Numerical-Analytical Assessment on Solar Chimney Power Plant

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Abstract

This study considers an appropriate expression to estimate the output power of solar chimney power plant systems (SCPPS). Recently several mathematical models of a solar chimney power plant were derived, studied for a variety of boundary conditions, and compared against CFD calculations. An important concern for modeling SCPPS is about the accuracy of the derived pressure drop and output power equation. To elucidate the matter, axisymmetric CFD analysis was performed to model the solar chimney power plant and calculate the output power for different available solar irradiation. Both analytical and numerical results were compared against the available experimental data from the Manzanares power plant. We also evaluated the fidelity of the assumptions underlying the derivation and present reasons to believe that some of the derived equations, specifically the power equation in this model, may require a correction to be applicable in more realistic conditions. This paper provides an approach to estimate the output power with respect to radiation available to the collector.

Keywords:
Solar chimney power plant, Pressure drop, Modeling and simulation (M&S), Analytical solution

Nomenclature

Variables

A cross-sectional area, m^2

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Preprint submitted to Elsevier *December 31, 2015*

A_r cross-sectional area of the collector ground, m^2
g acceleration due to gravity, m/s^2
h height, m
$\dot{m}$ air mass flow rate, kg/s
p pressure, N/m^2
$\dot{W}$ flow power, W
q heat transfer per unit mass, J/kg
q'' heat flux, W/m^2
R gas constant, $J/mol.K$
T temperature,K
ρ density, kg/m^3
u velocity, m/s
c_p specific heat capacity, $J/kg.K$

Subscripts
i inlet
o outlet
c collector
t tower
$turb$ turbine
atm atmospheric

1. Introduction

Although the idea of the solar chimney power plant (SCPP) can be traced to the early 20th century, practical investigations of solar power plant systems started in the late 1970s, around the time of conception and construction of the first prototype in Manzanares, Spain. This solar power plant operated between 1982 and 1989 and the generated electric power was used in the local electric network [1, 2]. With respect to the distinguished rise of R&D budget on renewable energy[3], studying different scenarios for solar power plant seems beneficial and vital.

The basic SCPP concept (Fig. 1) demonstrated in that facility is fairly striaghtforward. Sunshine heats the air beneath a transparent roofed collector structure surrounding the central base of a tall chimney. The hot air produces an updraft flow in the chimney. The energy of this updraft flow is harvested with a turbine in the chimney, producing electricity. Experiments with the prototype proved the concept to be viable, and provided data used by a variety of later researchers. A major motivation for subsequent studies lay in the need for reliable modeling of the operation of a large-scale power plant. The Manzanares prototype had a 200 m tall chimney and a 40,000 m^2 collector area. Proposals for economically competitive SCPP facilities usually feature chimneys on the scale of 1 km and collectors with multiple square kilometer areas.

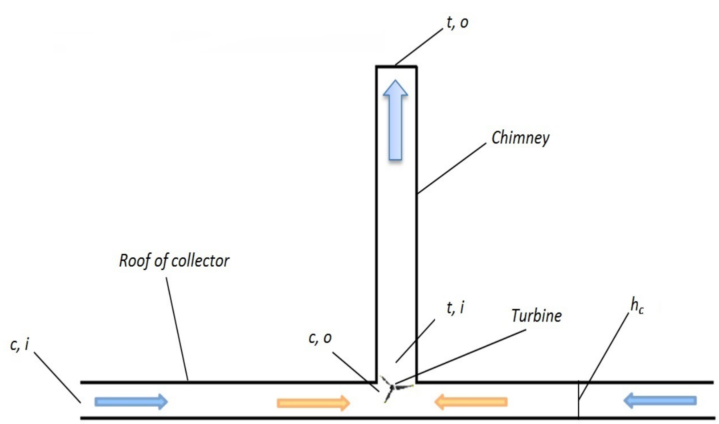

Figure 1: Schematic of SCPP with the applied variables and subscripts in the present analysis

Padki and Sherif [4] used the results from the Manzanares prototype to extrapolate the data to large scale models for SCPP. In 1991, Yan et al. [5] developed an SCPP model using a practical correlation. They introduced equations including air velocity, air flow rate, output power, and thermofluid efficiency. Von Backström and Fluri conducted a numerical study to determine the optimum ratio of pressure drop of the turbine as a fraction of the available pressure difference required to achieve the maximum power [6]. They noted that this ratio might lead to overestimating the flow passage in the plant and also designing a turbine without a sufficient stall margin. In other recent works, the SCPP concept involving an inflatable tower was examined, with all parts of the power plant modeled numerically [7, 8, 9]. A small-scale inflatable tower was fabricated for validation of these results, and code calibration was performed using the newly available experimental data [10, 11, 12].

To find the maximum power, different atmospheric pressure and temperature boundary conditions were applied for various tower heights and atmospheric lapse rates [13]. Theoretical analysis to study the effect of pressure drop in the SCPP turbine was performed by Koonsrisuk et al. [14]. The optimal pressure drop ratio was found numerically and analytically by Xu et al., around 0.9 for the Manzanares prototype. This inverstigation can be applied as an initial estimation for various SCPP turbines [15]. Tayebi et al. modeled and simulated the SCPP with a curved conjunction between tower and collector for different Rayleigh numbers [16].

Earlier modeling efforts [8] showed a keen sensitivity of the predictions of SCPP output to boundary conditions, in particular, pressure. Numerical simulations require careful validation and verification, and for that, analytical models are indispensable. A theoretical model was recently developed [17] to model the combined performance of the solar collector, chimney, and turbine. Here we will examine some of the assumptions and derivations in this model and present an alternative formulation for the energy equation.

2. Analytical Study

2.1. Collector

Solar Chimney Power Plants provide a reliable and conceptually straightforward way of energy generation from the solar irradiation[20]. A solar collector is the main and only component of this power plant to accumulate the available solar energy to heat up air in a greenhouse. The air escapes the collector through a tall chimney which connects the warm air flow of the collector with the cooler air above the ground. The temperature difference induces the natural convection, and turbine at the outlet of collector harvests the energy of the air flow. To model the collector, the simplified one dimensional mathematical analysis was performed to clarify the details and correct some points. the analytical correlation will be applied later to compare the CFD results against it. To derive the equations, we start from the collector. It is assumed that the flow through the collector is one-dimensional, steady-state, and compressible. Let us disregard the friction and assume the total heat from the solar irradiation is absorbed within the air filling the collector. For this one dimensional asymmetric compressible flow analysis, the mass conservation satisfies:

$$\frac{dA}{A} + \frac{d\rho}{\rho} + \frac{du}{u} = 0 \quad \text{(Continuity)} \tag{1}$$

Here A is the cross-sectional area of the collector that air goes through – $A = 2\pi r h_c$ and $dA = 2\pi r dh_c$.

Momentum equation is as follows [18]:

$$dp + \rho u du = 0 \quad \text{(Momentum)} \tag{2}$$

Consider the energy balance equation and the equation of state as follows:

$$c_p dT - dq + udu = 0 \quad \text{(Energy)} \tag{3}$$

$$dp = d(\rho RT) \quad \text{(State)} \tag{4}$$

To find dp we can apply Eq. (2) and substitute du/u from the continuity equation, Eq. (1).

$$dp = \rho u^2 \left(\frac{d\rho}{\rho} + \frac{dA}{A} \right) \tag{5}$$

From the equation of state we can find $d\rho/\rho$ and substitute in Eq. (5),

$$\frac{d\rho}{\rho} = \frac{dp}{p} - \frac{dT}{T} \tag{6}$$

$$dp = \rho u^2 \left(\frac{dp}{p} - \frac{dT}{T} + \frac{dA}{A} \right) \tag{7}$$

We can rewrite Eq. (7) as a function of T, A, u, p, ρ , where $\dot{m} = \rho A u$. Also by substitution dT from the energy equation on the base of dq, c_p and u, we obtain

$$dp = \frac{\dot{m}^2}{\rho} \left(\frac{dA}{A^3} - \frac{dq - udu}{A^2 T c_p} + \frac{dp}{A^2 p} \right) \tag{8}$$

For consistency with previous analyses, let us rewrite dq on the basis of heat flux per mass flow rate—$dq = q'' dA_r/\dot{m}$ where q has the units of J/kg. Here $A_r = \pi r^2$, therefore $dA_r = 2\pi r dr$. Note that $A = 2\pi r h_c$, where h_c is the collector height (roof height) that was assumed to be proportional to r –

$h_c = ar$, where a is a constant. By substituting A_r, dq and A in the second term on the RHS, we obtain

$$dp = \frac{\dot{m}^2}{\rho}\left(\frac{dA}{A^3} - \frac{q''(2\pi r)dr}{\dot{m}(2\pi r^2 a)^2 T c_p} + \frac{udu}{A^2 c_p T} + \frac{dp}{A^2 p}\right) \tag{9}$$

We can rewrite equation (9) and substitute udu of the third term on the RHS by applying momentum equation (2), $udu = -dp/\rho$ and $p = \rho RT$.

$$dp = \frac{\dot{m}^2}{\rho}\left[\frac{dA}{A^3} - \frac{q''dr}{2\pi \dot{m} r^3 a^2 c_p T}\right]\left[1 - \frac{u^2}{T}\left(\frac{1}{R} - \frac{1}{c_p}\right)\right]^{-1} \tag{10}$$

Equation (10) is the exact solutions for dp for the one-dimensional frictionless analysis of the collector. Since our fluid is air we can estimate c_p and rewrite Eq. (10).

$$dp \simeq \frac{\dot{m}^2}{\rho}\left(\frac{dA}{A^3} - \frac{q''dr}{2\pi \dot{m} r^3 a^2 c_p T}\right)\left(1 - \frac{2.494u^2}{T}\right)^{-1} \tag{11}$$

c_p, q'' and T are considered approximately constant as well. The last term on the RHS of (11) is very close to 1 with respect to the range of the velocities and the temperature unit in K unit. Therefore by integrating between the inlet and outlet of the collector without the last term of the RHS, pressure difference can be derived.

$$\int_{c,o}^{c,i} dp \simeq \int_{c,o}^{c,i}\left(\frac{\dot{m}^2 dA}{\rho A^3} - \frac{\dot{m} q'' dr}{2\pi r^3 a^2 \rho c_p T}\right) \tag{12}$$

$$p_{c,i} - p_{c,o} \simeq \left[\frac{\dot{m}^2}{2\rho}\left(\frac{1}{A_{c,o}^2} - \frac{1}{A_{c,i}^2}\right) - \frac{q''\dot{m}}{4\pi a^2 \rho c_p T}\left(\frac{1}{r_{c,o}^2} - \frac{1}{r_{c,i}^2}\right)\right] \tag{13}$$

2.2. Tower

The air flow in the chimney is considered as an adiabatic frictionless flow. The conservation equations for the one-dimensional steady state flow in variable-area tower are similar to collector except having the gravity term in momentum and energy equtions.

By following the same trend to find dp we get

$$dp = \left[-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3} + \rho u^2\left(\frac{dp}{p} - \frac{dT}{T}\right)\right] \tag{14}$$

By applying the energy equation, substitution $dT = (-gdz - udu)/c_p$ and $dp = -\rho(udu + gdz)$, we can rewrite the above equation as

$$dp = \left[-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3} + \rho u^2 \left(\frac{dp}{p} - \frac{dp}{\rho c_p T}\right)\right] \tag{15}$$

Also by considering the material properties of air the same way we did for the collector part,

$$dp \simeq \left[-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3}\right]\left[1 - \frac{2.494u^2}{T}\right]^{-1} \tag{16}$$

The above equation is the exact closed form solution for dp at any point as the function of variables ρ, T and q''. The last term on the RHS can be assumed to equal unity as we mentioned in the collector part. Let integrate between the inlet and outlet tower area to find the pressure difference of the chimney as,

$$\int_{t,0}^{t,i} dp \simeq \int_{t,0}^{t,i} \left(-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3}\right) \tag{17}$$

$$p_{t,i} \simeq p_{t,o} + \rho g h_t + \frac{\dot{m}^2}{2\rho}\left(\frac{1}{A_{t,o}^2} - \frac{1}{A_{t,i}^2}\right) \tag{18}$$

To calculate the output power, we can define the power on the basis of the pressure difference at the turbine – where it is normally utilized at the outlet of the collector and inlet of the tower.

$$\dot{W} \simeq \frac{\dot{m}(p_{c,o} - p_{t,i})}{\rho_{turb}} \tag{19}$$

Let $\rho_{turb} = (\rho_{c,o} + \rho_{t,i})/2$ and substitute equations $p_{c,o}$ and $p_{t,i}$ from (13) and (18). Hence for the flow power by assuming $p_{c,i} = p_{t,o} + \rho g h_t$, we have

$$\dot{W} = \frac{\dot{m}}{(\rho_{c,o} + \rho_{t,i})/2}\left[\frac{-\dot{m}^2}{2\rho}\left(\frac{1}{A_{c,o}^2} - \frac{1}{A_{c,i}^2}\right) + \frac{q''\dot{m}}{4\pi a^2 \rho c_p T}\left(\frac{1}{r_{c,o}^2} - \frac{1}{r_{c,i}^2}\right) - \frac{\dot{m}^2}{2\rho}\left(\frac{1}{A_{t,o}^2} - \frac{1}{A_{t,i}^2}\right)\right] \tag{20}$$

For area the following equations are used, where b and c are arbitrary positive real constants.

$$A_{c,i}^2 = bA_{c,o}^2, \ A_{t,o}^2 = cA_{t,i}^2 \tag{21}$$

The simplified form of equation (20) by applying the area correlations is,

$$\dot{W} \simeq \frac{\dot{m}}{(\rho_{c,o}+\rho_{t,i})/2}\left[\frac{-\dot{m}^2}{2\rho}\left(\frac{b-1}{bA_{c,o}^2}+\frac{1-c}{cA_{t,i}^2}\right)+\frac{q^{''}\dot{m}}{4\pi a^2\rho c_p T}\left(\frac{1}{r_{c,o}^2}-\frac{1}{r_{c,i}^2}\right)\right] \quad (22)$$

Assume ρ is constant before and after turbine, therefore

$$\rho_{turb}=\rho_{c,o}=\rho_{t,i}=\rho$$

Koonsrisuk et al. derived an equation in which the second term was neglected in comparison with the first term on the RHS of Eq. (22). However, Eq. (23) shows the derived power equation by them at the end is likely to exceed the expected amount by a factor of two.

$$\dot{W} \simeq \frac{-\dot{m}^3}{2\rho^2}\left(\frac{1-c}{cA_{t,i}^2}+\frac{b-1}{bA_{c,o}^2}\right) \quad (23)$$

To evaluate the derived anlaytical solution for the output power of SCPP, the available experimental data from Manzanares prototype was applied and extracted. The measured updraft velocity for 24 hours Manzanares power plant operation is imposed to the analytical solution and the analytical power compared against the experimental outpower from the turbine (Fig. 2).

3. Numerical Analysis

To perform computational fluid dynamics (CFD) modeling, the finite volume method was employed to model and simulate (M&S) the air flow as ideal gas under Boussinesq effect by solar irradiation. In the present modeling the mass flow rate, obtained from the CFD results, along with other parameters were used to evaluate the maximum mechanical power for each case. The flow of air in SCPPS was assumed steady (in the average flow sense) and axisymmetric with respect to the chimney centerline. The meshed SCPPS axisymmetric model is shown in Fig. 3 with the details of applied boundary condition. ANSYS ICEM (Integrated Computer Engineering and Manufacturing) CFD was employed to generate a quadrilateral cell mesh. To perform the CFD simulation, the standard $k-\epsilon$, which is classified as a two-equation turbulence model, was applied. In this model, with respect to the sensitivity of the pressure solver to the density change, the density of air is calculated from the ideal gas equation and the pressure boundary conditions

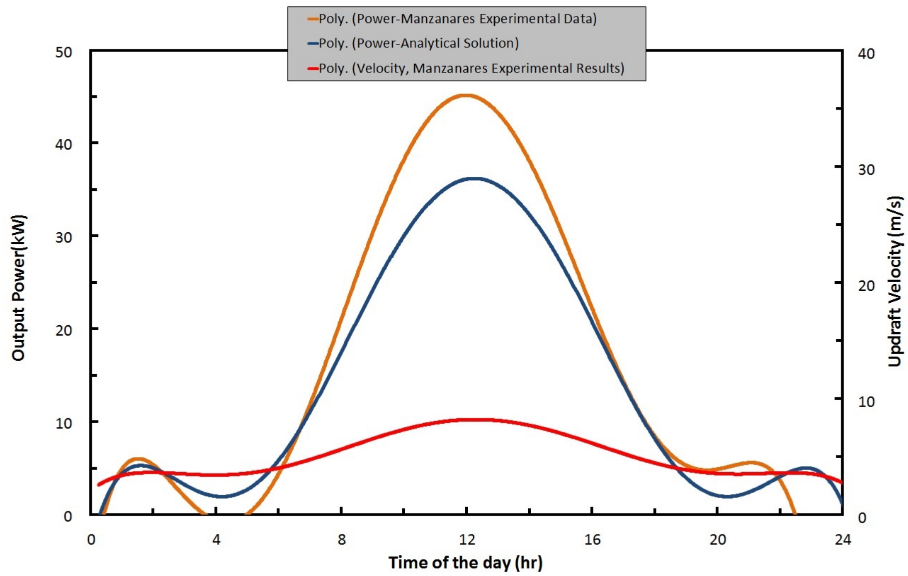

Figure 2: Analytical power results against measurement from Manzanares: updraft velocity and power output for a typical day

at the entrance of the collector and the outlet of the chimney were assumed as atmospheric pressure and identical. The chimney wall and the collector roof were considered adiabatic and the solar radiation was introduced to the ground as a constant heat flux. The residual criteria for all equations were set to be calculated and iterated not to exceed 10^{-6}. The calculations were done by using a 16-core, 32 G RAM computer.

To obtain the available power to rotate the turbine, the kinetic energy of the air flow at the outlet of the collector was calculated. For each CFD analysis, the mass flow rate and the average density at the turbine location are gained from the numerical simulation result and used to calculate the available kinetic energy per time. The calculated available power for different solar radiation are compared against the experimental data and the analytical solution in two cases (Fig. 5). Analytic-CFD presents the power where the average velocity and density at the the turbine location were imposed to the power correlation. Analytic-EXP shows the power where the average velocity were obtained from the available experimental data at the same amount of reported radiation. It is needless to say that details of available experimental data are not clear enough to report the data with all sources of uncertainty. The reliability of experimental data is suggested to measure for future works.

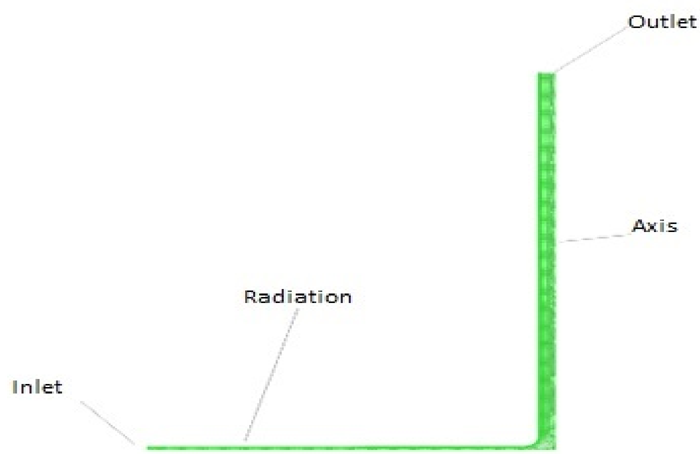

Figure 3: Computational domain and applied boundary conditions.

CFD results were performed under the ideal assumption of having no heat loss from the tower or collector. Also we imposed the available heat flux as a boundary condition. In other words in the CFD analysis we introduced the flow domain e.g. 1000 W/m^2 where in the experiment the amount of reported velocity at 1000 W/m^2 solar radiation is lower because of the absoptivity factor of the collector ground. Therefore, using the CFD velocity values in the analytical correlation gives a greater available power than other cases due to having a contant density assuption and having no friction in analytical solution. The available power at the turbine location, CFD in Fig. 5, by usignt the CFD values for density and velocity were calculated based on the rate of kinetic energy, $0.5\dot{m}u^2$. The difference between the available power from simulation results (CFD) and the experimental turbine power is due foremost to the turbine efficiency and then having no heat loass in the CFD model.

4. Conclusion

We presented a combined numerical-analytical analysis for solar chimney power plant, based on the Manzanares prototype. The harvestable power of Manzanares power plant was investigated as the function of available solar radiation. The modeling and simulation (M&S) was carried ou. Also One-dimensional analytical analysis was done with with attention to underlying assumptions and simplifications. We compared the numerical results

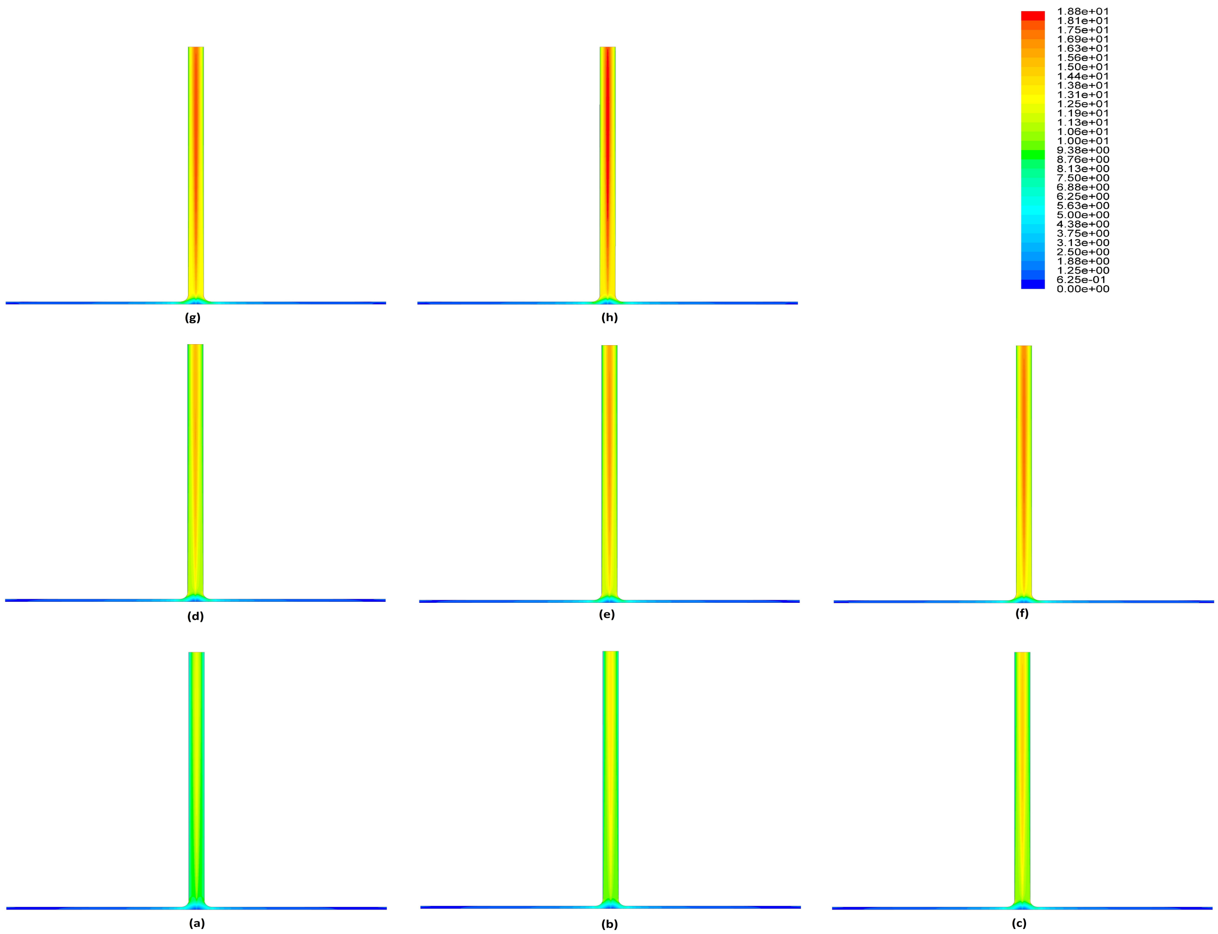

Figure 4: Velocity contour plot (m/s) for different available solar heat flux at the ground of collector, (a):300, (b):400, (c):500, (d):600, (e):700, (f):800, (g):900, (h):1000 W/m^2.

against the available limited raw experimental data from the prototype and also showed the range of reliability of the analytical solution. Where the inlet velocity values for analytical correlation were obtained from the experimental velocities we got lower available power than the output turbine power. That has several reasons as, (a) the one dimensional analytical solution has several-simplifications, including treatment of density and the heat flux term., (b) It is very important to pick the right source to impose the values to the analytical correlation., (c) Available experimental data are not just limited, but also not extensively characterized in terms of uncertainty and repeatablity, mak-

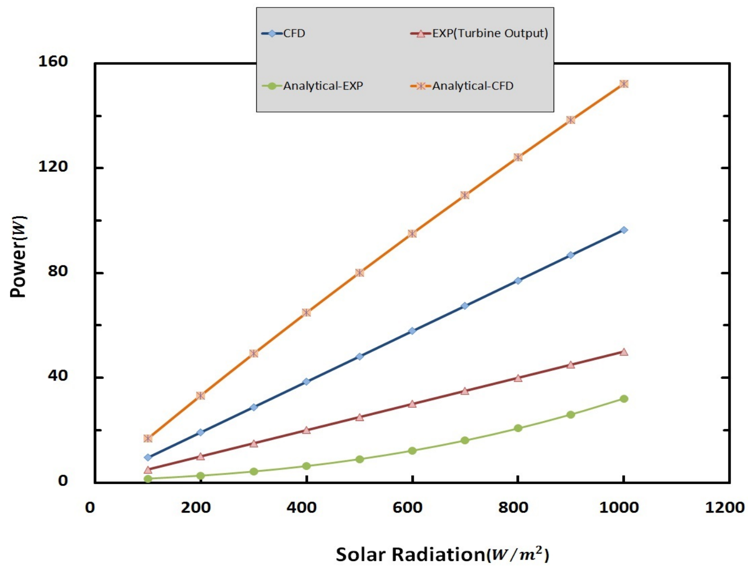

Figure 5: Power vs heat flux.

ing it difficult to produce error bars on experimental values for a prescribed level of confidence. To present the volatility of the analytical correlation, we selected two different sources to input values in this one-dimensional equation; I- imposing the experimental velocities and calculation densities with respect to average temperature., II- Applying the velocity and density values from the CFD analysis. During the verification and validation process, the modeler must ask two questions: "Am I modeling the physics correctly?" and "Am I modeling the correct physics?" Comparison with analytical models is important for answering both of these questions, and the only way to have them well-posed is to have correct physics in the analytics.

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