A Numerical Investigation Of The Significance Of Non-Dimensional Numbers On The Oscillating Flow Characteristics Of A Closed Loop Pulsating Heat Pipe

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Abstract

Pulsating Heat Pipe (PHP) is a two phase passive heat transfer device for low temperature applications. Even though it is a simple, flexible and cheap structure, its complex physics has not been fully understood and requires a robust, validated simulation tool. In the present work the basic theoretical model by H.B. Ma et al[11] has been updated with the inclusion of capillary forces in order to characterise the pulsating flow under the influence of various non dimensional quantities. The mathematical model is solved using explicit embedded Range-kutta method and proved that the Poiseuille number considered in the numerical analysis assumes more significance since it includes flow characteristics, geometry and fluid properties of a PHP.

Keywords : Pulsating heat pipe (PHP), slug flow, Mathematical modelling, Non -dimensional numbers.

1 Introduction

There has been a revolutionisation of Nanotechnology in the recent past with miniaturisation of electronic systems with high power utility coupled with high heat fluxes has been increased exponentially and hence it has necessitated to develop promising cooling technologies for their

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continuous operation. There have been many conventional cooling methods developed so far but none could match cooling requirements of micro electronics systems due to the reasons like space constraint and difficulties involved with adaptation of liquid jet etc. In line with the development of suitable cooling techniques for micro electronic systems, pulsating heat pipes (PHPs) have become effective entrants , in the family of passive two phase heat transfer systems. PHP was first proposed and patented by Akachi[1] of capillary dimensions filled with certain working fluid under vacuum. PHP works on the principle of fluid pressure oscillations that are created by means of differential pressure across vapour plugs from evaporator to condenser and back when the tube dimensions are maintained below the critical diameter. If the diameter of a PHP is less than its critical diameter, capillary slug flow exists otherwise, flow gets stratified. The critical diameter[2, 3] is the maximum inner diameter causing the capillary flow and is defined as

$$D_{crit} = 2\sqrt{\frac{\rho}{g(\rho_v - \rho_l}} \tag{1}$$

Basically, PHP is a simple meandering tube system, But it offers innumerable challenges in understanding of its phenomena and theory. Hence it has attracted the attention of researchers to derive the complete design rules and optimization procedures of PHPs. In the recent past, many attempts on PHPs have been carried out on theoretical modelling.

Xin-She Yang et al[4] developed a mathematical model for simple 1-D flow and predicted the start up characteristics of a Pulsating Heat Pipe. The model was solved using Finite difference method and could produce most of the known features of the system.

Moeuro Mameli[5] proposed an updated version of the flow from the basic numerical model by Holly and Faghri to make it suitable for variety of working fluids. The numerical results have the good agreement with experimental data obtained from the single loop PHP operating with ethanol.

Moeuro Mameli et al[6] adopted the separated flow model of single loop PHP by Holly and Faghri and has been derived an improvised numerical model with updated heat transfer correlations. The updated model could successfully predict the flow and heat transfer characteristics at all orientations of gravity fields.

Peng Cheng et al[7] developed a mathematical model and studied the oscillating motion in an OHP under influence of fill ratio operating temperature gravity effect and temperature difference between the evaporator and condenser. The mathematical model was considered to be the system of multi-degree oscillation of vapour bubbles and liquid slugs where vapour bubbles play a role similar to linear springs.

Shafi et al[8] developed a theoretical model to simulate the pulsating flow in both closed and open loop PHPs of two turns. The model solves for the pressure, temperature, plug location and heat transfer rates. An important conclusion drawn here is that majority of heat transfer(95%) is due to sensible heat for the slug flow mode of operation, but not due to latent heat of vaporisation. Latent heat serves to establish the pressure pulsations only.

W.Qu et al[9] conducted a theoretical analysis in order to determine primary factors affecting the start up characteristics of a PHP. Analytical relations were derived to estimate the super heat and heat flow level cavity size on the capillary inner surface and shape of the vapour bubbles formed considered to be which are critical factors for the start up of oscillating motion in PHP.

M.B. Shafi et al[10] developed an advanced heat transfer model for both looped and un looped pulsating heat pipes with multiple liquid slugs and vapour plugs considered. Parametric investigation of vapour plugs and liquid slugs in a PHP has been analysed with the incorporation of their film evaporation and condensation. The results show that the heat transfer in both types of PHPs is also primarily due to exchange of sensible heat.

In the present work, H.B. Ma et al[11] analytical model to describe the oscillating motion in a single loop PHP is taken as a base and updated it with the inclusion of capillary force in order to know the effect of non dimensional numbers on the flow and heat transfer characteristics of a pulsating flow. In his model, force balance was performed on a liquid slug by relating the pressure difference between the evaporator and condenser with temperature difference as driving potential using clasius-clapeyrons equation.

2 Theoretical model of the present work

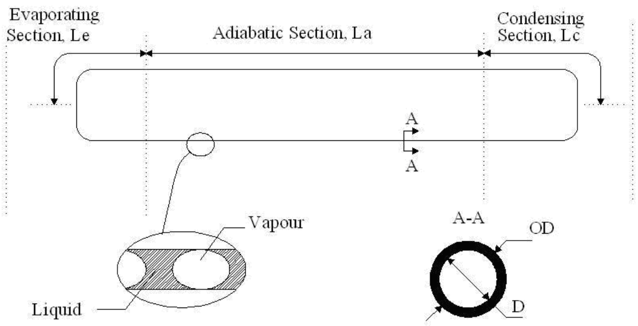

Figure 1: PHP schematic diagram considered for analysis

In the present work, an investigation of non dimensional numbers on the flow characteristics of a single closed loop PHP has been carried out through a numerical study. Most of the ongoing research works on modelling have focused on slug flow since slug flow is the primary flow pattern in PHPs.The present work adopts the mathematical model proposed by Ma et. al[11] and investigated the slug flow characteristics of PHP with the inclusion of capillary forces. The model is solved for various geometrical and operating characteristics using MATLAB with the nomenclature ODE 45[12] for the estimation of slug displacement and slug velocity. Figure 1 shows the physical model used for obtaining the governing differential equation.

The characteristic length of PHP consists of the length of evaporator section L_e, adiabatic section L_a and condenser section L_c. Hence,

$$L = L_e + L_a + L_c \tag{2}$$

As the fluid flows in a PHP, the fluid gets evaporated in the evaporator and condensed in the condenser and this results in the volume expansion and contraction of the bubbles. This causes an oscillating motion which affects the saturation temperature in the evaporator and condenser section. If the maximum and minimum temperature difference between the evaporator and condenser section are ΔT_{max} and ΔT_{min} respectively, then the temperature difference between the evaporator and condenser section will vary between ΔT_{max} and ΔT_{min} and is given by $\frac{\Delta T_{max}+\Delta T_{min}}{2}$. Considering the oscillating nature of PHP and system oscillation frequency as ω, the thermal driving potential can be expressed as

$$\Delta T = \frac{\Delta T_{max} + \Delta T_{min}}{2}[1 + \cos(\omega t)] \tag{3}$$

The pressure difference between evaporator and condenser can be related to the thermal driving potential using Clausius Clapeyron's equation.

$$\Delta p = \Delta T \frac{h_{fg}\rho_v}{T_e} \tag{4}$$

Thus the driving force causing the pulsating motion in a PHP is expressed as

$$F_d = \left(\frac{A h_{fg}\rho_v}{T_e}\right)\left(\frac{\Delta T_{max} + \Delta T_{min}}{2}\right)[1 + \cos(\omega t)] \tag{5}$$

This driving force overcomes the (i) viscous force which arises due to the interaction between liquid/vapour and the pipe walls (ii) force due to vapour pressure which arise due to volume contraction and expansion of bubbles and (iii) force due to inertia (iv) capillary force due to meandering tube diameter. From Newtons law, the governing equation for fluid flow in a PHP is

$$\begin{aligned}(\rho_l L_l + \rho_v L_v)A\frac{d^2x}{dt^2} + \left[32\left(\frac{\mu_l L_l + \mu_v L_v}{D^2}\right)\right]A\frac{dx}{dt} + \frac{A\rho_v R_g T}{L_v}x + N\frac{[2\sigma(-\cos\theta_A + \cos\theta_R)]A}{R} \\ = \left(\frac{A h_{fg}\rho_v}{T_e}\right)\left(\frac{\Delta T_{max} + \Delta T_{min}}{2}\right)[1 + \cos(\omega t)]\end{aligned} \tag{6}$$

The inertia force, the viscous force and the force due to vapour pressure in the above equation are given by

$$F_i = (\rho_l L_l + \rho_v L_v)A\frac{d^2x}{dt^2} \tag{7}$$

$$F_{vi} = \left[32 \left(\frac{\mu_l L_l + \mu_v L_v}{D^2} \right) \right] A \frac{dx}{dt} \tag{8}$$

$$F_{vp} = \frac{A \rho_v R_g T}{L_v} x \tag{9}$$

$$F_{cap} = N \frac{\left[2\sigma \left(-\cos\theta_A + \cos\theta_R \right) \right] A}{R} \tag{10}$$

The governing equation (6) for fluid flow in a PHP is same as that of the governing equation of forced damped mechanical vibration with the following initial conditions $x = 0$ and $\frac{dx}{dt} = 0$ at $t = 0$.

3 Results and Discussion: Significance of Non-Dimensional Numbers in PHP

3.1 Poiseuille Number

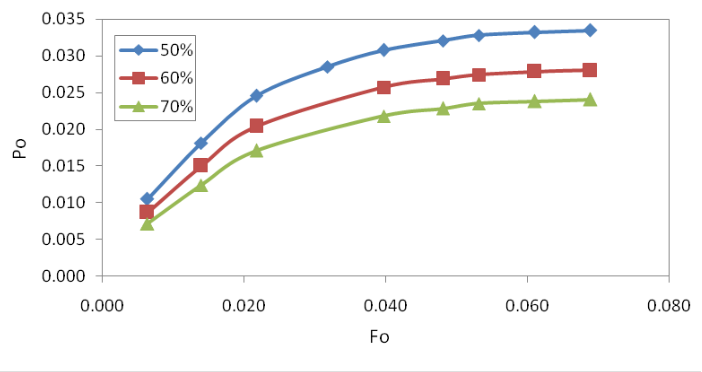

Figure 2: Effect of fill ratio on Poiseuille number (water, L=304.8mm,T= 60° C, D= 1.65mm,ΔT=5K)

The Poiseuille number is basically defined as the product of Reynolds number and friction factor. Poiseuille Number is one of the important non dimensional numbers defined in micro and mini channel flows with low values of Reynolds Number. The flow in a PHP is considered as a mini channel flow. That is the reason the analysis of oscillatory flow in a PHP with Poiseuille number is significant.

It is observed that in mini and micro channels, Poiseuille number is frequently used to study the effect of friction for single phase flows. Higher the Poiseuille number, higher is the velocity of

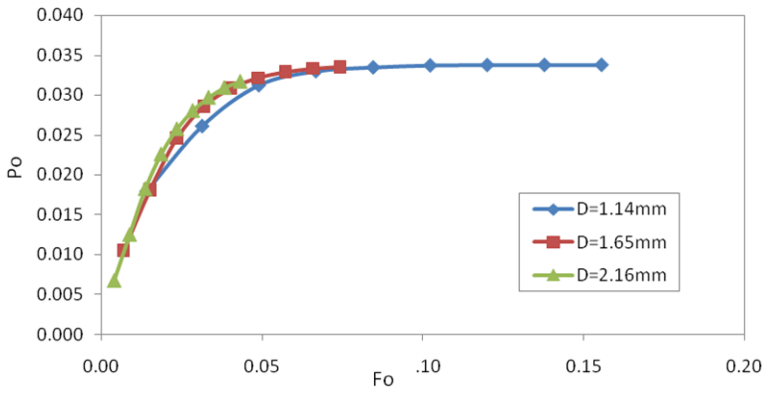

Figure 3: Effect of diameter on Poiseuille number(water, L=304.8mm,T= 60° C, $\phi = 50\%$, ΔT=5K)

the slug resulting in higher momentum transfer and better heat transport. The Poiseuille number is given by [2]

$$P_O = \frac{V_s \mu_l}{D^2 g(\rho_l - \rho_v)} \tag{11}$$

Considering the above equation, Poiseuille number is considered to be one of the important non dimensional numbers in the design of PHP because it includes flow characteristics, geometry and fluid characteristics of a PHP.

The Poiseuille number is determined from Equation (11) based on the RMS values of velocity of each cycle and plotted with respect to Fourier number which is a representation of non dimensional time. The Fourier number for a fluid flow is given by

$$F_O = \frac{\nu t}{D^2} \tag{12}$$

Figure 2 shows the variation of Poiseuille number with respect to Fourier number for different fill ratio with water as the working fluid for diameter of 1.65mm and operating temperature of 60° C. The Poiseuille number increases with increase in non dimensional time due to increase in the slug velocity . As the fill ratio increases, the displacement and velocity of the slug becomes smaller. Hence higher values of Poiseuille number are observed at lower fill ratio indicating better momentum transfer at lower fill ratio.

The effect of diameter on Poiseuille number with respect to Fourier number for water with fill ratio of 50% and operating temperature of 60° C is highlighted in Figure 3. The Poiseuille number increases with increase in Fourier number initially but reaches a value of 0.034 for Fourier number beyond 0.3. Hence, Poiseuille number of 0.034 can be used as the design parameter of PHP. Further it is evident from Figure 3 that Poiseuille number is independent of diameter in steady state.

3.2 Capilary Number

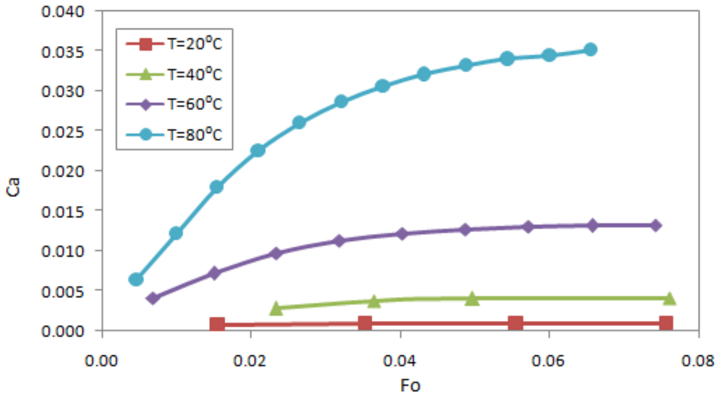

Figure 4: Effect of Operating Temperature on Capillary number (water, L=304.8mm, D=1.65mm, $\phi = 50\%$

The study of non-dimensional quantity such as Capillary number in PHP is highly recommended considering the fact that the smaller diameter tubes used in PHP contributes the larger significance of capillary forces. Capillary number represents the ratio of relative effect of viscous forces to surface tension acting across an interface between a liquid and a gas or between two immiscible liquids. It is defined as

$$Ca = \frac{\mu_l V_s}{\sigma} \tag{13}$$

Figure 4 shows the effect of operating temperature on the Capillary number with water as the fluid at diameter of 1.65 mm and fill ratio of 50%.It has been observed that the effect of capillary forces on the oscillatory motion of PHP reduces with increase in the operating temperature. This is due to increase in the thermal energy with increase in operating temperature results in higher slug velocity and higher Capillary number

Figure 5 shows the variation of Capillary number for different working fluids at fill ratio of 50%, diameter of 1.65mm and operating temperature of 60° C. Very low values of capillary number are observed in case of water and methanol. This is because of lower slug velocity and lower viscosity of these fluids. This is indicative to the fact that the capillary forces are higher in water as compared to acetone.

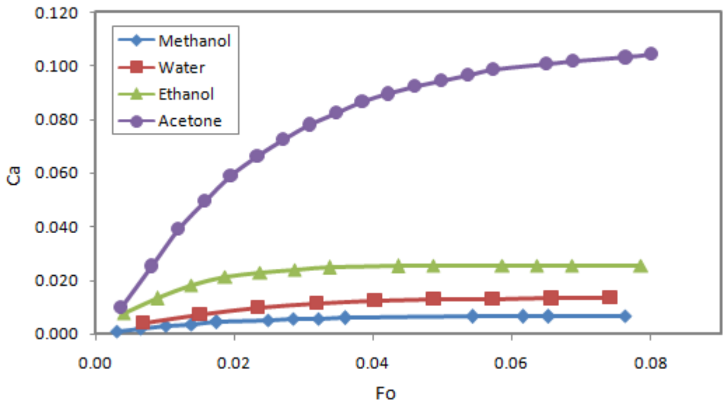

Figure 5: Effect of Working fluid on Capillary number (water, L=304.8mm, T=60° C, D=1.65mm, Δ T=5k)

3.3 Eckert Number

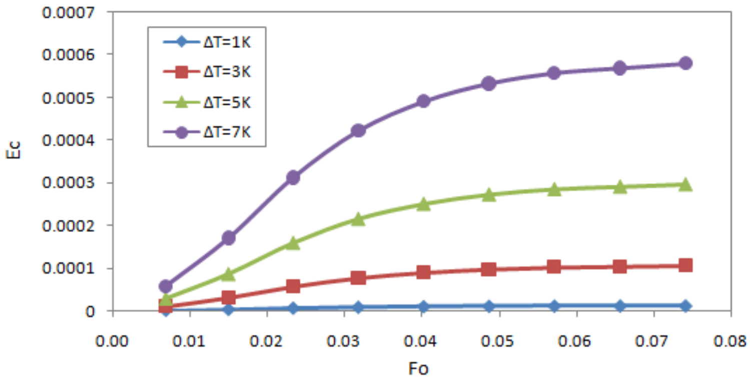

Figure 6: Effect of Temperature difference on Eckert number (water, L=304.8mm, T=60° C, D=1.65mm, $\phi = 50\%$)

A non dimensional number which characterizes the velocity of the slug and the temperature difference between the evaporator and the condenser becomes significant because there is a continuous evaporation and condensation of the working fluid in a PHP. Such a dimensionless number is Eckert number. It gives the relationship between the kinetic energy of the fluid and enthalpy and this is used to characterize dissipation. It is defined as

$$E_C = \frac{{V_s}^2}{\left(C_{pl} + C_{pv}\right)\Delta T} \tag{14}$$

The effect of temperature difference between evaporator and condenser on Eckert number for water at fill ratio of 50%, diameter of 1.65 mm and operating temperature of 60° C is shown in Figure 6. It is evident from the figure that high temperature difference between evaporator and condenser results in higher values of Eckert number.

4 Conclusions

The following conclusions have been made from the numerical study

- RMS values of slug velocities are used for determining the non dimensional numbers]
- Higher values of slug velocities are observed in case of acetone. This is the result of higher values of non dimensional numbers for acetone compared to other fluids. This shows that acetone exhibits better fluid flow characteristics of PHP compared to other fluids.
- The Poiseuille number considered in the numerical study assumes greater significance since it includes flow characteristics, geometry and fluid properties of a PHP.
- Higher values of Poisuille number are observed at lower fill ratio and higher operating temperature indicating better fluid flow characteristics of PHP.
- The effect of capillary forces are found to be significant at higher fill ratio, lower diameter, lower operating temperature and lower temperature difference between evaporator and condenser.
- Very low values of Eckert numbers have been observed in the case of water and methanol compared to acetone.

References

[1] H. Akachi, “Structure of a heat pipe,” May 1 1990, uS Patent 4,921,041.

[2] P. Charoensawan, S. Khandekar, M. Groll, and P. Terdtoon, “Closed loop pulsating heat pipes: Part a: parametric experimental investigations,” *Applied Thermal Engineering*, vol. 23, no. 16, pp. 2009 – 2020, 2003. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1359431103001595

[3] S. Khandekar, “Thermo-hydrodynamics of closed loop pulsating heat pipes,” Ph.D. dissertation, Universitt Stuttgart, Holzgartenstr. 16, 70174 Stuttgart, 2004. [Online]. Available: http://elib.uni-stuttgart.de/opus/volltexte/2004/1939

[4] X.-S. Yang and T. Luan, "Modelling of a pulsating heat pipe and startup asymptotics," *Procedia Computer Science*, vol. 9, no. 0, pp. 784 – 791, 2012, proceedings of the International Conference on Computational Science, {ICCS} 2012. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1877050912002050

[5] M. Mameli, M. Marengo, and S. Zinna, "Thermal simulation of a pulsating heat pipe: effects of different liquid properties on a simple geometry," *Heat Transfer Engineering*, vol. 33, no. 14, pp. 1177–1187, 2012.

[6] M. Mameli, M. Marengo, and S. Zinna, "Numerical investigation of the effects of orientation and gravity in a closed loop pulsating heat pipe," *Microgravity Science and Technology*, vol. 24, no. 2, pp. 79–92, 2012. [Online]. Available: http://dx.doi.org/10.1007/s12217-011-9293-2

[7] P. Cheng and H. Ma, "A mathematical model of an oscillating heat pipe," *Heat Transfer Engineering*, vol. 32, no. 11-12, pp. 1037–1046, 2011.

[8] M. B. Shafii, A. Faghri, and Y. Zhang, "Thermal modeling of unlooped and looped pulsating heat pipes," *Journal of Heat Transfer*, vol. 123, no. 6, pp. 1159–1172, 2001.

[9] W. Qu and H. Ma, "Theoretical analysis of startup of a pulsating heat pipe," *International Journal of Heat and Mass Transfer*, vol. 50, no. 1112, pp. 2309 – 2316, 2007. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0017931006006028

[10] M. Shafii, A. Faghri, and Y. Zhang, "Analysis of heat transfer in unlooped and looped pulsating heat pipes," *International Journal of Numerical Methods for Heat & Fluid Flow*, vol. 12, no. 5, pp. 585–609, 2002.

[11] H. Ma, M. Hanlon, and C. Chen, "An investigation of oscillating motions in a miniature pulsating heat pipe," *Microfluidics and Nanofluidics*, vol. 2, no. 2, pp. 171–179, 2006. [Online]. Available: http://dx.doi.org/10.1007/s10404-005-0061-8

[12] R. Ashino, M. Nagase, and R. Vaillancourt, "Behind and beyond the matlab {ODE} suite," *Computers & Mathematics with Applications*, vol. 40, no. 45, pp. 491 – 512, 2000. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0898122100001759