Invited paper

International Journal of Mechanical Engineering (IJME)
Volume 4 Issue 3 (Year : 2014) ISSN : 2277-7059
http://www.ijmejournal.com/

Tuning of a Feedforward Lag-Lead Second-Order Compensator used with a Highly Oscillating Second-order Process

Galal A. Hassaan

Emeritus Professor, Department of Mechanical Design & Production,
Faculty of Engineering, Cairo University, Giza, EGYPT
galalhassaan@ymail.com

Abstract

Lag-lead compensators are well known in automatic control engineering. They have 4 parameters to be adjusted (tuned) for proper operation. The frequency response of the control system or the root locus plot are traditionally used to tune the compensator in a lengthy procedure.

A highly oscillating second-order process has a time response to a unit step input of 85.4 % maximum overshoot and about 6 seconds settling time is controlled using a lag-lead compensator (through simulation). The lag-lead compensator is tuned by minimizing the sum of time multiplied by the absolute error (ITAE) of the control system using MATLAB. Three functional constrains are used to control the performance of the lag-lead compensated control system. The result was reducing the process oscillation to 6.926 % overshoot and a 1.413 seconds settling time. The steady-state characteristics of closed-loop control system using the lag-lead compensator were excellent. It is possible reduce the steady-state error to any desired value through one of the compensator parameters.

Keywords : Feedforward Compensators , Lag-lead Second-order Compensator , Compensator Tuning, Control System Performance..

1 Introduction

Lag-lead compensators can improve the performance of linear control systems through the proper tuning of the compensator parameters. There are two schools in designing lag-lead compensators. One of them uses the frequency response specifications of the compensated control system. The other uses its time response specifications. In the present work we follow the research school using the time response specifications. Still the subject is interested to automatic control researchers.

James, Frederick and Taylor (1987) discussed the application of expert system technique to the design of lead-lag compensators for linear SISO systems [1]. Loh, Cai and Tan (2004) studied the auto-tuning of phase lead-lag compensators using the frequency response of the plant using relays with hysteresis [2]. Chang (2004) used phase-lag and phase-lead compensators to control servo control systems [3]. Wang (2006) developed a non-trial-and-error procedure to design lag-lead compensators based on the idea of Yeung-Wang-Chen's graphical-based non-trial-and –error method for 3-parameters lag-lead compensators and Wang's result on the exact and unique solution for single lag and lead compensator design [4]. Zhang, Liu, Dang, Zhang and Ou (2006) used a lag-lead compensator to control asynchronous linear motors for better performance and application to active mass driver control system for vibration control of civil engineering structures [5].

Panda and Padhy (2007) used the genetic algorithm optimization technique to design thyristor controlled series compensator-based controller to enhance the power system stability [6]. Nassirharand (2008) developed an educational software utility for designing linear compensators based on the Youla parameterization technique and an exact model matching criterion [7]. Wang (2009) provided an approach for phase-lead/lag compensators to achieve the desired specifications of gain and phase margins for all-pole stable plant with time-delay [8]. Cao, Watkins and O'Brien (2009) discussed a compensator graphical user interface implemented by MATLAB to enable the user to design a continuous time compensator using the root-locus and Bode plot [9]. Li, Sheng and Chen (2010) derived the impulse response of the distributed order lead-lag compensator using MATLAB and compared with the numerical inverse Laplace Transformation method [10].

Zanasi and Cuoghi (2011) presented three different methods for the synthesis of lead-lag compensator meeting the phase margin and the gain crossover frequency [11]. Nandar (2012) proposed a robot power system stabilizer using genetic algorithm and a first-order lead-lag compensator [12]. Ntogramatzidis, Zanasi and Cuoghi (2012) presented a range of design techniques for the analysis of standard compensators (lead, lag, PID) in terms of the steady-state performance, the stability margins and the crossover frequencies [13]. Hassaan, Al-Gamil and Lashin (2013) used a second-order feedforward lag-lead compensator to control a difficult process consisting of a first-order with integrator process in a unity feedback loop having 67.3 % maximum percentage overshoot and a 12 seconds settling time. Through tuning the compensator, they succeeded to reduce the overshoot to only 2.4 % and the settling time to only 0.65 seconds [14]. Rodriguez, Guzman , Brenenguel and Hagglund (2014) presented a shaping design procedure with rules to obtain optimal feedforward controller for the case when the ideal compensator is not realizable due to right-half plane zeros in the process dynamics [15].

2 Analysis

Process:

The process is a second-order process having the transfer function:

$$G_p(s) = \omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \qquad (1)$$

The process has a 10 rad/s natural frequency and an 0.05 damping ratio. It has a time response to a unit step input shown in Fig.1 as generated by MATLAB:

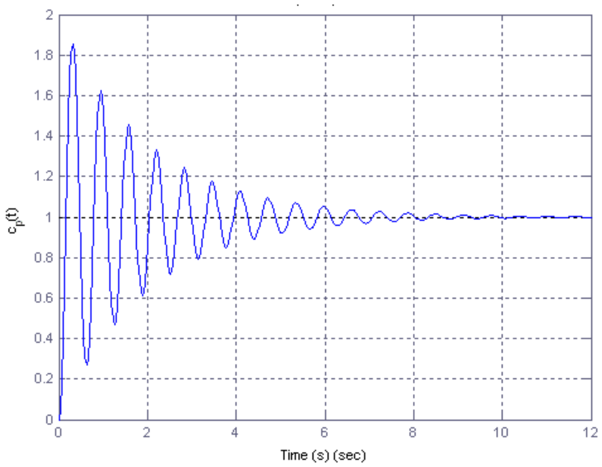

Fig.1 Step response of the process to a unit step input.

The severity of the output oscillation is measured by its maximum percentage overshoot. It has a maximum overshoot of 85.4 % and about 6 seconds settling time.

Lag-lead compensator:

A lag-lead compensator has 4 parameters [16]:

- First time constant, T_1.
- Second time constant, T_2.
- Time constant parameter, b.
- Gain, K_c used in controlling the steady state error of the closed loop control system.

The lag-lead compensator has a transfer function $G_c(s)$ given by [16]:

$$G_c(s) = K_c(1 + T_1s)(1+T_2s) / [(1 + T_1's)(1+T_2's)] \tag{2}$$

Where: $T_1' = (1/b)T_1$
$T_2' = bT_2$

Control System Transfer Function:

Assuming that the control system is a unit feedback one, its transfer function with $G_c(s)$ and $G_p(s)$ in its forward path is:

$$M(s) = (b_0s^2 + b_1s + b_2) / (a_0s^4 + a_1s^3 + a_2s^2 + a_3s + a_4) \tag{3}$$

where:

$b_0 = \omega_n^2K_cT_1T_2$

$b_1 = \omega_n^2K_c(T_1 + T_2)$

$b_2 = \omega_n^2K_c$

$a_0 = T_1'T_2'$

$a_1= T_1'+T_2' + 2\zeta\omega_n T_1'T_2'$

$a_2 = 1+ 2\zeta\omega_n (T_1'+T_2') + \omega_n^2T_1'T_2' + \omega_n^2K_cT_1T_2$

$a_3 = 2\zeta\omega_n + \omega_n^2(T_1 + T_2) + \omega_n^2K_c(T_1+T_2)$

$a_4 = \omega_n^2 + \omega_n^2K_c$

System Step Response:

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 3 providing the system response c(t) as function of time [17].

3 Compensator Tuning

The sum of time multiplied by the absolute error (ITAE) is used as an objective function, F of the optimization process. Thus:

$$F = \int t\, |c(t) - c_{ss}|\, dt \qquad (4)$$

where c_{ss} = steady state response of the system.

The performance of the control system is controlled using four functional constraints:

(a) The maximum percentage overshoot constraint, c_1:

$$c_1 = OS_{max} - OS_{des} \qquad (5)$$

Where OS_{des} is the desired maximum percentage overshoot of the control system.

(b) The settling time constraint, c_2:

$$c_2 = T_s - T_{sdes} \qquad (6)$$

Where T_{sdes} is the desired settling time of the control system.

The steady-state error constraint, c_3:

This constraint is written in the form:

$$c_3 = e_{ss} - e_{ssdes} \qquad (7)$$

where: e_{ss} is the steady-state error of the control system and e_{ssdes} is the desired value for the steady state error.

(c) The stability constraint:

The control system is a fourth order one which depending on the transfer functiom parameters may be unstable. Therefore, a constraint or more is required to be imposed to guarantee a stable system during the optimization process. The constraints for the case study in hand are assigned using Routh-Hurwitz criterion of the system characteristic equation:

- Positive compensator parameters.
- Certain function relating the compensator parameters. This function gives the third functional constaint of the optimization process. That is:

$$C_4 = a_0a_3 - a_1a_2 \qquad (8)$$

c_4 has to be positive for a stable control system.

Parameters Limits:

- A lower limit of 0.001 is set for the compensator parameters: T_1, T_2, b and K_c.
- An upper limit of 100 is set for the compensator parameters T_1 and T_2 and K_c.
- An upper limit of 10 is set for the compensator parameter b (this was because with all guessing values of the compensator parameters, b was less than 1).

4 Tuning Results

The MATLAB command "*fmincon*" is used to minimize the optimization objective function given by Eq.4 subjected to the functional inequality constraints given by Eqs. 5 through 8 to provide the lag-lead compensator parameters subjected to the limits mentioned in section 6 [18]. The results are as follows:

Controller parameters:

$T_1 =$ 0.332706 s
$T_2 =$ 0.394846 s
b = 48.57566
$K_c =$ 100

The time respone of the compensated system to a unit step input is shown in Fig.2.

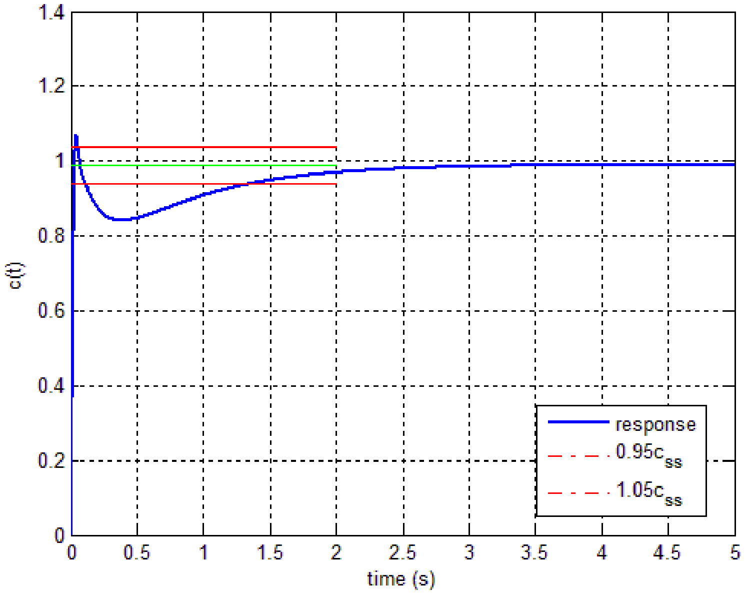

Fig.2 Step response of the lag-lead compensated system.

Characteristics of the control system using the tuned lag-lead compensator:

-	Maximum percentage overshoot:	6.926	%
-	Maximum percentage undershoot:	14.700	%
-	Settling time:	1.413	s

5 Conclusions

- The suggested tuning technique of lag-lead compensators used with a highly oscillating process is straight forward and simple compared with techniques based on classical parameter identification such as frequency response or root locus plot techniques.
- Through using a the proposed tuning technique, it was possible to reduce the maximum percentage overshoot of uncompensated process with unit feedback from 85.4 % to only about 7 %.
- Using the proposed tuning technique, it was possible to reduce the settling time from about 6 seconds to only 1.413 seconds.
- The undershoot was relatively high (14.7 %). In other compensating techniques, it is possible to eliminate completely both overshoot and undershoot.
- The ITAE objective function has given good characteristics compared with other functions tried by the author.

References

[1] J. James, D. Frederick and J. Taylor, "Use of expert-systems programming techniques for the design of lead-lag compensators", *IEE Proceedings,* Vol.134, No.3, May 1987, pp.137-144.

[2] A. Loh, X. Cai and W. Tan, "Auto-tuning of phase lead/lag compensators", *Automatica*, Vol.49, 2004, pp.423-429.

[3] T. Chang, "Servo control design", *Control Systems, Robotics and Automation*, Vol.VIII, Encyclopedia of Life Support Systems, 2004.

[4] F. Wang, "A new non-trial-and-error mrthod for lag-lead compensator design: A special case", *Int. J. Intelligent Control and Systems*, Vol.11, No.1, March 2006, pp.69-76.

[5] C. Zhang, C. Liu, H. Dong, G. Zhang and J. Ou, "Design of step response based lag-lead compensators for the asynchronous linear servo motors used in the AMD control system", *Pacific Science Review*, Vol.8, 2006, pp.109-114.

---

[6] S. Panda and N. Padhy, "Thyristor controlled series compensator-based controller design employing genetic algorithm: A comparative study", *Int. J. Electrical and Computer Engineering,* Vol.2, No.9, 2007, pp.608-617.

[7] A. Nassirharand, "A new software for design of linear compensators", *Advances in Engineering Software*, Vol.39, 2008, pp.132-136.

[8] D. Wang, "Synthesis of phase-lead/lag compensators with completer information on gain and phase margins", *Automatica*, Vol.45, 2009, pp.1026-1031.

[9] M. Cao, J. Watkins and B. O'Brien, "A graphical user interface for a unified approach for continuous time compensators design", *Computers in Education*, Vol.19, No.1, 2009, pp.49-59.

[10] Y. Li, H. Sheng and Y. Chen, "On distributed order lead-lag compensator", *Proceedings of the 4th IFAC Workshop*, Badajoz, Spain, October 18-20, 2010.

[11] R. Zanasi and S. Cuoghi, "Design of lead-lag compensators for robust control", *Int. Conference on Control and Automation*, Santiago, Cile, 19-21 December 2011.

[12] C. Nandar, "Design of robust power system stabilizer considering less control energy", *Int. J. Power Electronics and Drive System*, Vol.2, No.1, March 2012, pp.99-106.

[13] L. Ntogramatzidis, R. Zanasi and R. Cuoghi, "A unified analytical design method of standard compensators", *arXiv*: 1210.3812 v1, 14 October 2012.

[14] G. A. Hassaan, M. Al-Gamil and M. Lashin, "Tuning of a lag-lead compensator used with a first-order plus an integrator process", *International Journal of Mechanical and Production Engineering Research and Development*, Vol.3, No.5, 2013, pp.41-48.

[15] C. Rodriguez, J. Guzman, M. Berenguel and T. Hugglund, "Optimal feedforward compensator systems with right-half plane zeros", *Journal of Process Control*, Vol.24, No.4, 2014, pp.368-374.

[16] K. Ogata, "Modern control engineering", Prentice Hall Int., 1970, p.510.

[17] J. D'Azzo, C. Houpis and S. Sheldon, "Linear control system analysis and design with MATLAB programming", J. Wiley, 2009.

[18] P. Venkataraman, "Applied optimization with MATLAB programming", J. Wiley, 2009.

International Journal of Mechanical Engineering (IJME)
Volume 4 Issue 3 (Year : 2014) ISSN : 2277-7059
http://www.ijmejournal.com/

Biography

Galal Ali Hassaan

- Professor of System Dynamics and Automatic Control.
- Faculty of Engineering, Cairo University, EGYPT
- Research on Automatic Control, Mechanical Vibrations and Mechanism Synthesis.
- Author of 10's of research papers.
- Author of a book on Experimental Systems Control.