Collection of Math Notes

Mathematical Constants , Formulas

Edgar Valdebenito

Abstract

Collection of formulas involving mathematical constants

Contenido

SERIES PARA CONSTANTES CLASICAS QUE CONTIENEN LA SUCESION $x_n = \sqrt{n+1} - \sqrt{n}$ $, n = 1,2,3,...$

EDGAR VALDEBENITO V.
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Resumen

Se muestran series para algunas constantes clásicas como son: π, $ln\,2$, e, $\zeta(3)$, $\zeta^*\left(\frac{1}{2}\right)$, G .Las series contienen la sucesión $x_n = \sqrt{n+1} - \sqrt{n} \quad n \in \mathbb{N}$.

1. INTRODUCCIÓN.

Las constantes π, $ln\,2$, e, $\zeta(3)$, $\zeta^*\left(\frac{1}{2}\right)$, G, se definen como sigue:

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.14159265...$$

$$ln\,2 = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 0.69314718...$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 2.71828182...$$

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} = 1.20205690...$$

$$\zeta^*\left(\frac{1}{2}\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} = 0.60489864...$$

$$G = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} = 0.91596559...$$

En esta nota se muestran series para las constantes anteriores, que contienen la sucesión:

$$x_n = \sqrt{n+1} - \sqrt{n} \qquad n \in \mathbb{N}$$

$$x_n = \left\{\sqrt{2}-1, \sqrt{3}-\sqrt{2}, \sqrt{4}-\sqrt{3}, \sqrt{5}-\sqrt{4}, ...\right\}$$

2. SERIES.

2.1.

$$\pi = 4 - 8\sum_{n=1}^{\infty}\sum_{k=1}^{n}(-1)^{n+1}\, x_k^{4n-4k+2}$$

2.2.

$$\zeta^*\left(\frac{1}{2}\right) = 2\sum_{n=1}^{\infty}\sum_{k=1}^{n}(-1)^{k-1}\, x_k^{2n-2k+1}$$

2.3.

$$\ln 2 = 8\sum_{n=1}^{\infty}\sum_{k=1}^{n}(-1)^{k-1}(n-k+1)\, x_{2k}^{2n-2k+1}$$

2.4.

$$e = 1 + 4\sum_{n=1}^{\infty}\sum_{k=1}^{n}\frac{(n-k+1)}{(k-1)!}\, x_k^{2n-2k+2}$$

2.5.

$$\pi = 16\sum_{n=1}^{\infty}\sum_{k=1}^{n}(-1)^{k-1}(n-k+1)\, x_{2k-1}^{2n-2k+2}$$

2.6.

$$\zeta(3) = 64\sum_{n=1}^{\infty}\sum_{k=1}^{n}\frac{(6)_{n-k}}{(n-k)!}\, x_k^{2n-2k+6}$$

2.7.

$$G = 4\sum_{n=1}^{\infty}\sum_{k=1}^{n}\frac{(-1)^{k-1}\, k\,(n-k+1)}{(2k-1)^2}\, x_k^{2n-2k+2}$$

2.8.

$$G = 1 + 4\sum_{n=1}^{\infty}\sum_{k=1}^{n}(-1)^{n}(n-k+1)\, x_k^{4n-4k+4}$$

2.9. Para $m \in \mathbb{N}$:

$$\pi^{2m} = \frac{(2m)!\, 2^{2m+1}}{\left(2^{2m}-1\right)B_m}\left(\frac{1}{2^{2m}} + \sum_{n=1}^{\infty}\sum_{k=1}^{n}\frac{(-1)^{n-k}\,(2m)_{n-k}}{(n-k)!}\, x_k^{4n-4k+4m}\right)$$

donde B_m son los números de Bernoulli: $B_m = \left\{\frac{1}{6}, \frac{1}{30}, \frac{1}{42}, \frac{1}{30}, \frac{5}{66}, \frac{691}{2730}, \ldots\right\}$

3. REFERENCIAS.

1. Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover, 1965.

2. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey), Academic Press, New York, London, and Toronto, 1980.

3. M. R. Spiegel, Mathematical Handbook, McGraw-Hill Book Company, New York, 1968.

4. E. Valdebenito, Pi Handbook, manuscript, unpublished, 1989 , (20000 fórmulas).

PRODUCTO INFINITO PARA π

EDGAR VALDEBENITO V.
(1991)

Resumen

Se muestra un producto infinito para el número $\pi = 3.14159265...$

1. INTRODUCCIÓN.

El número π se define por la serie: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$, en esta nota se muestra un producto infinito para la constante π, además se muestran algunas variantes de dicho producto.

2. PRODUCTO INFINITO PARA π :

2.1.

$$\pi = 4\prod_{n=1}^{\infty}\left(1+\frac{(-1)^n}{2n+1}\left(\sum_{k=1}^{n}\frac{(-1)^{k-1}}{2k-1}\right)^{-1}\right)$$

2.2.

$$\pi = 4\prod_{n=1}^{\infty}\left(1+\frac{(-1)^n(2n)!}{(2n+1)2^n n!\,A_n}\right)$$

donde

$$A_{n+1} = (2n+1)A_n + (-1)^n\frac{(2n)!}{2^n n!} \quad , \ n\in\mathbb{N}, \ A_1 = 1$$

2.3.

$$\pi = 4\prod_{n=1}^{\infty}\left(1+\frac{(-1)^n(2n)!}{B_n}\right)$$

donde

$$B_{n+1} = 2(n+1)(2n+3)\left(B_n + (-1)^n(2n)!\right) \quad , \ n\in\mathbb{N} \ , \ B_1 = 6$$

2.4.

$$\pi = 4\prod_{n=1}^{\infty}\left(1+\frac{(-1)^n}{2n+1}\left(1-\sum_{k=1}^{n-1}(-1)^k C_k\right)\right)$$

donde

$$C_k = \left(\frac{(2k)!}{2^k\, k!} \right)^2 \frac{1}{A_k A_{k+1}}$$

$$A_{k+1} = (2k+1) A_k + (-1)^k \frac{(2k)!}{2^k\, k!} \quad , \; k \in \mathbb{N} \quad , \quad A_1 = 1$$

2.5.

$$\pi = 4 \prod_{n=1}^{\infty} \left(1 + \frac{(-1)^n a_n}{(2n+1) b_n} \right)$$

donde

$$a_{n+1} = (2n+1) a_n$$

$$b_{n+1} = (2n+1) b_n + (-1)^n a_n \quad , \quad a_1 = b_1 = 1$$

2.6.

$$\pi = 4 \prod_{n=1}^{\infty} \left(1 + \frac{(-1)^n (2n)!}{(2n+1) D_n} \right)$$

donde

$$D_n = \sum_{k=0}^{n-1} (-1)^k (2k)! (2k+2) \cdots (2n) \quad , \quad n \in \mathbb{N}$$

3. REFERENCIAS.

1. Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover, 1965.

2. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey), Academic Press, New York, London, and Toronto, 1980.

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PRODUCTOS INFINITOS PARA LA CONSTANTE e^e

EDGAR VALDEBENITO V.
(1991)

Resumen

Se muestran algunos productos infinitos para la constante $e^e = 15.15426224147926...$,donde $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.71828182845904...$

1. INTRODUCCIÓN.

El número e se define por:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

$$e = \lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{k!}$$

$$e = \sum_{k=0}^{\infty}\frac{1}{k!}$$

En esta nota se muestran algunos productos infinitos para el número e^e .

2. PRODUCTOS INFINITOS.

2.1.

$$e^e = 4\prod_{n=1}^{\infty}\left(\frac{n^{n(n+1)^{2n}}\,(n+2)^{n^n(n+2)^{n+1}}}{(n+1)^{n(n+1)^{2n}+n^n(n+2)^{n+1}}}\right)^{\left(\frac{1}{n(n+1)}\right)^n}$$

2.2.

$$e^e = \left(\frac{3}{2}\right)^4\sqrt{\frac{3}{2}}\prod_{n=1}^{\infty}\left(\frac{\left(1+2^{-n-1}\right)^{2\left(1-\left(2^{n+1}+1\right)^{-2}\right)^{-2^n}}}{1-2^{-n}}\right)^{2^n\left(1+2^{-n}\right)^{2^n}}$$

2.3.

$$e^e = 4\prod_{n=1}^{\infty}\left(\left(\frac{n+2}{n+1}\right)^{\frac{1}{n!}}\left(\frac{n^n\,(n+2)^{n+1}}{(n+1)^{2n+1}}\right)^{\sum_{k=0}^{n}\frac{1}{k!}}\right)$$

2.4.Para $n \in \mathbb{N}$ sea a_n la sucesión definida por la recurrencia: $a_{n+1} = (n+1)a_n + 1$, $a_1 = 2$.
Algunos valores de la sucesión son: $a_n = \{2,5,16,65,326,1957,13700,...\}$.

$$e^e = 4\prod_{n=1}^{\infty}\left[\left(\frac{n!}{a_n}\right)^{(n+1)^{2n+1}}\left(\frac{a_{n+1}}{(n+1)!}\right)^{n^n(n+2)^{n+1}}\right]^{\frac{1}{n^n(n+1)^{n+1}}}$$

$$e^e = 4\prod_{n=1}^{\infty}\left[\left(\frac{n!}{a_n}\right)^{a_n}\left(\frac{a_{n+1}}{(n+1)!}\right)^{\frac{a_{n+1}}{n+1}}\right]^{\frac{1}{n!}}$$

2.5.Para $n \in \mathbb{N}$ sea a_n la sucesión definida por la ecuación: $a_n = n^{1-n}(n+1)^n$ .Algunos valores de la sucesión son: $a_n = \{2, 2^{-1}3^2, 3^{-2}4^3, 4^{-3}5^4, ...\}$.

$$e^e = 4\prod_{n=1}^{\infty}\left[\frac{\left(1+\frac{2}{n}\right)^{a_{n+1}}}{\left(1+\frac{1}{n}\right)^{a_n+a_{n+1}}}\right]$$

3. REFERENCIAS.

1. Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover, 1965.

2. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey), Academic Press, New York, London, and Toronto, 1980.

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EL NÚMERO $\nu = e^{-e^{-e^{-\cdots}}} = 0.5671432...$

EDGAR VALDEBENITO V.
(1991)

Resumen

Se muestran algunas fórmulas que involucran el número ν.

1. INTRODUCCIÓN.

El número $\nu = e^{-e^{-e^{-\cdots}}} = 0.5671432....$, donde $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.7182818...$, satisface la ecuación $\nu e^{\nu} = 1$. En esta nota se muestran algunas fórmulas en las que aparece el número ν.

2. FÓRMULAS.

2.1. $$\ln 2 = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left(1-\frac{2^{2n-1}\left(2^{2n}-1\right)B_n}{(2n)!}\right)\nu^{2n} + \sum_{n=1}^{\infty}\frac{(1-\nu)^n}{n}$$

2.2. $$\nu^{\gamma} = \Gamma(1+\nu)\prod_{n=1}^{\infty}\left(1+\frac{\nu}{n}\right)\sqrt[n]{\nu}$$

2.3. $$\frac{1}{\pi} = \left(\nu+\frac{1}{\nu}\right)\sum_{n=1}^{\infty}\frac{(-1)^{n-1}(2n-1)}{3(2n-1)^2\zeta(2)+2\nu^2}$$

2.4. $$\nu^{\pi}+\nu^{-\pi} = 2\prod_{n=1}^{\infty}\left(1+\frac{4\nu^2}{(2n-1)^2}\right)$$

2.5. $$\frac{\nu^{-\pi}-\nu^{\pi}}{\pi} = 2\nu\prod_{n=1}^{\infty}\left(1+\frac{\nu^2}{n^2}\right)$$

3. REFERENCIAS.

1. Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover, 1965.

2. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey), Academic Press, New York, London, and Toronto, 1980.

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UNA FÓRMULA QUE CONTIENE EL NÚMERO π

EDGAR VALDEBENITO V.
(1991)

Resumen
Se muestra una fórmula que contiene el número $\pi = 3.14159265...$

1. INTRODUCCIÓN.

El número π se define por la serie: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$, en esta nota se muestra una fórmula que contiene la constante π .

2. FÓRMULA.

$$\frac{1}{16}\ln\left[\left(\frac{20+3\sqrt{6}+3\sqrt{2}}{20-3\sqrt{6}-3\sqrt{2}}\right)^{\sqrt{6}+\sqrt{2}}\left(\frac{10+3\sqrt{2}}{10-3\sqrt{2}}\right)^{2\sqrt{2}}\left(\frac{20+3\sqrt{6}-3\sqrt{2}}{20-3\sqrt{6}+3\sqrt{2}}\right)^{\sqrt{6}-\sqrt{2}}\right]+$$

$$+\frac{1}{8}\left(\left(\sqrt{6}-\sqrt{2}\right)\tan^{-1}\frac{3\left(\sqrt{6}-\sqrt{2}\right)}{16}-2\sqrt{2}\tan^{-1}\frac{3-\sqrt{2}}{3}\right)+$$

$$+\frac{1}{8}\left(\left(\sqrt{6}+\sqrt{2}\right)\tan^{-1}\frac{3\left(\sqrt{6}+\sqrt{2}\right)}{16}-2\sqrt{2}\tan^{-1}\frac{3\left(3-\sqrt{2}\right)}{7}\right)+$$

$$+\frac{\sqrt{2}}{8}\pi = \sum_{n=0}^{\infty}\frac{(-1)^n}{(12n+1)3^{12n}} \qquad (1)$$

Algunas variantes de la fórmula (1) son:

$$\frac{\ln A}{16}-\frac{\sqrt{6}}{8}\tan^{-1}\frac{55\sqrt{6}}{144}-\frac{\sqrt{2}}{16}\tan^{-1}\frac{2428416\sqrt{2}}{10339103}+\frac{\sqrt{6}+\sqrt{2}}{16}\pi = \sum_{n=0}^{\infty}\frac{(-1)^n}{(12n+1)3^{12n}} \qquad (2)$$

$$\frac{\ln A}{16}+\frac{\sqrt{6}}{8}\tan^{-1}\frac{6481-2840\sqrt{6}}{431}-\frac{\sqrt{2}}{16}\tan^{-1}\frac{2428416\sqrt{2}}{10339103}+$$

$$+\frac{\sqrt{6}+2\sqrt{2}}{32}\pi = \sum_{n=0}^{\infty}\frac{(-1)^n}{(12n+1)3^{12n}} \qquad (3)$$

$$\frac{\ln A}{16}-\frac{\sqrt{6}}{8}\tan^{-1}\frac{3\sqrt{6}}{20}-\frac{\sqrt{6}}{8}\tan^{-1}\frac{334\sqrt{6}}{1935}-\frac{\sqrt{2}}{16}\tan^{-1}\frac{2428416\sqrt{2}}{10339103}+$$

$$+\frac{\sqrt{6}+\sqrt{2}}{16}\pi=\sum_{n=0}^{\infty}\frac{(-1)^n}{(12n+1)3^{12n}} \qquad (4)$$

$$\frac{\ln A}{16}-\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\left(\frac{3^{2n+2}}{2^{3n+4}5^{2n+1}}+\frac{2^{3n-1}167^{2n+1}}{3^{3n+1}5^{2n+1}43^{2n+1}}+\frac{2^{19n+6}3^{4n+2}17^{2n+1}31^{2n+1}}{991^{2n+1}10433^{2n+1}}\right)+$$

$$+\frac{\sqrt{6}+\sqrt{2}}{16}\pi=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)3^{12n}} \qquad (5)$$

En todos los casos:

$$A=\left(\frac{20+3\sqrt{6}+3\sqrt{2}}{20-3\sqrt{6}-3\sqrt{2}}\right)^{\sqrt{6}+\sqrt{2}}\left(\frac{10+3\sqrt{2}}{10-3\sqrt{2}}\right)^{2\sqrt{2}}\left(\frac{20+3\sqrt{6}-3\sqrt{2}}{20-3\sqrt{6}+3\sqrt{2}}\right)^{\sqrt{6}-\sqrt{2}} \qquad (6)$$

3. REFERENCIAS.

1. Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover, 1965.

2. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey), Academic Press, New York, London, and Toronto, 1980.

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FÓRMULAS QUE CONTIENEN EL NÚMERO π Y LAS FUNCIONES $E(q), V(q), R(q), J(q)$

EDGAR VALDEBENITO V.
(1991)

Resumen

Se muestran algunas fórmulas que involucran el número $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.141592...$, y las funciones $E(q)$, $V(q)$, $R(q)$, $J(q)$.

1. INTRODUCCIÓN.

Para $|q| < 1$ las funciones $E(q)$, $V(q)$, $R(q)$, $J(q)$ se definen por :

$$E(q) = 1 + \sum_{n=1}^{\infty}(-1)^{n-1}\left(q^{(4n-3)(6n-4)} - q^{(4n-1)(6n-2)} - q^{2n(12n-1)} - q^{2n(12n+1)}\right)$$

$$E(q) = 1 + q^2 - q^{12} - q^{22} - q^{26} - q^{40} + q^{70} + ...$$

$$V(q) = \sum_{n=1}^{\infty}(-1)^{n-1}\left(q^{(4n-3)(6n-5)} - q^{(2n-1)(12n-7)} + q^{(2n-1)(12n-5)} - q^{(4n-1)(6n-1)}\right)$$

$$V(q) = q - q^5 + q^7 - q^{15} - q^{35} + q^{51} - q^{57} + ...$$

$$R(q) = 1 + 2\sum_{n=1}^{\infty}\left(q^{(4n-2)^2} + q^{(4n)^2}\right)$$

$$R(q) = 1 + 2\left(q^4 + q^{16} + q^{36} + q^{64} + ...\right)$$

$$J(q) = 2\sum_{n=1}^{\infty}\left(q^{(4n-3)^2} + q^{(4n-1)^2}\right)$$

$$J(q) = 2\left(q + q^9 + q^{25} + q^{49} + ...\right)$$

En esta nota se muestran algunas fórmulas (sumas de arcotangentes) que involucran la constante π .

2. FÓRMULAS.

2.1.Para $|q| < 1$ se tiene:

$$\tan^{-1}(q)=\tan^{-1}\left(\frac{V(q)}{E(q)}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{\left(1-q^2\right)q^{4n-1}}{1+q^{8n}}\right)$$

$$\tan^{-1}(q)=\frac{1}{2}\tan^{-1}\left(\frac{J(q)}{R(q)}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{\left(1-q^2\right)q^{4n-1}}{1+q^{8n}}\right)$$

$$\tan^{-1}(q)=\tan^{-1}\left(\frac{J(q)}{R(q)}\right)-\tan^{-1}\left(\frac{V(q)}{E(q)}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{\left(1-q^2\right)q^{4n-1}}{1+q^{8n}}\right)$$

2.2.

$$\frac{\pi}{12}=\tan^{-1}\left(\frac{V\left(2-\sqrt{3}\right)}{E\left(2-\sqrt{3}\right)}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{2\sqrt{3}\left(2-\sqrt{3}\right)^{4n}}{1+\left(2-\sqrt{3}\right)^{8n}}\right)$$

$$\frac{\pi}{8}=\tan^{-1}\left(\frac{V\left(\sqrt{2}-1\right)}{E\left(\sqrt{2}-1\right)}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{2\left(\sqrt{2}-1\right)^{4n}}{1+\left(\sqrt{2}-1\right)^{8n}}\right)$$

$$\frac{\pi}{6}=\tan^{-1}\left(\frac{V\left(1/\sqrt{3}\right)}{E\left(1/\sqrt{3}\right)}\right)+\sum_{n=1}^{\infty}\tan^{-1}\left(\frac{2\sqrt{3}\cdot 3^{2n-1}}{3^{4n}+1}\right)$$

3. REFERENCIAS.

1. Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover, 1965.

2. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey), Academic Press, New York, London, and Toronto, 1980.

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COLECCIÓN DE SERIES QUE INVOLUCRAN LA CONSTANTE π

EDGAR VALDEBENITO V.
(1991)

Resumen
Se muestra una colección de series que contienen la constante Pi ,

$$\pi = 4\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.14159265...$$

1. INTRODUCCIÓN.

En esta nota mostramos una colección de series que involucran la clásica constante π , y que corresponden a casos particulares de la siguiente fórmula general:

$$\left(p\left(\frac{a^2}{2} - \frac{1}{4} \right) + qa \right) sen^{-1}(a) + \frac{pa\sqrt{1-a^2}}{4} + q\sqrt{1-a^2} - q =$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n n!} \left(p\frac{a^{2n+3}}{2n+3} + q\frac{a^{2n+2}}{2n+2} \right)$$

donde $\quad p,q \in \mathbb{R} \qquad 0 < a < 1$

Escogiendo los valores de p,q,a , de manera adecuada podemos obtener muchas series que involucran la constante π .A continuación mostramos algunos ejemplos.

2. EJEMPLOS DE SERIES.

2.1.Para $a = \frac{\sqrt{6}-\sqrt{2}}{4}$, $q = a$, $p \in \mathbb{R}$ se tiene:

$$A + B\pi = \sum_{n=0}^{\infty} \frac{(2n)!\left(2(p+1)n+2p+3\right)}{16^n \left(2+\sqrt{3}\right)^n n!(n+1)!(2n+1)(2n+3)} =$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2 \left(2(p+1)n+2p+3\right)}{\left(\frac{3}{2}\right)_n 4^n \left(2+\sqrt{3}\right)^n (n+1)!(2n+3)}$$

donde $A=\left(\frac{5p}{2}+10\right)\sqrt{2}-8\sqrt{3}+\left(\frac{3p}{2}+6\right)\sqrt{6}-16$

$$B=\frac{\left(5+3\sqrt{3}\right)\left(4-\left(p+2\right)\sqrt{3}\right)}{6\sqrt{2}}$$

2.1.1.Caso $p=0$:

$$A+B\pi=\sum_{n=0}^{\infty}\frac{(2n)!}{16^n\left(2+\sqrt{3}\right)^n n!(n+1)!(2n+1)}=$$

$$=\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n 4^n\left(2+\sqrt{3}\right)^n (n+1)!}$$

$$A=10\sqrt{2}-8\sqrt{3}+6\sqrt{6}-16 \qquad B=\frac{1+\sqrt{3}}{3\sqrt{2}}$$

2.1.2.Caso $p=-1$:

$$A+B\pi=\sum_{n=0}^{\infty}\frac{(2n)!}{16^n\left(2+\sqrt{3}\right)^n n!(n+1)!(2n+1)(2n+3)}=$$

$$=\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n 4^n\left(2+\sqrt{3}\right)^n (n+1)!(2n+3)}$$

$$A=\frac{15\sqrt{2}}{2}-8\sqrt{3}+\frac{9\sqrt{6}}{2}-16 \qquad B=\frac{11+7\sqrt{3}}{6\sqrt{2}}$$

2.1.3.Caso $p=-4$:

$$A+B\pi=-\sum_{n=0}^{\infty}\frac{(2n)!(6n+5)}{16^n\left(2+\sqrt{3}\right)^n n!(n+1)!(2n+1)(2n+3)}=$$

$$=-\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2(6n+5)}{\left(\frac{3}{2}\right)_n 4^n\left(2+\sqrt{3}\right)^n (n+1)!(2n+3)}$$

$$A=-8\sqrt{3}-16 \qquad B=\frac{19+11\sqrt{3}}{3\sqrt{2}}$$

2.1.4.

$$A+\pi=\frac{1}{3}\sum_{n=0}^{\infty}\frac{(2n)!\left(a(n+1)+3\right)}{16^n\left(2+\sqrt{3}\right)^n n!(n+1)!(2n+1)(2n+3)}=$$

$$=\frac{1}{3}\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2\left(a(n+1)+3\right)}{\left(\frac{3}{2}\right)_n 4^n\left(2+\sqrt{3}\right)^n (n+1)!(2n+3)}$$

$$A=\frac{19\sqrt{6}}{2}-10\sqrt{3}+11\sqrt{2}-16 \qquad a=30\sqrt{6}+8\sqrt{3}-54\sqrt{2}-6$$

2.1.5.

$$B\pi=\sum_{n=0}^{\infty}\frac{(2n)!\left(a(n+1)+1\right)}{16^n\left(2+\sqrt{3}\right)^n n!(n+1)!(2n+1)(2n+3)}=$$

$$=\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2\left(a(n+1)+1\right)}{\left(\frac{3}{2}\right)_n 4^n\left(2+\sqrt{3}\right)^n (n+1)!(2n+3)}$$

$$B=\frac{11\sqrt{6}-16\sqrt{3}+19\sqrt{2}-24}{6} \qquad a=8\sqrt{6}-8\sqrt{2}-6$$

2.2.Para $a=q=\frac{1}{2}$, $p\in\mathbb{R}$:

$$\frac{(2-p)}{3}\pi+(p+4)\sqrt{3}-8=\sum_{n=0}^{\infty}\frac{(2n)!\left(2(p+1)n+2p+3\right)}{16^n n!(n+1)!(2n+1)(2n+3)}=$$

$$=\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2\left(2(p+1)n+2p+3\right)}{\left(\frac{3}{2}\right)_n 4^n (n+1)!(2n+3)}$$

2.2.1.Caso $p=0$:

$$\pi+6\sqrt{3}=12+\frac{3}{2}\sum_{n=0}^{\infty}\frac{(2n)!}{16^n n!(n+1)!(2n+1)}=$$

$$=12+\frac{3}{2}\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n 4^n (n+1)!}$$

2.2.2.Caso $p=-1$:

$$\pi+3\sqrt{3}=8+\sum_{n=0}^{\infty}\frac{(2n)!}{16^n\,n!(n+1)!(2n+1)(2n+3)}=$$

$$=8+\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n 4^n\,(n+1)!(2n+3)}$$

2.2.3.Caso $p=-4$:

$$\pi=4-\frac{1}{2}\sum_{n=0}^{\infty}\frac{(2n)!(6n+5)}{16^n\,n!(n+1)!(2n+1)(2n+3)}=$$

$$=4-\frac{1}{2}\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2(6n+5)}{\left(\frac{3}{2}\right)_n 4^n\,(n+1)!(2n+3)}$$

2.3.Para $a=q=\frac{1}{\sqrt{2}}$, $p\in\mathbb{R}$:

$$\frac{\pi}{\sqrt{2}}+\frac{(p+4)\sqrt{2}-8}{2}=\sum_{n=0}^{\infty}\frac{(2n)!\left(2(p+1)n+2p+3\right)}{8^n\,n!(n+1)!(2n+1)(2n+3)}=$$

$$=\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2\left(2(p+1)n+2p+3\right)}{\left(\frac{3}{2}\right)_n 2^n\,(n+1)!(2n+3)}$$

2.3.1.Caso $p=0$:

$$\frac{\pi}{\sqrt{2}}+2\sqrt{2}=4+\sum_{n=0}^{\infty}\frac{(2n)!}{8^n\,n!(n+1)!(2n+1)}=$$

$$=4+\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n 2^n\,(n+1)!}$$

2.3.2.Caso $p=-4$:

$$\frac{\pi}{\sqrt{2}}=4-\sum_{n=0}^{\infty}\frac{(2n)!(6n+5)}{8^n\,n!(n+1)!(2n+1)(2n+3)}=$$

$$= 4 - \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2 (6n+5)}{\left(\frac{3}{2}\right)_n 2^n (n+1)!(2n+3)}$$

2.3.3.Caso $p = -1$:

$$\frac{\pi + 3}{\sqrt{2}} = 4 + \sum_{n=0}^{\infty} \frac{(2n)!}{8^n n!(n+1)!(2n+1)(2n+3)} =$$

$$= 4 + \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n 2^n (n+1)!(2n+3)}$$

2.3.4.Caso $p = 4\left(\sqrt{2} - 1\right)$:

$$\frac{\pi}{\sqrt{2}} = \sum_{n=0}^{\infty} \frac{(2n)!\left[\left(8\sqrt{2} - 6\right)(n+1) + 1\right]}{8^n n!(n+1)!(2n+1)(2n+3)} =$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2 \left[\left(8\sqrt{2} - 6\right)(n+1) + 1\right]}{\left(\frac{3}{2}\right)_n 2^n (n+1)!(2n+3)}$$

2.4.Para $a = q = \frac{\sqrt{3}}{2}$, $p \in \mathbb{R}$:

$$\frac{2(p+6)}{9\sqrt{3}}\pi + \frac{p\sqrt{3} - 12}{9} = \sum_{n=0}^{\infty} \frac{(2n)!\left(2(p+1)n + 2p + 3\right)}{n!(n+1)!(2n+1)(2n+3)} \left(\frac{3}{16}\right)^n =$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2 \left(2(p+1)n + 2p + 3\right)}{\left(\frac{3}{2}\right)_n (n+1)!(2n+3)} \left(\frac{3}{4}\right)^n$$

2.4.1.Caso $p = 0$:

$$\frac{4\pi}{3\sqrt{3}} = \frac{4}{3} + \sum_{n=0}^{\infty} \frac{(2n)!}{n!(n+1)!(2n+1)} \left(\frac{3}{16}\right)^n =$$

$$= \frac{4}{3} + \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n (n+1)!} \left(\frac{3}{4}\right)^n$$

2.4.2.Caso $p=-1$:

$$\frac{10\pi}{3\sqrt{3}}-\frac{1}{\sqrt{3}}=4+3\sum_{n=0}^{\infty}\frac{(2n)!}{n!(n+1)!(2n+1)(2n+3)}\left(\frac{3}{16}\right)^{n}=$$

$$=4+3\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n}^{2}}{\left(\frac{3}{2}\right)_{n}(n+1)!(2n+3)}\left(\frac{3}{4}\right)^{n}$$

2.4.3.Caso $p=4\sqrt{3}$:

$$\left(2+\sqrt{3}\right)\pi=\frac{9}{4}\sum_{n=0}^{\infty}\frac{(2n)!\left(\left(8\sqrt{3}+2\right)(n+1)+1\right)}{n!(n+1)!(2n+1)(2n+3)}\left(\frac{3}{16}\right)^{n}=$$

$$=\frac{9}{4}\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n}^{2}\left(\left(8\sqrt{3}+2\right)(n+1)+1\right)}{\left(\frac{3}{2}\right)_{n}(n+1)!(2n+3)}\left(\frac{3}{4}\right)^{n}$$

2.5.Para $a=q=\frac{\sqrt{6}+\sqrt{2}}{4}$, $p\in\mathbb{R}$:

$$A+B\pi=\sum_{n=0}^{\infty}\frac{(2n)!\left(2(p+1)n+2p+3\right)}{n!(n+1)!(2n+1)(2n+3)}\left(\frac{2+\sqrt{3}}{16}\right)^{n}=$$

$$=\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n}^{2}\left(2(p+1)n+2p+3\right)}{\left(\frac{3}{2}\right)_{n}(n+1)!(2n+3)}\left(\frac{2+\sqrt{3}}{4}\right)^{n}$$

$$A=\frac{\sqrt{2}\left(3\sqrt{3}-5\right)\left(p+4-4\sqrt{6}-4\sqrt{2}\right)}{2}$$

$$B=\frac{\sqrt{2}\left(3\sqrt{3}-5\right)\left(20+5(p+2)\sqrt{3}\right)}{12}$$

2.5.1.Caso $p=0$:

$$A+B\pi=\sum_{n=0}^{\infty}\frac{(2n)!}{n!(n+1)!(2n+1)}\left(\frac{2+\sqrt{3}}{16}\right)^{n}=$$

$$=\sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)_{n}^{2}}{\left(\frac{3}{2}\right)_{n}(n+1)!}\left(\frac{2+\sqrt{3}}{4}\right)^{n}$$

$$A = 6\sqrt{6} + 8\sqrt{3} - 10\sqrt{2} - 16 \qquad B = \frac{5\sqrt{2}\left(\sqrt{3}-1\right)}{6}$$

2.6.Para $a = \frac{1}{2}$, $p = 2$, $q \in \mathbb{R}$:

$$\frac{(2q-1)\pi}{3} + (4q+1)\sqrt{3} - 8q = \sum_{n=0}^{\infty} \frac{(2n)!\left(2(q+1)n+3q+2\right)}{16^n n!(n+1)!(2n+1)(2n+3)} =$$

$$= \sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2 \left(2(q+1)n+3q+2\right)}{\left(\frac{3}{2}\right)_n 4^n (n+1)!(2n+3)}$$

2.6.1.Caso $q = 0$:

$$\sqrt{3} - \frac{\pi}{3} = 2\sum_{n=0}^{\infty} \frac{(2n)!}{16^n (n!)^2 (2n+1)(2n+3)} =$$

$$= 2\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2}{\left(\frac{3}{2}\right)_n 4^n n!(2n+3)}$$

2.6.2.Caso $q = \frac{2\sqrt{3}+3}{4}$:

$$\left(2\sqrt{3}+1\right)\pi = \frac{3}{2}\sum_{n=0}^{\infty} \frac{(2n)!\left(\left(4\sqrt{3}+4\right)n + 6\sqrt{3}+17\right)}{16^n n!(n+1)!(2n+1)(2n+3)} =$$

$$= \frac{3}{2}\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^2 \left(\left(4\sqrt{3}+4\right)n + 6\sqrt{3}+17\right)}{\left(\frac{3}{2}\right)_n 4^n (n+1)!(2n+3)}$$

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π - FÓRMULAS

EDGAR VALDEBENITO V.
(1991)

Resumen
Se muestran algunas fórmulas que involucran la constante $\pi = 3.14159265...$

1. INTRODUCCIÓN.

EL número Pi se define por la serie: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$, en esta nota se muestran algunas fórmulas que contienen la constante π .

2. FÓRMULAS.

2.1.

$$a = \frac{2-\sqrt[3]{1+3\sqrt[3]{1+3\sqrt[3]{1+...}}}}{2+\sqrt[3]{1+3\sqrt[3]{1+3\sqrt[3]{1+...}}}} = \frac{\pi^2}{324}\prod_{n=1}^{\infty}\left(\frac{2n-1}{2n}\right)^4\left(\frac{(18n)^2-1}{4(9n-5)(9n-4)}\right)^2$$

2.2.

$$a = \sum_{n=1}^{\infty}\frac{\pi^{2n}}{3^{4n}}\sum_{k=1}^{n}\frac{\left(2^{2k}-1\right)\left(2^{2n-2k}-1\right)B_k B_{n-k}}{(2k)!(2n-2k)!}$$

B_k son los números de Bernoulli $B_k = \left\{\frac{1}{6},\frac{1}{30},\frac{1}{42},\frac{1}{30},\frac{5}{66},...\right\}$

2.3.

$$a = -1+\frac{81}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{1}{(9n-5)^2}+\frac{1}{(9n-4)^2}\right)$$

2.4.

$$a = \frac{81}{\pi^2}\sum_{n=1}^{\infty}\sum_{k=1}^{n}\frac{1}{(9k-5)(9k-4)(9n-9k-5)(9n-9k-4)}$$

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NÚMERO e , NÚMEROS HARMONICOS H_n

EDGAR VALDEBENITO V.
(1991)

Resumen
Se muestran tres fórmulas que relacionan la constante $e = 2.71828182...$, y los números harmonicos $H_n = \sum_{k=1}^{n} \frac{1}{k}$

1. INTRODUCCIÓN.

La constante e de Euler se define por:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = \sum_{k=0}^{\infty}\frac{1}{k!}$$

Los números harmonicos $H_n = \sum_{k=1}^{n}\frac{1}{k}$, $n \in \mathbb{N} = \{1,2,3,...\}$, aparecen en una infinidad de fórmulas matemáticas y tienen propiedades interesantes.A continuación mostramos algunas relaciones que involucran números harmonicos:

$$H_{n+1} = H_n + \frac{1}{n+1}$$

$$H_{n+m} = H_n + \sum_{k=1}^{m}\frac{1}{n+k} = H_m + \sum_{k=1}^{n}\frac{1}{m+k}$$

$$H_n + H_m = 2H_n + \sum_{k=n+1}^{m}\frac{1}{k} = 2H_m - \sum_{k=n+1}^{m}\frac{1}{k} \qquad n < m$$

$$H_n H_m = H_n^2 + H_n \sum_{k=n+1}^{m}\frac{1}{k} = H_m^2 - H_m \sum_{k=n+1}^{m}\frac{1}{k} \qquad n < m$$

$$H_n = \frac{a_n}{b_n} \Leftrightarrow \begin{cases} a_{n+1} = (n+1)a_n + b_n \\ b_{n+1} = (n+1)b_n \end{cases} \quad a_1 = b_1 = 1$$

$$H_n = \left\{1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}, \frac{363}{140}, ...\right\}$$

En esta nota se muestran tres fórmulas que involucran el número e y los números H_n .

2. FÓRMULAS.

$$e = \lim_{n\to\infty} n^{1/H_n}$$

$$e = 1 + \sum_{n=1}^{\infty} \left(\left(n+1\right)^{1/H_{n+1}} - n^{1/H_n} \right)$$

$$e = \prod_{n=1}^{\infty} \left(\left(\frac{n+1}{n}\right) \left(\frac{1}{n}\right)^{1/\left((n+1)H_n\right)} \right)^{1/H_{n+1}}$$

3. REFERENCIAS.

1. Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover, 1965.

2. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey), Academic Press, New York, London, and Toronto, 1980.

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INTEGRAL QUE RELACIONA EL NÚMERO π Y LOS NÚMEROS $x(n)=\sqrt[n]{1+\sqrt[n]{1+\sqrt[n]{1+\ldots}}}$ $\quad n\in\mathbb{N}-\{1\}$

EDGAR VALDEBENITO V.
(1991)

Resumen

Se muestra una fórmula integral que involucra la constante $\pi = 3.141592\ldots$, y los números

$x(n)=\sqrt[n]{1+\sqrt[n]{1+\sqrt[n]{1+\ldots}}}$, $n\in\mathbb{N}-\{1\}$.

1. INTRODUCCIÓN.

EL número Pi se define por la serie: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$, los números $x(n)=\sqrt[n]{1+\sqrt[n]{1+\sqrt[n]{1+\ldots}}}$ $n\in\mathbb{N}-\{1\}$, satisfacen la ecuación $\left(x(n)\right)^n - x(n) - 1 = 0$.

En esta nota se muestra una fórmula integral que relaciona el número π y los números $x(n)$.

2. INTEGRAL.

$$\pi\sqrt[n]{1+\sqrt[n]{1+\sqrt[n]{1+\ldots}}} = \frac{1}{4}\int_0^{2\pi}\frac{n\left(3+e^{it}\right)^n - 2^{n-1}e^{it} - 3\cdot 2^{n-1}}{\left(3+e^{it}\right)^n - 2^{n-1}e^{it} - 2^{n-1}}e^{it}\,dt$$

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1. Abramowitz, M. e I.A. Stegun, Handbook of Mathematical Functions. Nueva York: Dover, 1965.

2. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (A. Jeffrey), Academic Press, New York, London, and Toronto, 1980.

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SERIES PARA π UTILIZANDO LOS VERTICES DE UN TRIANGULO

EDGAR VALDEBENITO V.
(1991)

Resumen. Usando los vértices de un triangulo y la serie de Taylor para la función $arcsen(x)$, se pueden obtener series para el número π.

1. INTRODUCCIÓN.

Consideremos un triángulo con vértices en los puntos:

$$V1=(a_1,a_2),V2=(b_1,b_2),V3=(c_1,c_2)$$

$$A=d(V1,V2),B=d(V2,V3),C=d(V1,V3)$$

$$\measuredangle V1=\beta \;,\measuredangle V2=\gamma \;,\measuredangle V3=\alpha$$

La condición para que $V1,V2,V3$ sean los vértices de un triángulo es:

$$\begin{vmatrix} a_1 & a_2 & 1 \\ b_1 & b_2 & 1 \\ c_1 & c_2 & 1 \end{vmatrix} = a_1b_2+b_1c_1+c_2b_1-b_2c_1-a_2b_1-a_1c_2 \neq 0$$

Por trigonometría sabemos que:

$$A^2=B^2+C^2-2BC\cos(\alpha)$$

$$B^2=A^2+C^2-2AC\cos(\beta)$$

$$C^2=A^2+B^2-2AB\cos(\gamma)$$

poniendo: $P=cos(\alpha),Q=cos(\beta),R=cos(\gamma)$, se tiene:

$$P=\frac{B^2+C^2-A^2}{2BC}\;,Q=\frac{A^2+C^2-B^2}{2AC}\;,R=\frac{A^2+B^2-C^2}{2AB}$$

Se tienen dos casos:

Caso1. P,Q,R todos positivos.

Caso2. Alguno de los números P,Q,R es negativo.supondremos: $P>0,Q>0,R<0$.

Caso1. $P>0,Q>0,R>0$:

Se tiene que: $\pi=\alpha+\beta+\gamma$, y por trigonometría:

$$\frac{\pi}{2}=sen^{-1}(P)+sen^{-1}(Q)+sen^{-1}(R)$$

usando la serie de Taylor para la función $arcsen(x)=sen^{-1}(x)$, se tiene:

$$\frac{\pi}{2}=\sum_{n=0}^{\infty}\frac{(2n)!}{4^n(n!)^2(2n+1)}\left(P^{2n+1}+Q^{2n+1}+R^{2n+1}\right)$$

Caso2. $P>0,Q>0,R<0$:

Se tiene que: $\pi=\alpha+\beta+\gamma$, y por trigonometría:

$$\frac{\pi}{2}=sen^{-1}(P)+sen^{-1}(Q)-sen^{-1}(-R)$$

usando la serie de Taylor para la función $arcsen(x)=sen^{-1}(x)$, se tiene:

$$\frac{\pi}{2}=\sum_{n=0}^{\infty}\frac{(2n)!}{4^n(n!)^2(2n+1)}\left(P^{2n+1}+Q^{2n+1}-(-R)^{2n+1}\right)$$

2. EJEMPLOS.

2.1. Caso1. $(a_1,a_2)=(3,4),(b_1,b_2)=(1,1),(c_1,c_2)=(4,1)$

$$A^2=13\ ,B^2=9\ ,C^2=10$$

$$P=\frac{1}{\sqrt{10}}\ ,Q=\frac{7}{\sqrt{130}}\ ,R=\frac{2}{\sqrt{13}}$$

$$\frac{\pi}{2}=\sum_{n=0}^{\infty}\frac{(2n)!}{4^n(n!)^2(2n+1)}\left(\left(\frac{1}{\sqrt{10}}\right)^{2n+1}+\left(\frac{7}{\sqrt{130}}\right)^{2n+1}+\left(\frac{2}{\sqrt{13}}\right)^{2n+1}\right)$$

2.2. Caso2. $(a_1,a_2)=(2,2),(b_1,b_2)=(1,1),(c_1,c_2)=(5,1)$

$$A^2=2\ ,B^2=16\ ,C^2=10$$

$$P = \frac{3}{\sqrt{10}} \, , Q = -\frac{1}{\sqrt{5}} \, , R = \frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2} = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2 (2n+1)} \left(\left(\frac{3}{\sqrt{10}} \right)^{2n+1} - \left(\frac{1}{\sqrt{5}} \right)^{2n+1} + \left(\frac{1}{\sqrt{2}} \right)^{2n+1} \right)$$

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LA CONSTANTE $Log(2)$

EDGAR VALDEBENITO V.
(1991)

Resumen. Se muestran algunas fórmulas para la constante $Log(2) \equiv ln(2) = 0.69314...$

1. INTRODUCCIÓN.

Recordamos la clásica serie para $ln(2)$:

$$ln(2) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - ....$$

En esta nota se muestran algunas series trigonométricas para $ln(2)$.

2. FÓRMULAS.

2.1. Para $0 < x \leq \frac{\pi}{2}$, se tiene:

$$ln(2) = \sum_{n=1}^{\infty} \frac{(cos(x))^{2n} - (cos(2x))^{n}}{n}$$

2.2. Para $0 \leq x < \frac{\pi}{2}$, se tiene:

$$ln(2) = \sum_{n=1}^{\infty} \frac{(sen(x))^{2n} - (-1)^{n} (cos(2x))^{n}}{n}$$

2.3. Para $0 < x < \frac{\pi}{2}$, se tiene:

$$ln(2) = \frac{1}{2} \sum_{n=1}^{\infty} \frac{(sen(x))^{2n} + (cos(x))^{2n} - (cos(2x))^{2n}}{n}$$

2.4. Para $0 < x < \frac{\pi}{8}$, se tiene:

$$ln(2)=\sum_{n=1}^{\infty}\frac{(1-tan(x))^n-(tan(x))^{2n}-(1-tan(2x))^n}{n}$$

3. CASOS PARTICULARES.

3.1.

$$ln(2)=\sum_{n=1}^{\infty}\frac{\left(1/\sqrt{2}\right)^{2n}}{n}=\sum_{n=1}^{\infty}\frac{1}{n2^n}$$

$$ln(2)=\sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\sqrt{2+\sqrt{2}}\right)^{2n}-\left(\frac{1}{\sqrt{2}}\right)^n}{n}$$

$$ln(2)=\sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{2n}-\left(\frac{1}{2}\sqrt{2+\sqrt{2}}\right)^n}{n}$$

3.2.

$$ln(2)=\sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\sqrt{2-\sqrt{2}}\right)^{2n}-(-1)^n\left(\frac{1}{\sqrt{2}}\right)^n}{n}$$

$$ln(2)=\sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)^{2n}-(-1)^n\left(\frac{1}{2}\sqrt{2+\sqrt{2}}\right)^n}{n}$$

$$ln(2)=\sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}\right)^{2n}-(-1)^n\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^n}{n}$$

3.3.

$$ln(2)=\frac{1}{2}\sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\sqrt{2-\sqrt{2}}\right)^{2n}+\left(\frac{1}{2}\sqrt{2+\sqrt{2}}\right)^{2n}-\left(\frac{1}{\sqrt{2}}\right)^{2n}}{n}$$

$$ln(2)=\frac{1}{2}\sum_{n=1}^{\infty}\frac{\left(\frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}\right)^{2n}+\left(\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}\right)^{2n}-\left(\frac{1}{2}\sqrt{2+\sqrt{2}}\right)^{2n}}{n}$$

$$ln(2)=\frac{1}{2}\sum_{n=1}^{\infty}\frac{\left(\sqrt{\frac{5-\sqrt{5}}{8}}\right)^{2n}+\left(\frac{\sqrt{5}+1}{4}\right)^{2n}-\left(\frac{\sqrt{5}-1}{4}\right)^{2n}}{n}$$

3.4.

$$ln(2)=\sum_{n=1}^{\infty}\frac{\left(2-\sqrt{2}\right)^{n}-\left(\sqrt{2}-1\right)^{2n}}{n}$$

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