Three conjectures on the numbers of the form p(p+4n)-60n where p and p+4n primes

Abstract. In this paper I present three conjectures on the numbers of the form p*(p + 4*n) – 60*n, where p and p + 4*n are primes, more accurate a general conjecture and two particular ones, on the numbers of the form p*(p + 4) – 60 respectively p*(p + 20) - 300.

Note: The numbers of the form p*(p + 4*n) – 60*n, where p and p + 4*n are primes, seem to have special attributes.

Conjecture 1: There exist an infinity of primes of the form p*(p + 4*n) – 60*n, where p and p + 4*n are primes, for any n non-null positive integer.

1.

Let's take the positive numbers of the form p*q – 60, where p and q = p + 4 are both primes:

: for (p, q) = (7, 11) is obtained 17, prime;
: for (p, q) = (13, 17) is obtained 161 = 7*23;
: for (p, q) = (19, 23) is obtained 377 = 13*29;
: for (p, q) = (37, 41) is obtained 1457 = 31*47;
[...]
: for (p, q) = (104323, 104327) is obtained 73*101*1033*1429 (we note the prime factors with a, b, c, d, a < b < c < d, and it can be seen that b*c – a*d = 16;
: for (p, q) = (104239, 104243) is obtained 61*1709*104233 (it can be seen that a*b – c = 16);
: for (p, q) = (104707, 104711) is obtained 10963974617 = 104701*104717 (it can be seen that b – a = 16);

Conjecture 2: For any composite number of the form p*q – 60, where p and q = p + 4 are both primes, is true that its prime factors can be divided in two sets in such a way such that the result of the subtraction of the product of some of them (or one of them) from the product of the others (or the other one of them) is equal to 16.

2.

Let's take the positive numbers of the form p*q – 120, where p and q = p + 8 are both primes: the sequence of primes of this form is 83, 953, 3833, 8513, 10889, 18089 (...), obtained for (p, q) = (11, 19), (29, 37), (59, 67), (89, 97), (101, 109), (131, 139)...

3.

Let’s take the positive numbers of the form p*q – 180, where p and q = p + 12 are both primes: the sequence of primes of this form is 73, 313, 409, 1009, 1993, 2593, 4273, 5113 (...), obtained for (p, q) = (11, 23), (17, 29), (19, 31), (29, 41), 41, 53), (47, 59), (61, 73), (67, 79)...

4.

Let’s take the positive numbers of the form p*q – 240, where p and q = p + 16 are both primes: the sequence of primes of this form is 137, 1217, 1721, 6257 (...), obtained for (p, q) = (13, 29), (31, 47), (37, 53), (73, 89)...

5.

Let’s take the positive numbers of the form p*q – 300, where p and q = p + 20 are both primes:

: for (p, q) = (11, 31) is obtained 41, prime;
: for (p, q) = (17, 37) is obtained 329 = 7*47;
: for (p, q) = (23, 43) is obtained 689 = 13*53;
: for (p, q) = (41, 61) is obtained 2201 = 31*71;
[...]
: for (p, q) = (104681, 104701) is obtained 7*19*787*104711 (we note the prime factors with a, b, c, d, a < b < c < d and it can be seen that d – a*b*c = 40;
: for (p, q) = (104639, 104659) is obtained 7*17*47*131*14947 (it can be seen that b*c*d – a*e = 40);
: for (p, q) = (104471, 104491) is obtained 7*31*3371*14923 (it can be seen that b*c – a*d = 40);
: for (p, q) = (104327, 104347) is obtained 11*53*73*179*1429 (it can be seen that a*b*d – c*e = 40);

Conjecture 3: For any composite number of the form p*q – 300, where p and q = p + 20 are both primes, is true that its prime factors can be divided in two sets in such a way such that the result of the subtraction of the product of some of them (or one of them) from the product of the others (or the other one of them) is equal to 40.