Dimensions of the Universe

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I Introduction

In this paper, we will present a new perspective of certain dimensions that manifest physically and therefore can be studied. Theoretical ground work will be set within General Relativity since it provides the highest accuracy in cosmology, but we will also "bridge" to quantum mechanics near the end of the paper.

Using the notion of "physical dimensions" we will explain some aspects that have puzzled physicists so far, including what is known as "the arrow of time". It will be proven theoretically that there is no such thing and that time has no direction since temporal motion requires only expansion and a velocity of that expansion, time expands in all directions and influences the expansion of space, hence inflating space and forming the spacetime continuum from the earliest age of the Universe, the Big Bang, to present time.

We will first take a relatively short part of the paper to introduce the reader in the necessary subjects.

1. Mathematical Dimensions

The dimension of a mathematical space/object is informally defined as the minimal number of coordinates needed to specify any point within it.

In mathematics the dimension of an object is an instrict property, independent of the space in which the object is embedded. This is not the case with physical dimensions.

The dimension of Euclidean n-space (E^n) is (n).

Using a standard inner product on (R^n), for two real n-vectors (x) and (y), we define the distances between points and angles, between the lines or vectors and form a set of points as Euclidean space:

$$(1)\; x \cdot y = \sum_{i=1}^{n} x_i \cdot y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + \cdots + x_n y_n$$

where ($x_i y_i$) are i-th coordinates of vectors (x) and (y), respectively. The result must always be a real number.

2. Dimensions in Physics

In physics, we will base in General Relativity, which describes the Universe with the most accurate cosmological models. In General, and subsequently Special, theory of Relativity the structure of the Universe is described as consisting of three spatial and one dimension of time, which form the space-time continuum (further on it will be called spacetime).

Manifolds are used to form adequate cosmological models but the specific type used in General Relativity is known as “Lorentzian manifold” and the cosmological model of Special Relativity is known as “Minkowski spacetime”.

2.1. Manifolds

A manifold is a space which is similar to Euclidean space in local aspects, meaning that it can be converted by coordinate patches therefore having a structure that permits differentiation to be defined. However, the structure does not distinguish intrinsically between different coordinate systems; hence the only concepts defined by the manifold structure are those which are independent of the choice of a coordinate system.

A pair (M, C) such that (M) is an arbitrary set and (C) is a family of functions, such that $(C = (S \subset C)_M)$, is called a differential space.

If $(\ C_0 \coloneqq \ \{f_1, f_2, f_3, \dots, f_n\})$ is a family of real functions on (M), and $(C = (S \subset C_0)_M)$ then the pair (M, C) would be called differential space generated by (C_0), denoted by $(C = genC_0)$. Functions $(f_1, f_2, f_3, \dots, f_n)$ are then called generators.

Now we have two differential spaces (M, C) and (N, D).

Mapping $(F: M \to N)$ can be called smooth if:

$$(2)\ \forall_{f \in D}\ f \cdot F \in C$$

(F) can be called diffeomorphism if it is bijective and both (F) and (F^{-1}) are smooth.

In such a case that there exists some fixed $(n \in N)$ and a countable, or finite, covering $(\{A\}_{i \in I})$ of (M), such that for all $(i \in I)$ there exists diffeomorphism:

$$(3)\ F_i: \left(A_i, C_{A_i}\right)\ \to (R^n, C^\infty(R^n))$$

then (M, C) is called a manifold.

If (M, C) is a differential space then any $(f\ \in C)$ is called a smooth function.

2.2. Lorentzian manifolds

A Lorentzian manifold is a pair (M, g) where (M) is an n-dimensional smooth manifold and (g) is a Lorentzian metric which means that (g) associates with each point $(p \in M)$ a Lorentzian scalar product (g_p) on the tangent space $(T_p M)$.

It is needed that (g_p) depends smoothly on (p); this means that for any choice of local coordinates $(x = (x_1, \dots, x_n): U \to V)$, where $(U \subset M)$ and $(V \subset R^n)$ are open subsets, and for any $(a, b = 1, \dots, n)$ the functions $(g_{ab}: V \to R)$ defined by:

$$(4)\ g_{ab} = (\frac{\partial}{\partial x_a}, \frac{\partial}{\partial x_b})$$

are smooth.

Here $(\frac{\partial}{\partial x_a})$ and $(\frac{\partial}{\partial x_b})$ denote the coordinate vector fields; with respect to these coordinates we write:

$$(5)\ g = \sum_{a,b} g_{ab}\ dx_a \otimes dx_b$$

or shortly:

$$(6)\ g = \sum_{a,b} g_{ab} dx_a dx_b$$

Let (N, h) be a connected manifold and $(I \subset R)$ an open interval. For any $(t \in I)$, $(p \in N)$ we identify:

$$(7)\ T_{(t,p)}(I \times N) = T_t I \oplus T_p N$$

Then for any smooth positive function $(f : I \to (0, \infty))$ the Lorentzian metric $(g = -dt^2 + f(t)^2 \cdot h)$ on $(I \times M)$ is defined as follows:

For any $(\varepsilon_1, \varepsilon_2 \in T_{(t,p)}(I \times N))$ we write $(\varepsilon_1 = (\alpha_a \frac{d}{dt}) \oplus \mathcal{S}_a)$ with $(\alpha_a \in R)$ and $(\mathcal{S}_a \in T_p N)$, $(a = 1{,}2)$ and we have:

$$(8)\ g(\varepsilon_1, \varepsilon_2) = -\alpha_1 \cdot \alpha_2 + f(t)^2 \cdot h(\mathcal{S}_1, \mathcal{S}_2)$$

Such a Lorentzian metric is called “warped product metric”.

This example covers the Robertson-Walker spacetimes, where it is required additionally that (N, h) is complete and that it has constant curvature.

Friedmann cosmological models are of this type; they are used in General Relativity to discuss Big Bang, expansion of the Universe and cosmological redshifts.

3. Physical dimensions

Physical dimensions, unlike mathematical dimensions, have influence and manifest in the physical world that is in nature and hence can be directly or indirectly observed. We will define only one type of physical dimensions, a temporal type that will be named “String dimensions”. The name is derived to honor the String theory.

3.1. String dimensions

String dimensions are temporal physical dimensions. There is only one global String dimension, a dimension of time and many, many local ones but although temporal they are not dimension of time but rather dimensions of time dilatation. The global one is, as it will be later explained, responsible for the entire nature of our Universe, including the expansion of time and the speed of that expansion. Local String dimensions are a product of celestial bodies curving spacetime which, due to "global-to-local" influence of String dimensions, manifests as gravitation; they are local axis dimensions of time dilatation caused by mass of celestial bodies, they are not dimensions of time itself.

Local String dimensions function as an axis to every curvature in spacetime caused by mass of matter fields. After mass curves the spacetime, the curvature gains an axis that is a local String dimension, which led to every celestial body having elliptical shape and an axis of rotation.

This can be witnessed even with modern observations during the process of star formation, for example: as the gas clouds start forming and therefore gaining "centralized mass", they begin to curve spacetime and hence an axis of that curvature, a local string dimension (t_l), forms and thus the curvature dictates the gas cloud to take elliptical shape while (t_l) interacts with the global temporal dimension (t_G) forcing the now elliptical body to gain momentum and rotate around its axis. This will be further debated and proven theoretically in the latter chapter of the paper, since we require much other work, mostly from General Relativity, in order to form "dimensional equations"; we will however develop two simple equations that will be crucial in those latter chapters.

We will use the simple equation:

$$(9)\ (x, y, z) = (l, m, n)$$

Meaning that ($x = l; y = m; z = n$). Further on we define that:

$$(10)\ l = e^{t(x)}; m = e^{t(y)}$$

we do not define (n) since it is not necessary. We form the two equations we need:

$$(11)\ t(x) = log_e\, x;\ t(y) = log_e\, y\ \Rightarrow t(x) = \ln x\, ; t(y) = \ln y$$

These simple equations are all that we need for the time being.

II General Relativity

We will now enter the domain of the most accurate and beautiful scientific theory. One could even name it "the divine theory" since it allows us to create accurate cosmological models of the Universe. Using nothing other than natural logic, mathematics and imagination one can consciously develop a functioning model of a universe which, in return, helps us understand our own Universe. We will begin by using a manifold.

1. Manifold

The model we will use for spacetime is a Lorentzian manifold, as we mentioned before that is the manifold type used in General Relativity. We have a pair (M, g) where (M) is a connected $(4 + n)$-dimensional Hausdorf (C^{∞}) manifold and (g) is a Lorentzian metric with a signature $(+2)$ on (M).

Note*: We will temporarily neglect the $(+n)$ part and treat (M) as a connected four-dimensional Hausdorf (C^{∞}) manifold.*

(M, g) and (M', g') will be taken as equivalent if they are isometric, meaning that there should be a diffeomorphism $(\theta: M \to M')$ which would take the metric (g) into the metric (g'), that is:

$$(12)\ \theta_{*}\, g = g'$$

The metric (g) enables the non-zero vectors at a point $(p \in M)$ to be divided into three classes: a non-zero vector $(X \in T_p)$ that is timelike, spacelike or null if $(g(X, X))$ is negative, positive or zero, respectively.

The order of differentiability (r) of the metric should be sufficient for the field equations to be defined. They can be defined in a distributional sense if the metric coordinate components (g_{ab}) and (g^{ab}) are continuous and have locally square integrable generalized first derivatives with respect to local coordinates. A set of functions $(f_{;a})$ on (R^n) is said to be a generalized derivative of a function (f) on (R^n) if for any (C^{∞}) function (Ψ) on (R^n) with compact support:

$$(13)\ \int f_{;a}\, \Psi\, d^n x = \int f \left(\frac{\partial \Psi}{\partial x^a}\right) d^n x$$

However, this condition is too weak since it does not guarantee neither the existence nor the uniqueness of geodesics. We will now assume that the metric is at least (C^2). The (C^r) pair (M', g') is a (C^r)-extension of (M, g) if there is an isometric (C^r) imbedding $(\mu: M \to M')$.

We require that the model (M, g) is (C^r)-inextensible, meaning that there is no (C^r) extension (M', g') of (M, g) where $(\mu(M))$ does not equal (M').

A pair (M, g) is (C^r) locally inextensible if there is no open set $(U \subset M)$ with non-compact closure in (M), such that the pair $(U, {}^{g}/_{n})$ has an extension (U', g') in which closure of the image of (U) is compact.

2. Matter fields

We denote the matter fields as:

$$\Psi(i)^{a \dots b}{}_{c \dots d}$$

where the sub-script (i) numbers the fields considered.

The following two postulates on the nature of the equations obeyed by the $(\Psi(i)^{a \dots b}{}_{c \dots d})$ are common to both Special and General Relativity.

2.1. The first postulate: Local causality

The equations governing the matter fields must be such that if (U) is a convex normal neighborhood and (p) and (q) are points in (U), then a signal can be sent in (U) between (p) and (q) if and only if (p) and (q) can be joined by a (C^1) curve lying entirely in (U), whose tangent vector is everywhere non-zero and is either timelike or null; hence called “non-spacelike”.

Whether the signal is sent from (p) to (q), or vice versa, will depend on the direction of time in (U).

This postulate is what sets apart the metric (g) from other fields on (M) and gives it its distinctive geometrical character.

If $(\{x^a\})$ are normal coordinates in (U) about (p), then we can conclude that the points that can be reached from (p) by non-spacelike curves in (U) are those whose coordinates satisfy:

$$(14)\ (x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 \leq 0$$

The boundary of these points is formed by the image of the null cone of (p) under the exponential map, that is the set of all null geodesics through (p). Therefore by observing which points can communicate with (p), we can determine the null cone (N_p) in (T_p). Once the (N_p) is known, the metric at (p) may be determined up to a conformal factor.

2.2. The second postulate: Local conservation of energy and momentum

The equations governing the matter fields are such that there exists a symmetric tensor (T^{ab}), known as the energy momentum tensor, which depends on the fields, their covariant derivatives and the metric; all of which has properties:

1) (T^{ab}) vanishes on an open set (U) if and only if all the matter fields vanish on (U).

2) (T^{ab}) obeys the equation:

$$(15)\ T^{ab}{}_{;b} = 0$$

3. Lagrangian formulation

Let (L) be a Lagrangian which is a scalar function of the fields $(\Psi(i)^{a\ldots b}{}_{c\ldots d})$, their first covariant derivatives and the metric. We obtain the equations of the fields by requiring that the action:

$$(16)\ \int_D L\, dv$$

be stationary under variations of the fields in the interior of a compact four-dimensional region (D). By variation of the fields $(\Psi(i)^{a\ldots b}{}_{c\ldots d})$ we mean a one-parameter family of fields $(\Psi(i)(u,r))$ where $(u \in (-\varepsilon, \varepsilon))$ and $(r \in M)$, such that:

1) $(17)\ \Psi(i)(0,r) = \Psi(i)(r)$
2) $(18)\ \Psi(i)(u,r) = \Psi(i)(r)$ when $(r \in M - D)$

We denote $(\partial\Psi(i)(u,r)/\partial u|_{u=0})$ by $(\Delta\Psi(i))$.

Then:

$$(19)\ \left.\frac{\partial I}{\partial u}\right|_{u=0} = \sum_{(i)} \int_D \left(\frac{\partial L}{\partial\Psi(i)^{a\ldots b}{}_{c\ldots d}} \Delta\Psi(i)^{a\ldots b}{}_{c\ldots d} + \frac{\partial L}{\partial\Psi(i)^{a\ldots b}{}_{c\ldots d;e}} \Delta(\Psi(i)^{a\ldots b}{}_{c\ldots d;e})\right) dv$$

where $(\Psi(i)^{a\ldots b}{}_{c\ldots d;e})$ are the components of the covariant derivatives of $(\Psi(i))$ but $(\Delta\left(\Psi(i)^{a\ldots b}{}_{c\ldots d;e}\right) = \Delta(\Delta\Psi(i)^{a\ldots b}{}_{c\ldots d;e}))$, hence the second term can be expressed as:

$$(20)\ \sum_{(i)} \int_D \left[\left(\frac{\partial L}{\partial\Psi(i)^{a\ldots b}{}_{c\ldots d;e}} \Delta\Psi(i)^{a\ldots b}{}_{c\ldots d}\right)_{;e} - \left(\frac{\partial L}{\partial\Psi(i)^{a\ldots b}{}_{c\ldots d;e}}\right)_{;e} \Delta\Psi(i)^{a\ldots b}{}_{c\ldots d}\right] dv$$

The first term in this expression can be written as:

$$(21)\ \int_D Q^a{}_{;a}\, dv = \int_{\partial D} Q^a\, d\sigma_a$$

Where (Q) is a vector whose components are:

$$(22)\ Q^e = \sum_{(i)} \frac{\partial L}{\partial\Psi(i)^{a\ldots b}{}_{c\ldots d;e}} \Delta\Psi(i)^{a\ldots b}{}_{c\ldots d}$$

This integral is zero as condition two states that ($\Delta\Psi(i)$) vanishes at the boundary (∂D). Hence in order that ($\partial I/\partial u|_{\partial u=0}$) should vanish for all variations on all volumes (D), it is necessary and sufficient that the Euler-Lagrange equations:

$$(23)\ \frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d}} - \left(\frac{\partial L}{\partial \Psi(i)^{a\ldots b}{}_{c\ldots d;e}}\right)_{;e} = 0$$

hold for all (i).

We obtain the energy momentum tensor from the Lagrangian by considering the change in the action induced by a change in the metric.

4. Field equations

To determine what the field equations should be we shall determine the Newtonian limit. Since the Newtonian gravitational field equation does not include time, correspondence with the Newtonian theory should be made with a metric that is static, meaning a metric that admits a timelike Killing vector field (K) which is orthogonal to a family of spacelike surfaces, which can be regarded as surfaces of constant time and may be labeled by the parameter (t).

We define the unit timelike vector (V) as ($f^{-1}K$) where ($f^2 = -K^aK_a$). Then ($V^a{}_{;b} = -\dot{V}_aV_b$) where ($\dot{V}^a = V^a{}_{;b}V^b = f^{-1}f_{;b}g^{ab}$) represents the departure from geodesity of the integral curves of (V), which are also the curves of (K). Note that ($\dot{V}^aV_a = 0$).

These integral curves define the static frame of reference. We can derive an equation for the Newtonian gravitational potential by considering the divergence of ($\dot{V}^a$):

$$(24)\ \dot{V}^a{}_{;a} = \left(V^a{}_{;b}V^b\right)_{;a} = V^a{}_{;b;a}V^b + V^a{}_{;b}V^b{}_{;a} = R_{ab}V^aV^b + \left(V^a{}_{;b}\right)_{;b}V^b + \left(V_b\dot{V}^b\right)^2$$
$$= R_{ab}V^aV^b$$

But:

$$(25)\ \dot{V}^a{}_{;a} = \left(f^{-1}f_{;b}g^{ab}\right)_{;a} = -f^{-2}f_{;a}f_{;b}g^{ab} + f^{-1}f_{;ba}g^{ab}$$

and:

$$(26)\ f_{;ab}V^aV^b = -f_{;a}V^a{}_{;b}V^b = -f^{-1}f_{;a}f_{;b}g^{ab}$$

So we find:

$$(27)\ f_{;ab}\left(g^{ab} + V^aV^b\right) = f\ R_{ab}V^aV^b$$

We therefore obtain agreement with the Newtonian theory in the limit of a weak field, when ($f \cong 1$), if the term on the right is equal to ($4\pi G$) times the matter density plus terms which are small in the weak field limit. This will be the case if there is a relation:

$$(28)\ R_{ab} = K_{ab}$$

where (K_{ab}) is a tensorial function of the energy momentum tensor and the metric, which is such that ($(4\pi G)^{-1}\ K_{ab}V^aV^b$) is equal to the matter density plus terms which are small in the Newtonian limit since (R_{ab}) satisfies the contracted Bianchie identities ($R_a{}^b{}_{;b} = \frac{1}{2}R_{;a}$), implies that:

$$(29)\ K_a{}^b{}_{;b} = \frac{1}{2}K_{;b}$$

which shows that the apparently natural equation ($K_{ab} = 4\pi G T_{ab}$) cannot be correct due to the equation above (eq29) and the conservation equations ($T_a{}^b{}_{;b} = 0$) would imply ($T_{;a} = 0$).

The only first order identities satisfied by the energy-momentum tensor are the conservation equations. The only tensorial function (K_{ab}) of the energy-momentum tensor and the metric, which obeys the identities ($K_a{}^b{}_{;b} = \frac{1}{2}K_{;b}$) for all energy-momentum tensors is the:

$$(30)\ K_{ab} = k\left(T_{ab} - \frac{1}{2}T\ g_{ab}\right) + \lambda g_{ab}$$

where (k) and (λ) are constants. The values of these constants can be determined from the Newtonian limit, for example a perfect fluid with energy density (μ) and pressure (p) whose flow lines are the integral curves of the Killing vector, hence:

$$(31)\ f_{;ab}(g^{ab} + V^aV^b) = f\left(\frac{1}{2}k(\mu + 3p) - \lambda\right)$$

In the Newtonian limit the pressure is usually very small compared to energy density. We would then obtain approximate agreement with Newtonian theory if ($k = 8\pi G$) and if (λ) is very small. We shall use the units of mass where ($G = 1$), in these units a mass of ($10^{28}g$) corresponds to a length of ($1cm$).

We then integrate the previous equation over a compact region (F) at the three surface where ($t = constant$), and transform the left hand into an integral of the gradient of (f) over the bounding two-surface (∂F):

$$(32)\ \int_F\ f\big(4\pi(\mu + 3p)\big)d\sigma = \int_F\ f_{;ab}(g^{ab}+V^aV^b)d\sigma = \int_{\partial F} f_{;a}(g^{ab} + V^aV^b)d\tau_b$$

where $(d\sigma)$ is the volume element of the three-surface where $(t = const)$, in the induced metric, and $(d\tau_b)$ is the surface element of the two-surface (∂F) in the three surface.

Here we realize two important differences from the Newtonian case:

1) A factor (f) appears in the integral on the right-hand side.
2) The pressure contributes to the total mass.

Hence we form the equations:

$$(33)\ R_{ab} = 8\pi\left(T_{ab} - \frac{1}{2}Tg_{ab}\right) + \lambda g_{ab}$$

known as "Einstein field equations" and can also be written in the form:

$$(34)\ \left(R_{ab} - \frac{1}{2}Rg_{ab}\right) + \lambda g_{ab} = 8\pi T_{ab}$$

Since both sides are symmetric, these form a set of ten coupled non-linear partially differential equations in the metric and its first and second derivatives. However, the covariant divergence of each side vanishes identically:

$$(35)\ \left(R^{ab} - \frac{1}{2}Rg^{ab} + \lambda g^{ab}\right)_{;b} = 0 \quad and\ T^{ab}{}_{;b} = 0$$

hold independent of the field equations, which means that the field equations really provide only six independent differential equations of the metric, which is the correct number of equations to determine the spacetime, since four of the ten components of the metric can be given arbitrary values by use of the four degrees of freedom to make coordinate transformations. Einstein equations can also be determined by requiring that the action:

$$(36)\ I = \int_D (A(R - 2\lambda) + L)dv$$

be stationary under variations of (g_{ab}), where (L) is the matter Lagrangian and (A) is a suitable constant for:

$$(37)\ \Delta\big((R-\lambda)dv\big) = \left((R-2\lambda)\frac{1}{2}g^{ab}\Delta g^{ab} + R_{ab}\Delta g^{ab} + g^{ab}\Delta R_{ab}\right)dv$$

which can be written as:

$$(38) g^{ab}\Delta R_{ab}dv = g^{ab}\big((\Delta\Gamma^c{}_{ab})_{;c} - (\Delta\Gamma^c{}_{ab})_{;b}\big)dv = (\Delta\Gamma^c{}_{ab}g^{ab} - \Delta\Gamma^d{}_{ad}g^{ac})_{;c}dv$$

Thus it can be transformed into an integral over the boundary (∂D), which vanishes as $(\Delta\Gamma^a{}_{bc})$ vanishes the boundary:

$$(38)\ \left.\frac{\partial I}{\partial u}\right|_{u=o} = \int_D \left\{A\left(\left(\frac{1}{2}r-\lambda\right)g^{ab}-R^{ab}\right)+\frac{1}{2}T^{ab}\right\}\Delta g_{ab}dv$$

hence if $(\partial I/\partial u)$ vanishes for all (Δg_{ab}), we obtain the Einstein equations on the setting $(A = (16\pi)^{-1})$.

5. Dimensional equations

We will define the global String dimension for a flat expanding spacetime and then a local String dimension as an axis of a local curvature in spacetime caused by a celestial body.

We use the two equations from before $(t(x) = \log_e x\,; t(y) = \log_e y)$ and input an equation:

$$(39)\ \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \lim_{a\to 0}\left(1+\frac{1}{\alpha}\right)^{\frac{1}{\alpha}} = e$$

We now naturalize the equation for both cases individually.

5.1. Global String dimension

The global String dimension is definitive for the whole Universe, since the point of singularity to this, present, time. It defines the "temporal motion" which means that time is expanding without a specific direction, in all directions positive to the observer, and it also causes spatial expansion which will be shown later as well as that it accelerates the spatial expansion.

We naturalize the equation by defining that there are two angles (α_x, α_y) formed by the intersection of an imaginary line that will represent (t_G) and another imaginary line (d) that will represent a diameter of space. The two angles are always:

$$(40)\ \alpha_x = \alpha_y = 90^\circ$$

These two imaginary angles will represent the difference between the present and the ongoing "temporal motion" that is the future. We will represent this temporal motion with an (↠) symbol, in honor of the previous idea of the "arrow of time". The second part about local String dimensions will deal with "gravitational time dilatation" which too will be very important, but this part will deal with the nature of time itself. The temporal motion will be defined on a quantum level in the latter section, for now we will define it only with the example:

We have two events (e_1) and (e_2), both events are on the same spatial position (x, y, z) thus $(e_1(x, y, z, t_1))$, $(e_2(x, y, z. t_2))$ since the second event is after the first:

$$(41)\ t_2 = t_1 + \twoheadrightarrow$$

This is the simplest way to describe the temporal motion.

However, it is far too simple to be accepted hence we will need to also define the expansion of time, we denote it as $(a(t))$ which is the scale factor that depends on time and it will describe the expansion of time, that influences the spatial expansion, hence the expansion of the whole Universe. We declare that $(a(t))$ is heading to a *realistic maximum* $(rmax)$. It is not certain if this maximum exists so we also define that $(rmax = \infty)$ if there is no maximum or $(rmax \neq \infty)$ if there is a maximum.

Now we define a set of equations to explain how the global String dimension influences the space to expand:

$$(42)\ t_G \begin{cases} t_G(x) = \log_{\lim\limits_{x \to \infty}\left(\frac{\twoheadrightarrow}{x} + \twoheadrightarrow\right)^x}(x) \\ t_G(y) = \log_{\lim\limits_{y \to \infty}\left(\frac{\twoheadrightarrow}{y} + \twoheadrightarrow\right)^y}(y) \\ t_G(z) = z + \twoheadrightarrow \end{cases}$$

This is the reason why our spacetime is nearly flat. If the (z) spatial dimension were to expand as the (x) and (y) do, our spacetime would be shaped like a saddle.

Or in other words, we imagine the spacetime as:

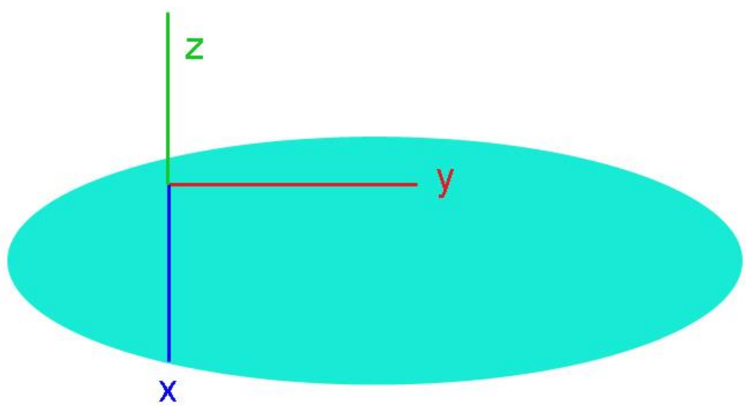

Figure 1: the flat spacetime

Meaning that the (x) and (y) are expanding while spacetime is "moving on", or "expanding on", the (z) dimension. On an image it would look like this:

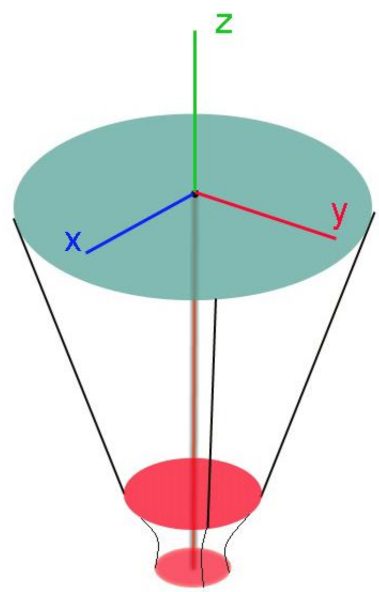

Figure 2: spacetime from the Big Bang to the present

Now we rewrite the equations as:

$$(43)\ t_G \begin{cases} t_G(x) = \log_{\lim\limits_{a(t)\to rmax}\left(\frac{\twoheadrightarrow}{\alpha_x}+\twoheadrightarrow\right)^{\frac{\twoheadrightarrow}{\alpha_x}}}(x) \\ t_G(y) = \log_{\lim\limits_{a(t)\to rmax}\left(\frac{\twoheadrightarrow}{\alpha_y}+\twoheadrightarrow\right)^{\frac{\twoheadrightarrow}{\alpha_y}}}(y) \\ t_G(z) = z+\twoheadrightarrow \end{cases}$$

Conclusion: *The global String dimension expands in all directions as represented by* $(a(t))$*, therefore making an observer feel like time is "passing". The global String dimension also forces the spatial dimensions to expand hence forming the spacetime continuum and by expanding in all directions we could say that time inflates space and it has done so from the soul beginning of the Big Bang.*

5.2. Local String dimensions

As we mentioned before, local String dimensions function as an axis of a curvature in spacetime.

In order to represent them we proclaim a celestial body as a sub-coordinate system (x', y', z') of the coordinate system (x, y, z, t) used in General Relativity. Physical dimensions do not count among the mathematical dimensions; there must always be only one mathematical temporal dimension in General Relativity (t), but physical dimensions do have to be accounted for which is why we used a $(4+n)$-dimensional manifold at the beginning of this chapter. The (t) mathematical dimension is the global String dimension, described in a different way, therefore we respect that there is only one dimension of time.

We also respect that the celestial body has a mass (M) and the size of the sub-coordinate system (x', y', z') accounts for the bodies volume.

***Note**: It is easier in practice to represent the (x', y', z') in spherical coordinates to account for the volume of the celestial body but it is not necessary in this example.*

$$(44)\ t_l \begin{cases} t_l(x') = \log_{\lim\limits_{a(t)\to rmin}\left(\frac{\twoheadrightarrow}{\alpha_{x\prime}}+\twoheadrightarrow\right)^{\frac{\twoheadrightarrow}{\alpha_{x\prime}}}}(x) \\ t_l(y') = \log_{\lim\limits_{a(t)\to rmin}\left(\frac{\twoheadrightarrow}{\alpha_{y\prime}}+\twoheadrightarrow\right)^{\frac{\twoheadrightarrow}{\alpha_{y\prime}}}}(y) \\ t_l(z') = \dfrac{z+\twoheadrightarrow}{GM} \end{cases}$$

Where ($rmin$) stands for *realistic minimum* and ($\alpha_{x\prime}$) and ($\alpha_{y\prime}$) are angles formed by a diameter of the equator of the celestial body and (t_l), that is the axis. The axis (t_l) will be parallel to (z) if and only if there is no axial tilt, represented by an angle (θ_{at}).

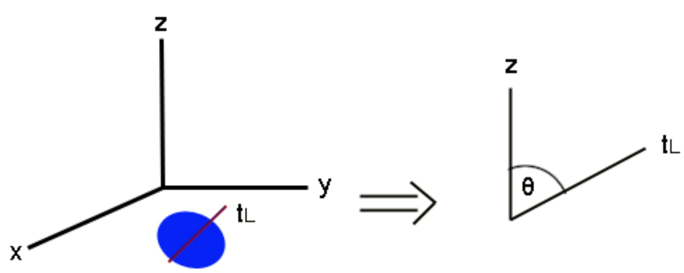

Figure 3:$(t_l \parallel z)\ if\ and\ only\ if\ (\theta_{at} = 0^\circ)$.

Observing the third equation ($t_l(z') = \frac{z+\twoheadrightarrow}{GM}$) we can see that gravitational time dilatation occurs do to the mass of the celestial body that curves spacetime. However this is only a farce.

By observing the first two equations we see that celestial bodies decrease the expansion of time and influence temporal motion, decelerating it, and by observing the third equation we see that it is due to the mass of the celestial body, meaning that the deceleration of temporal motion and decreasing of the expansion of time are only at a local level, in the approximate vicinity of the celestial body, appearing as a “curvature”.

This dilates time at the local level and since, as mentioned before, time influences spatial expansion, it also manifests as a curvature in spacetime around the celestial body. This means that the local String dimension is an axis dimension of time dilatation not of time itself. After it is formed it acts as an axis of the curvature in spacetime, hence the curvature forces the body to have an elliptical shape which leads to the body gaining momentum and rotating around its axis and therefore it has gravitational influence. Every celestial body with an axis causes time dilatation to some extent. In the simplest of terms one could say that gravity is a product of time dilatation, not the reverse. Time dilatation is a reduced rate of the expansion of time and decelerated temporal motion on a local level due to a massive body. Time dilatation caused by velocity will be explained later on in the paper.

Bodies that are more massive cause higher levels of time dilatation and therefore have stronger gravitational influence.

Conclusion: *After the local String dimension is formed it is unchanging, meaning that the angles ($\alpha_{x\prime}$) and ($\alpha_{y\prime}$) formed by the intersection of the axis (t_l) and the diameter of the equator (d) must always be approximately (90°). This often causes the axial tilt, various gravitational influences tilt the body and the axis doesn't remain parallel to (z) hence the angle (θ_{at}). Even if the body is to vanish, such as a star going supernova, the local String dimension will remain and preserve the curvature to some extent which causes a gravitational anomaly that gradually becomes a black hole.*

6. Temporal motion

Unlike spatial motion, temporal motion requires no direction. Instead of a trajectory it needs expansion and it needs a velocity. Time expands in all direction and it influences spatial expansion, hence it "inflates space", which forms the spacetime continuum.

We define that on the quantum level:

$$(45)\ \delta \twoheadrightarrow = \delta \int d\text{Д}\ L(a(t), \dot{a}(t))$$

where the ($d\text{Д}$) is the accelerating quantum and ($\dot{a}(t)$) is the velocity. We also define that:

$$(46)\ \dot{a}(t) = c$$

Where (c) is "the speed of light". This is the reason for time dilatation caused by velocity and why (c) is the speed necessary to achieve maximal time dilatation. What (c) actually is, is the speed of temporal motion. Any velocity will cause time dilatation to some extent.

The accelerating quantum ($d\text{Д}$) constantly tries to accelerate temporal motion but it cannot since temporal motion cannot exceed (c), which leads the accelerating quantum to influence the expansion of space instead, manifesting as "dark energy" on the large scale and accelerating spatial expansion. Spatial expansion of the Universe is well beyond the speed (c) since dark energy has been massing to the point that it has an overwhelming presence in the Universe.

However, the accelerating quantum can be neglected since it is irrelevant to the nature of time on the quantum level, it is however exceptionally important for spatial expansion on the large scale of the Universe, as explained above.

Finally:

$$(47)\ \delta \twoheadrightarrow = \ \delta \int L(a(t), c)$$

***Suggestion**: In this form temporal motion can be called "Einstein-Hawking motion" to honor my two favorite physicists.*

III Cosmological model

We will form a model in two steps that represent the Universe on the large (global) and the local scale.

1. Expanding flat spacetime

The Universe will be represented as homogenous and isotropic. Isotropy means that the metric must be diagonal since it will be show that space is allowed to be curved. Therefore we will use spherical coordinates to describe the metric.

The metric is given by the following line element:

$$(48)\ ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

where we measure (θ) from the north pole and at the south pole it will equal (π).

In order to simplify the calculations, we abbreviate the term between the brackets as:

$$(49)\ d\omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$

because it is a measure of angle, which can be thought of as “on the sky” from the observers point of view. It is important to mention that the observers are at the center of the spherical coordinate system.

Due to the isotropy of the Universe the angle between two galaxies, for the observers, is the true angle from the observers’ vantage point and the expansion of the Universe does not change this angle.

Finally, we represent flat space as:

$$(50)\ ds^2 = dr^2 + r^2 d\omega^2$$

Robertson and Walker proved that the only alternative metric that obeys both isotropy and homogeneity is:

$$(51)\ ds^2 = dr^2 + f_K(r)^2 d\omega^2$$

where ($f_K(r)$) is the curvature function given by:

$$(52)\ f_K(r) = \begin{cases} K^{-1/2}\ for\ K > 0 \\ r\ for\ K = 0 \\ K^{-1/2}\sin h\left(K^{1/2} r\right)\ for\ K < 0 \end{cases}$$

which means that the circumference of a sphere around the observers with a radius (r) is, for ($K \neq 0$), not anymore equal to ($C = 2\pi r$) but smaller for ($K > 0$) and larger for ($K < 0$).

The surface area of that sphere would no longer be ($S = \left({}^{4\pi}/_{3}\right)r^3$) but smaller for ($K > 0$) and larger for ($K < 0$). If ($r$) is ($r \ll |K|^{-1/2}$) the deviation from ($C = 2\pi r$) and ($S = \left({}^{4\pi}/_{3}\right)r^3$) is very small, but as (r) approaches ($|K|^{-1/2}$) the deviation can become rather large.

The metric in the equation (47) can also be written as:

$$(53)\ ds^2 = \frac{dr^2}{1 - Kr^2} + r^2 d\omega^2$$

If we determine an alternative radius (r) as:

$$(54)\ r \equiv f_K(r)$$

This metric is different only in the way we chose our coordinate (r); other than that there is no physical difference with the equation (47).

1.1. Friedmann equations

We can now build our model by taking for each point in time a RW space. We allow the scale factor and the curvature of the RW space to vary with time. This gives the generic metric:

$$(55)\ ds^2 = -dt^2 + a(t)^2[dx^2 + f_K(x)^2 x^2 d\omega^2]$$

the function ($a(t)$) is the scale factor that depends on time and it will describe the expansion of it, that influences spatial expansion, hence the expansion of the whole Universe. We use (x) instead of (r) because the radial coordinate, in this form, no longer has meaning as a true distance.

We now insert equation (55) into the Einstein equations and after calculus, we obtain two equations:

$$(56)\ \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\lambda}{3}$$

$$(57)\ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\lambda}{3}$$

These equations are known as Friedmann equations and the first equation is from the “00 component” and the second from the “ii component” of the Einstein equations.

The two equations can be combined to make the adiabatic equation:

$$(58)\ \frac{d}{dt}(\rho a^3 c^2) + p\frac{d}{dt}(a^3) = 0$$

1.2. Scaling of relativistic and non-relativistic matter

Cold matter is matter for which the pressure ($p \ll \rho c^2$) leads us to reduce the equation (58) to:

$$(59)\ \frac{d}{dt}(\rho a^3) = 0$$

meaning that the equation of state for such cold matter is:

$$(60)\ \rho \propto \frac{1}{a^2}$$

If we look at the other limiting case, of ultra-relativistic matter, we have the maximum possible relativistic isotropic pressure:

$$(61)\ p = \frac{\rho c^2}{3}$$

for radiation (ultra-hot matter).

Equation (58) is now reduces to:

$$(62)\ \frac{d}{dt}(\rho a^3 c^2) + \frac{\rho c^2}{3}\frac{d}{dt}(a^3) = 0$$

hence:

$$(63)\ \rho \propto \frac{1}{a^4}$$

for radiation.

1.3. Critical density

Critical velocity turns into a critical density; the best way to define this is to start from the first Friedmann equation and rewrite it as:

$$(64)\ H^2 = \frac{8\pi G}{3}(\rho + \rho_\lambda) - \frac{Kc^2}{a^2}$$

with the Hubble constant ($H = \dot{a}/a$), and we have (λ) as (ρ_λ) according to:

$$(65)\ \rho_\lambda = \frac{\lambda}{8\pi G}$$

The density (ρ) can be written as contributions from matter, meaning baryons, cold dark matter and radiation:

$$(66)\ \rho = \rho_m + \rho_r$$

where baryonic and cold dark matter are:

$$(67)\ \rho_m = \rho_b + \rho_{cdm}$$

for matter density (ρ_m).

We write that radiation consists of photons and neutrinos:

$$(68)\ \rho_r = \rho_\gamma + \rho_v$$

The first Friedmann equation becomes:

$$(69)\ H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\lambda) - \frac{Kc^2}{a^2}$$

If we define (ρ_{crit}) as:

$$(70)\ \rho_{crit} = \frac{3H^2}{8\pi G}$$

then we see that if the total density ($\rho_m + \rho_r + \rho_\lambda$) equals the critical density, then ($K = 0$), which means that the Universe is flat. By the equivalence of curvature and expansion rate, it would also mean that the Universe expands critically. Therefore the critical density is the density at which the Universe expands critically, given the value for (H).

1.4. Dimensionless Friedmann equation

We define (H_0) as the Hubble constant at the present time and ($\rho_{crit;0}$) as the critical density at the present time, forming the first Friedmann equation as:

$$(71)\ H^2 = {H_0}^2\left(\frac{\rho_m}{\rho_{crit;0}} + \frac{\rho_r}{\rho_{crit;0}} + \frac{\rho_\lambda}{\rho_{crit;0}}\right) - \frac{Kc^2}{a^2}$$

Allowing us to introduce the following dimensionless densities:

$$(72)\ \Omega_m(a) = \frac{\rho_m(a)}{\rho_{crit}(a)}$$

$$(73)\ \Omega_r(a) = \frac{\rho_r(a)}{\rho_{crit}(a)}$$

$$(74)\ \Omega_\lambda(a) = \frac{\rho_\lambda(a)}{\rho_{crit}(a)}$$

The values of these quantities are denoted as ($\Omega_{m;0}$), ($\Omega_{r;0}$) and ($\Omega_{\lambda;0}$).

At this point we introduce ($\Omega_K(a)$), respectively ($\Omega_{K;0}$).

If we consider the first Friedmann equation at the present time:

$$(75)\ H^2 = {H_0}^2\left(\Omega_{m;0} + \Omega_{r;0} + \Omega_{\lambda;0}\right) - Kc^2$$

We can evaluate the curvature:

$$(76)\ Kc^2 = {H_0}^2\left(\Omega_{m;0} + \Omega_{r;0} + \Omega_{\lambda;0} - 1\right)$$

We define the curvature density ($\Omega_{K;0}$) as:

$$(77)\ \Omega_{K;0} \equiv -\frac{Kc^2}{{H_0}^2} = 1 - \Omega_{m;0} - \Omega_{r;0} - \Omega_{\lambda;0}$$

concluding that all (Ω_s) add up to (1). We can define ($\Omega_K(a)$) in terms of a "curvature density":

$$(78)\ \Omega_K(a) = \frac{\rho_K(a)}{\rho_{crit}(a)}$$

The (Ω) symbol can be used to rewrite the Friedmann equations. The matter density goes as ($1/a^3$), the radiation as ($1/a^4$) and the (Ω_λ) stays constant. The (Ω_K) is, according to equation (77), ($1/a^2$). Now we can write:

$$(79)\ H^2 = {H_0}^2\left(\frac{\Omega_{m;0}}{a^3} + \frac{\Omega_{r;0}}{a^4} + \Omega_{\lambda;0} + \frac{\Omega_{K;0}}{a^2}\right) = {H_0}^2E^2(a)$$

at present time ($a = 1$).

2. The Standard Model

The current understanding of the Universe tells us that it is flat ($\Omega_{K;0} \simeq 0$), but that it contains matter, radiation and that it has a non-zero cosmological constant. We are currently dominated by (λ) by a factor of three, which means a phase of exponential growth.

But before that, around ($z \gtrsim 0.5$), the Universe was dominated by could matter and before that, around ($z \gtrsim 3200$), the Universe was dominated by radiation.

The late Universe, ($z = few$) until ($z = 0$), in which both matter and (λ) are important, but radiation is not important, can also be integrated analytically. During this period, most of the structure formation in the Universe occurred.

We take ($0 < \Omega_{m;0} < 1$) and ($\Omega_{\lambda;0} = 1 - \Omega_{m;0}$) and set ($\Omega_r = 0$) and ($\Omega_K = 0$). Then we have:

$$(80)\ \frac{1}{a}\frac{da}{dt} = H_0\sqrt{\frac{\Omega_{m;0}}{a^3} + \Omega_{\lambda;0}}$$

which can be integrated as:

$$(81)\ t = \frac{1}{H_0}\int_0^a \frac{da'}{a'\sqrt{\frac{\Omega_{m;0}}{a'^3} + \Omega_{\lambda;0}}} = \frac{1}{H_0}\int_0^a \frac{\sqrt{a'}da'}{\sqrt{\Omega_{m;0} + \Omega_{\lambda;0} \cdot a'^3}}$$

by submitting ($x = a^{3/2}$) we integrate to:

$$(82)\ t = \frac{2}{3H_0\sqrt{1-\Omega_{m;0}}}\operatorname{arcsin} h\left(\sqrt{\frac{1-\Omega_{m;0}}{\Omega_{m;0}}}a^{3/2}\right)$$

if (a) has a small value, the formula above approaches the equation for the matter dominated era, which is:

$$(83)\ a(t) \simeq \left(\frac{3}{2}H_0\sqrt{\Omega_{m;0}}t\right)^{2/3}$$

and that for ($a \gg 1$) this formula describes an exponentially expanding Universe.

The equation (82) is accurate for all redshifts up to around ($z \simeq 1000$), meaning that it can be used for the estimation of age of the Universe by inserting ($a = 1$) into the equation (82) hence we obtain ($\Omega_{m;0} = 0.273$) an age of ($1376Gyr$).

3. Initial inflation

In order to prove that time inflates space and has inflated it since the beginning of the Big Bang, causing “cosmic inflation”, we go back to the early Universe in the period known as the “radiation dominated era”.

As ($t \to 0$) and ($a \to 0$) the only term is the radiation one, meaning that the Universe was dominated by radiation, around ($z \gtrsim 3200$).

For the early, radiation dominated era we can approximate a solution:

$$(84)\ a(t) \simeq \left(2H_0\sqrt{\Omega_{r;0}}t\right)^{1/2}$$

The early, radiation dominated Universe expanded as ($a \propto \sqrt{t}$).

We use the initial data to solve the dimensional equations for the global string dimension, that is for time. We used (x) to represent true distance and we define that the angles (α_x, α_y) are represented by one angle (α_{xy}) since they are equal. Therefore:

$$(85)\ t(x) = \log_{\lim_{a \to 0}\left(\frac{\twoheadrightarrow}{\alpha_{xy}} + \twoheadrightarrow\right)^{\frac{\twoheadrightarrow}{\alpha_{xy}}}}(x)$$

Where we used $(t(x))$ instead of $(t_G(x))$ for simplicity and where temporal motion $(\twoheadrightarrow)$ is:

$$(86)\ \delta \twoheadrightarrow = \delta \int d\text{Д}\, L(a(t), c)$$

Which means that time inflates space and has always inflated it, causing “cosmic inflation”. With time the accelerating quantum influenced space so that the Universe is now dominated by the (λ) factor, or in other words “dark energy” is dominant in the present Universe.

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