# Conjecture on an infinity of triplets of primes generated by each 3-Poulet number

**Abstract.** In this paper I present the following conjecture: for any 3-Poulet number (Fermate pseudoprime to base two with three prime factors) P = x*y*z is true that there exist an infinity of triplets of primes [a, b, c] such that x*a + a – x = y*b + b – y = z*c + c – z.

**Conjecture:**

For any 3-Poulet number (Fermate pseudoprime to base two with three prime factors) P = x*y*z is true that there exist an infinity of triplets of primes [a, b, c] such that x*a + a – x = y*b + b – y = z*c + c – z.

The sequence of 3-Poulet numbers is: 561, 645, 1105, 1729, 1905, 2465, 2821, 4371, 6601, 8481, 8911, 10585, 12801, 13741, 13981, 15841 (...). See the sequence A215672 that I posted on OEIS.

**Examples:**

For P = 561 = 3*11*17,
we need to find [a, b, c] such that 4*a – 3 = 12*b – 11 = 18*c – 17; for this, [a, b, c] must be of the form [9*n + 1, 3*n + 1, 2*n + 1], where n can’t be odd, can’t be of the form 3*k + 1 and also can’t have the last digit 2, 6 or 8. The least n for which [a, b, c] are all three primes is n = 20 which gives us [a, b, c] = [181, 61, 41]. The following such triplet is [a, b, c] = [487, 163, 109] corresponding to n = 54.

For P = 645 = 3*5*43,
we need to find [a, b, c] such that 4*a – 3 = 6*b – 5 = 44*c – 43; for this, [a, b, c] must be of the form [33*n + 1, 22*n + 1, 3*n + 1], where n can’t be odd, can’t be of the form 3*k + 2 and also can’t have the last digit 2 or 8. The least n for which [a, b, c] are all three primes is n = 4 which gives us [a, b, c] = [133, 89, 13]. The following such triplet is [a, b, c] = [199, 133, 19] corresponding to n = 6.

For P = 1105 = 5*13*17,
we need to find [a, b, c] such that 6*a – 5 = 14*b – 13 = 18*c – 17; for this, [a, b, c] must be of the form [21*n + 1, 9*n + 1, 7*n + 1], where n can’t be odd, can’t be of the form 3*k + 2 and also can’t have the last digit 2, 4 or 6. The least n for which [a, b, c] are all three

primes is n = 18 which gives us [a, b, c] = [379, 163, 127]. The following such triplet is [a, b, c] = [631, 271, 211] corresponding to n = 30.

For P = 1729 = 7*13*19,

we need to find [a, b, c] such that 8*a – 7 = 14*b – 13 = 20*c – 19; for this, [a, b, c] must be of the form [35*n + 1, 20*n + 1, 14*n + 1], where n can't be odd, can't be of the form 3*k + 1 and also can't have the last digit 6. The least n for which [a, b, c] are all three primes is n = 2 which gives us [a, b, c] = [71, 41, 29]. The following such triplet is [a, b, c] = [491, 281, 197] corresponding to n = 14.

For P = 1905 = 3*5*127,

we need to find [a, b, c] such that 4*a – 3 = 6*b – 5 = 128*c – 127; for this, [a, b, c] must be of the form [96*n + 1, 64*n + 1, 3*n + 1], where n can't be odd, can't be of the form 3*k + 2 and also can't have the last digit 4, 6 or 8. The least n for which [a, b, c] are all three primes is n = 12 which gives us [a, b, c] = [1153, 769, 37]. The following such triplet is [a, b, c] = [2113, 1409, 67] corresponding to n = 22.

For P = 2465 = 5*17*29,

we need to find [a, b, c] such that 6*a – 5 = 18*b – 17 = 30*c – 29; for this, [a, b, c] must be of the form [15*n + 1, 5*n + 1, 3*n + 1], where n can't be odd, can't be of the form 3*k + 1 and also can't have the last digit 8. The least n for which [a, b, c] are all three primes is n = 2 which gives us [a, b, c] = [31, 11, 7]. The following such triplet is [a, b, c] = [181, 61, 37] corresponding to n = 12.

For P = 2821 = 7*13*31,

we need to find [a, b, c] such that 8*a – 7 = 14*b – 13 = 32*c – 31; for this, [a, b, c] must be of the form [28*n + 1, 16*n + 1, 7*n + 1], where n can't be odd, can't be of the form 3*k + 2 and also can't have the last digit 2, 4 or 8. The least n for which [a, b, c] are all three primes is n = 16 which gives us [a, b, c] = [449, 257, 113]. The following such triplet is [a, b, c] = [841, 481, 211] corresponding to n = 30.