An Experiment for Light Position in Special Relativity

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Abstract

An experiment is proposed in Special Relativity (SR) such that two given coordinate systems are in relative motion and a light pulse is emitted from the origins along the positive x-axis when they are common. In a specific configuration of the two coordinate systems, it will be shown that the output of the Lorentz transformations (LT) for the position of light in the primed frame does not match the position of light according to the light postulate (LP) in the primed frame.

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I. Introduction

The purpose of LT is to correctly translate an event in a source coordinate system to an event in a target coordinate system that is moving in relative motion. Now, since SR is a math theory, it should be expected to provide the mathematical machinery necessary to prove that the LT translation matches what the target frame agrees is true. So for example, given LT translates light event E to E', other than by blind acceptance. how do we know E' is actually the correct answer in the primed frame?

SR does not lend itself well to such introspection, however any scientific theory should have no problem being subjected to mathematical proof analysis. Now, it is true that the translated light event E' does measure c and that can proven mathematically. However, there are many space-time coordinates that measure c such as $\left(x', y', z', \sqrt{x'^2+y'^2+z'^2}/c\right)$. So, the fact that E' measures c is not a sufficient condition to prove that the primed frame agrees E' is the correct specific space-time coordinate for the circumstances.

This entire analysis depends on the ability to create common agreed upon circumstances for both frames. This can be achieved by considering configurations of the two coordinate systems. So, an experiment is designed below such that the start of the experiment is the configuration where the two origins of the frames are common and a light pulse is emitted from the common origins in the positive direction of the common x-axis. The end of the experiment occurs when the primed coordinate $(-v/c,0,0)$ is co-located with the unprimed frame origin.

At the end of the experiment, the light pulse is located at some unique position on the positive x-axis of the primed frame coordinates. In the primed frame, the light's position is determined by LP. In the unprimed frame, the light's position is first determined by LP and then translated to primed frame coordinates by LT. If the result of LT does not match the primed frame LP result then LT got the answer wrong.

The next section gives the details and math for the experiment.

II. Experiment

For the experiment, assume two frames F and F' are in relative motion in the standard configuration. Assume further when the two origins M and M' are common, a light pulse is emitted along the common x-axis in the positive direction. Finally assume a primed frame observer is located at $C'=(-v/c,0,0)$.

Now a simple question is proposed. If the coordinate systems are in the configuration such that the origin M of the F frame and C' are co-located, which will be named event E, where is the light positioned in the coordinates of the F’ frame? It will be shown that LT calculates an answer that differs from the F’ frame’s light postulate position for the light. See figure 1 for details.

For the initial conditions, the origins of the two frames are co-located and a light flash is emitted from the two origins along the positive x-axis.

If F and F’ are in the configuration above with M and C’ co-located, LT in F places the light pulse at O’$_2$ for the F’ frame. The F’ frame light postulate, on the other hand, places it at O’$_1$ with O’$_1 \neq$ O’$_2$.

Figure 1

The F’ frame makes the calculation as follows. The time for M to travel from M' to C' while moving at a constant velocity v must be determined in order to figure out where the light is located in the F’ frame. Since C' is located at $(-v/c,0,0)$, then the time it takes for M to move from M' to C' is calculated as follows.

$$t' = (v/c)/v = 1/c \tag{1}$$

Hence, by the light postulate,

$$x'_{LP} = ct' = c/c = 1. \tag{2}$$

So the light is located at the F’ space-time coordinate

$$O'_1 = (1,0,0,1/c). \tag{3}$$

Now it will be shown that LT provides a different answer. For the F frame calculation, LT must be used to translate the location of the light from the F frame to the F’ frame coordinate. The F frame makes the calculation as follows. The time for C' to travel to M must be determined in order to figure out where the light is located in the F frame. Then, we can apply LT to translate where the light is in the F’ frame.

We know the location for C' is $(-v/c,0,0)$ in F’ frame coordinates and obviously M is located at $(0,0,0)$ in the F frame. So use the LT $x' = (x - vt)\gamma$ with $x = 0$ and $x' = -v/c$ then solve for t. In that case,

$$t = 1/c\gamma\ . \tag{4}$$

So, it takes $1/c\gamma$ seconds for C' to travel to M .

Now that we have the time in the F frame, apply the light postulate in the F frame. Hence, the light is located in the F frame at

$$ct = 1/\gamma\,. \tag{5}$$

Next, apply LT to determine the light's corresponding location in the F' frame. So, use $x' = (x - vt)\gamma$ with $x = 1/\gamma$ and $t = 1/c\gamma$. Then,

$$x'_{LT} = 1 - v/c\,. \tag{6}$$

By applying $t' = (t - vx/c^2)\gamma$ then

$$t' = (1 - v/c)/c\,. \tag{7}$$

Hence, if the two frames are in the configuration such that M/C' are co-located then according to LT, the light is physically located at the F' space-time coordinate

$$O'_2 = ((1 - v/c), 0, 0, (1 - v/c)/c)\,. \tag{8}$$

Note that O'_2 satisfies $c^2t'^2 = x'^2 + y'^2 + z'^2$. However, it is the wrong vector. Therefore given M/C' are co-located, the LT calculated space-time position for the light pulse in the coordinates of the F' frame (equation 8) disagrees with the F' LP calculated space-time position for the light pulse in the coordinates of the F' frame (equation 3). Obviously, given M/C' are co-located, one light pulse cannot be located at two different space-time coordinates along the positive x-axis of the F' frame.

One may argue this discrepancy is a result of the relativity of simultaneity (ROS). Under ROS, since the F' frame views the events E and O'_1 as simultaneous, then the F frame will not. Further, the F frame instead considers E and O'_2 as simultaneous events. Therefore it is built into ROS, given the two coordinate systems are in the configuration with M/C' co-located, that the LT mapped light position based on the F frame light position will not match the F' light position as mandated by the F' light postulate.

However as mentioned above, the purpose of LT is to correctly translate unprimed frame coordinates into primed frame coordinates. Given M/C' are co-located, the light pulse is at one unique position in the coordinates of the F' frame. LT either matches that correct answer or it does not. The mathematical analysis above proved that LT does not match the correct answer.

IV. Conclusions

This article presented a simple experiment in which it was proven for a specific configuration of two frame coordinates systems, the position of light calculated by LT for the F' frame did not match the position of light as provided by the F' light postulate.

V. References

[1] A. Einstein., *Annalen der Physik* **17**, 891-921 (1905); translated in English in A. Einstein, *The Principle of Relativity*, Dover, New York, p. 37-65, 1952.