality constraints occurring in practical problem.The FLFP problem is converted into a equivalent crisp linear fractional programming (CLFP) problem and transform into crisp linear programming (CLP) problem.This CLP problem is solved by simplex method. We showed that the method proposed in this paper is very effective. Some possible future work also be done based on this paper, further proposed method can be extended for fully fuzzy linear fractional programming problem. To illustrate the proposed method two numerical applications one of real life applications and one example are solved. We also shows the efficient of proposed method via above graph.

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