A roadmap to some dimensionless constants of physics

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It is well known that nature's dimensionless constants variously take the form of mass ratios, coupling constants, and mixing angles. What is not generally known is that by considering a subset of these constants in a particular order (following a roadmap if you will) one can easily find accurate, but compact, approximations for each member of this subset, with each compact expression pointing the way to the next. Specifically, if the tau-muon mass ratio, the muon-electron mass ratio, the neutron-electron mass ratio, the fine structure constant, and the three largest quark and lepton mixing angles are considered in that order, one can readily find a way of compressing them into a closely-related succession of compact mathematical expressions.

I. THE ROADMAP

It is well known that nature's dimensionless constants variously take the form of mass ratios, coupling constants, and mixing angles. What is not generally known is that by considering a subset of these constants in a particular order — following a "roadmap" if you will — one can easily find accurate, but compact, approximations for each member of this subset, with each compact expression pointing the way to the next. Specifically, if the following dimensionless constants:

- the tau-muon mass ratio
- the muon-electron mass ratio
- the neutron-electron mass ratio
- the fine structure constant
- the three largest quark and lepton mixing angles

are considered in that order, one can readily find a way of compressing them into a closely-related succession of compact mathematical expressions (see [1–9] for related treatment of these same constants).

II. THE TAU-MUON MASS RATIO

So, if we take as our starting point the 2014 CODATA recommended value [10] for the tau-muon mass ratio 16.8167 (15), we immediately find that

$$\frac{M_\tau}{M_\mu} \approx 4.1^2 \quad , \tag{1}$$

which proves accurate to one part in $\sim$2500 (i.e., it is off by a factor of $\sim$0.9996).

III. THE MUON-ELECTRON MASS RATIO

And, if we then take the 2014 CODATA value for the muon-electron mass ratio 206.768 282 6 (46) we almost automatically find that

$$\frac{M_\mu / M_\mathrm{e}}{4.1^2} = 12.300\,314\ldots \quad , \tag{2a}$$

which immediately suggests this approximate formula

$$\frac{M_\mu}{M_\mathrm{e}} \approx 3 \times 4.1^3 \quad , \tag{2b}$$

which is accurate to one part in $\sim$40 000 [1, 4]. With a little additional effort we find

$$\frac{M_\mu}{M_\mathrm{e}} \approx \frac{4.1^3 - 0.1^3}{0.333\,319\,980\,8\ldots} \quad , \tag{2c}$$

which suggests the following still more precise approximation

$$\frac{M_\mu}{M_\mathrm{e}} \approx \frac{4.1^3 - 0.1^3}{0.333\,32} \quad , \tag{2d}$$

accurate to one part in $\sim$17 000 000 [3].

IV. THE NEUTRON-ELECTRON MASS RATIO

We can now experiment with the above denominator on the 2014 CODATA value for the neutron-electron mass ratio 1838.683 661 58 (90) to find

$$\frac{M_\mathrm{n}}{M_\mathrm{e}} \times 0.333\,32 = 612.870\,038\ldots \quad , \tag{3a}$$

which is an *un*interesting result: *a dead end.* But by modifying the value 0.333 32 only slightly — we simply need to add three — we get this *interesting* result

$$\frac{M_\mathrm{n}}{M_\mathrm{e}} \times 3.333\,32 - 4.1^3 = 6060.000\,022\,8 \quad , \tag{3b}$$

involving a value close to the integer

$$6060 = 6000 \times (1 + 10^{-2}) \quad . \tag{3c}$$

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Rearranging terms gives

$$\frac{M_{\rm n}}{M_{\rm e}} \approx \frac{4.1^3 + 6060}{3.333\,32} \quad , \tag{3d}$$

which proves accurate to one part in $\sim$270 000 000.

V. THE DIFFERENCE IN THE MUON AND NEUTRON DENOMINATORS

Now compare the denominator used in this *exact* formula for the 2014 CODATA muon-electron mass ratio

$$\frac{4.1^3 - 0.1^3}{0.333\,319\,980\,8\ldots} = 206.768\,282\,6 \tag{4a}$$

against the denominator used in this *exact* formula for the 2014 CODATA neutron-electron mass ratio

$$\frac{4.1^3 + 6060}{3.333\,319\,987\,5\ldots} = 1838.683\,661\,58 \quad . \tag{4b}$$

Specifically, observe that subtracting the above "muon-denominator" from the above "neutron-denominator" gives

$$\begin{array}{r} 3.333\,319\,987\,5 \\ -\ 0.333\,319\,980\,8 \\ \hline 3.000\,000\,006\,7 \end{array} \tag{4c}$$

a result that differs from three by just one part in $\sim$450 000 000. One naturally wonders if, physically, this value is not *exactly* three.

Note: The muon-electron mass ratio is poorly measured compared to the neutron-electron mass ratio. Conveniently, however, the decimal portion of 0.333 319 980 8 is only about four times less accurate than the decimal portion of 3.333 319 987 5 [10].

VI. USING THE NEUTRON DENOMINATOR WITH THE MUON-ELECTRON MASS RATIO

If we now solve for x in

$$\frac{4.1^3 + x}{-3 + 3.333\,319\,987\,5\ldots} = 206.768\,282\,6\,(46) \quad , \tag{5a}$$

whose denominator is exactly three *less* than the *neutron*-denominator used in Eq. (4b), we get

$$x \approx -0.000\,998\,6\,(15) \quad , \tag{5b}$$

where the 1σ uncertainty (15) in the last two digits is determined by the 1σ uncertainty (46) in the last two digits of the muon-electron mass ratio.

First question: Why should the above equation, which uses the *neutron*-denominator to help produce the *muon*-electron mass ratio, employ $-0.000\,998\,6$, a value so close to -10^{-3}? In fact, 0.000 998 6 differs from 10^{-3} by just 1 part in $\sim$700.

VII. THE FINE STRUCTURE CONSTANT

We will now turn our attention to the fine structure constant reciprocal $1/\alpha$, whose 2014 CODATA value is 137.035 999 139 (31). The obvious way to use 10 and 3 to approximate this value is

$$\begin{aligned} \frac{1}{\alpha} &\approx \frac{10^3}{3^3} + 10^2 \\ &= 137.\overline{037} \quad , \end{aligned} \tag{6a}$$

which is accurate to one part in $\sim$130 000. But how to make this formula still more accurate? To this end, let

$$\frac{10^3 + y}{3^3} + 10^2 + y = 137.035\,999\,139\,(31) \tag{6b}$$

and solve for y to get

$$y \approx -0.001\,000\,830\,(30) \quad , \tag{6c}$$

where the 1σ uncertainty (30) in the last two digits of y is determined by the 1σ uncertainty (31) in the last two digits of the fine structure constant reciprocal. Equation (6c) suggests the following more accurate approximation

$$\frac{10^3 - 10^{-3}}{3^3} + 10^2 - 10^{-3} = 137.036 \quad , \tag{6d}$$

which is accurate to one part in $\sim$160 000 000.

Second question: Why should solving for y produce $-0.001\,000\,830$, a value so close to -10^{-3}? This solution appears to "automatically reuse" -10^{-3} from the earlier muon-electron mass ratio approximation. In fact, 0.001 000 830 differs from 10^{-3} by just 1 part in $\sim$1200.

See [7–9] for how 137.036 relates to the cubic equation.

VIII. USING THE MUON DENOMINATOR WITH THE NEUTRON-ELECTRON MASS RATIO

If we now solve for z in

$$\frac{4.1^3 + z}{3 + 0.333\,319\,980\,8\ldots} = 1838.683\,661\,58\,(90) \quad , \tag{7a}$$

whose denominator is exactly three *more* than the *muon*-denominator used in Eq. (4a), we get

$$\begin{aligned} z &\approx 6059.999\,987\,63 \\ &\approx 6000 \times (1 + 0.009\,999\,997\,9) \quad . \end{aligned} \tag{7b}$$

Third question: Why should the above equation, which uses the *muon*-denominator to help produce the *neutron*-electron mass ratio, employ 0.009 999 997 9 (5), a value so close to 10^{-2}? In fact, 0.009 999 997 9 differs from 10^{-2} by just 1 part in $\sim$5 000 000.

Note that, above, the 1σ uncertainty (5) in the last digit of 0.009 999 997 9 is determined by the 1σ uncertainty (90) in the last two digits of the neutron-electron mass ratio.

IX. THE COMPOSITE FORMULA

This article's muon- and neutron-electron mass ratio formulas invite the following "composite" formula

$$\frac{\begin{Bmatrix} M_\mu \\ M_\text{n} \end{Bmatrix}}{M_\text{e}} \approx \frac{\mathbf{4.1^3} + \begin{Bmatrix} \mathbf{-10^{-3}} \\ 6060 \end{Bmatrix}}{\begin{Bmatrix} 0 \\ 3 \end{Bmatrix} + 0.333\,32} \approx \begin{Bmatrix} 206.768\,270\,730 \\ 1838.683\,654\,734 \end{Bmatrix} \tag{8}$$

which recovers either Eq. (2d) or (3d), depending on whether one chooses the upper or lower values in braces.

Now observe that the composite formula's similarity to two previous formulas is underscored by $\mathbf{4.1^3}$ and $\mathbf{-10^{-3}}$ appearing in boldface, where:

- the tau-muon mass ratio formula used $\mathbf{4.1^2}$.
- the fine structure constant formula used $\mathbf{-10^{-3}}$.

Because the composite formula shares $\mathbf{4.1^n}$ and $\mathbf{-10^{-3}}$ with the above two formulas, the composite formula is all the more likely not to be purely coincidental. Nor is the composite formula's use of 6060 unique. The Particle Data Group [11] gives the J/ψ-electron mass ratio as

$$\begin{aligned} \frac{M_{J/\psi}}{M_\text{e}} &= \frac{3096.916\,(11)\text{ MeV}}{0.510\,998\,928\,(11)\text{ MeV}} \\ &\approx 6060.514\,(22) \quad . \end{aligned} \tag{9}$$

X. THE THREE LARGEST QUARK AND LEPTON MIXING ANGLES

We will now turn our attention to the less accurately measured quark and lepton mixing angles, or, more specifically, the sines squared of the three largest mixing angles.

Firstly, experiment [5, 12] tells us that

$$\begin{aligned} \sin^2 L12 &= 0.304^{+0.013}_{-0.012} \\ &\approx 0.3 \quad . \end{aligned} \tag{10a}$$

This value is readily interpreted as 3/10, a ratio employing the same two small integers we have already seen repeatedly.

Secondly, experiment [5, 13] tells us that

$$\sin Q12 = 0.225\,36 \pm 0.000\,61 \quad ,$$

which gives

$$\begin{aligned} \sin^2 Q12 &= 0.050\,78 \pm 0.000\,28 \\ &\approx 0.05 \quad . \end{aligned} \tag{10b}$$

Now, experiment [5, 12] also tells us that

$$\sin^2 L23 = 0.452^{+0.052}_{-0.028} \quad .$$

If we then round off this value as follows

$$\sin^2 L23 \approx 0.5 \quad ,$$

we get

$$\begin{aligned} \frac{\sin^2 Q12}{\sin^2 L23} &\approx \frac{0.05}{0.5} \\ &\approx 10^{-1} \quad , \end{aligned} \tag{10c}$$

another power of 10. In and of itself these results are not so remarkable (these values are, after all, poorly measured). But the above relations lead to an economical mixing model where all six mixing angles fit experiment, which is not so trivial. This model is explained in [5, 9].

XI. ORIGINS OF THE ROADMAP

As it happened the author did not discover the above formulas by following the roadmap exactly: The muon- and neutron-electron mass ratio formulas were the first found, with Eqs. (2b)–(3d) identified in that order, over a span of about ten minutes (after a year of effort more than a decade and a half ago). A month later the value 137.036 was investigated, and the fine structure constant approximation was quickly found. Only much later was the tau-muon mass ratio even considered. In retrospect, it appears that the tau-muon mass ratio actually provided the most favorable starting point for finding this article's formulas.

Moreover, it appears that the two biggest obstacles to progress were: firstly, *not knowing which dimensionless constants to consider*; and, secondly, *not knowing in what order to consider them.* It now appears that a researcher who knew enough to investigate *just the constants of the roadmap in just the order specified* might well be able to discover all of this article's formulas on his own with comparative ease.

XII. THE ROADMAP BRANCHES OUT

Now suppose that the roadmap's formulas are, in fact, non-coincidental — i.e., that they underlie physical laws. One might then expect them to provide a convenient jumping off point to mathematics that is interesting in its own right. In fact, the roadmap's formulas do lead to some interesting mathematics, as shown by the following articles on:

- a substitution map [6].
- a nonstandard cubic equation [8].

Similarly, one might expect the roadmap's formulas to lead to more general formulas that mathematically model a wider range of physical results. In fact, they do, as shown by the following articles on:

- all six mixing angles [5, 9].
- the Weinberg angle [7].

In the first instance, a quark and lepton mixing model embracing all six mixing angles is built, by exploiting an intrinsic property of rotation matrices to constrain the mixing angles two additional ways. In the second instance, the Weinberg angle is added to the list of dimensionless constants reproduced, by building on an unusually economical solution to a nonstandard cubic equation.

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