Create polygon through fans suitable for parellel calculations

Kang Yang, Kevin yang, Shuang-ren Ren Zhao

Imrecons Inc, Toronto Canada

Abstract

There are many method for finding whether a point is inside a polygon or not. The congregation of all points inside a polygon can be referred point congregation of polygon. Assume on a plane there are N points. Assume the polygon have M vertexes. There are O(NM) calculations to create the point congregation of polygon. Assume N>>M, we offer a parallel calculation method which is suitable for GPU programming. Our method consider a polygon is consist of many fan regions. The fan region can be positive and negative.

I. INTRODUCTION

There are mathod to find out whether a point is inside a polygon or not[1–23]. All points inside a polygon are the point congregation of polygon. If a plane have N points and the polygon have M vertexes, If N>>M, Found all points inside the polygon need O(MN) calculations. If N is very big, the above method to create the polygon is time consume. In case we have GPU, would like to find a mathod can parallel find all points inside the polygon.

By notice that a polygon can be build by positive/negitive fan regions, we offers the folloing mathod. In this article w use Julia programming language to test our ideas.

II. HALF PLANE

Any two points can create a line, all points at the right of the line is a half plane. We use Julia programming language to test our idea. The following is to start the julia program,

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```
using TestImages
using Images, Colors, FixedPointNumbers, ImageView
```

Assume there are 2 points R_0 and R_1, R_0, R_1 are vector. R_0 and R_1can create a line. Any point in the plane is Point P. P is vector. P has two components $P[1] = i$, $P[2] = j$. Hence $P = [i, j]$. Assume M is the direction vector from R_0 to R_1, i.e.,

$$M = R_1 - R_0$$

The normal vector on right side of the line is N,

$$N = [-M[2], M[1]]$$

Hence there is,

$$N = [-R_1[2] - R_0[2], R_1[1] - R_0[1]]$$

Define a vector X,

$$X = P - R_0$$

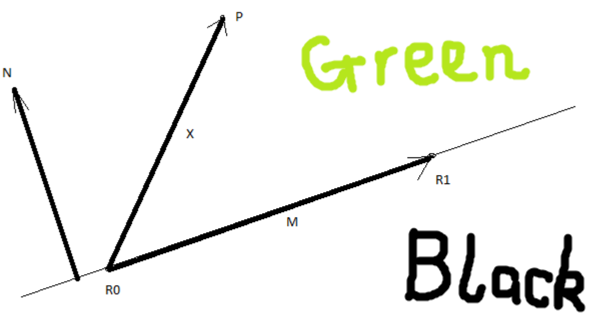


Figure 1: Half plane is created from 2 points R_0 and R_1.

We define if the value v

$$v = N \cdot X \geq 0$$

the point is P is inside the half plane. Other points are not inside the half plane. Here "·" inner product of two vector. We will use color green to show the point inside the half plane. We use color red to show the point outside the half plane. This half plane is at the right side of the line. We also give a value 1 to all green point. The other point give a vaule 0. The following gives the Julia programming code for the half plane, see Figure (1).

The following is the function of half plane $H(R_0, R_1)$ which have 3 parameters. The first Point R_0, the second point R_1, and the image size *imsize*. The boundary line is includes inside the half plane

---

```
function half_plane(R0,R1,imsize)
 n_vector=[-(R1[2]-R0[2]),R1[1]-R0[1]]
 B=zeros(imsize)
 for jjj=1:imsize[2]
  for iii=1:imsize[1]
   x_vector=[iii-R0[1],jjj-R0[2]]
   value=n_vector'*x_vector
   if value[1,1]>=0.
    B[iii,jjj]=1.0
   else
```

```
      B[iii,jjj]=0.
    end
  end
end
copy(B)
end
```

If the bourndary line does not include inside the half pline, the above formula need to be adjusted as following.

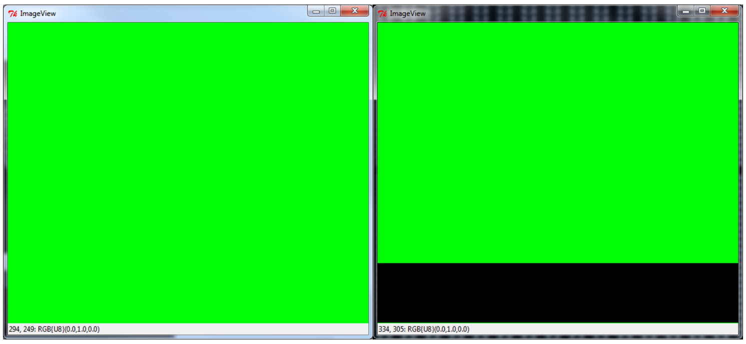
$$v = N \cdot X > 0$$

We call this is half plane less $HL(R_0, R_1)$. The source code after this change becomes,

---

```
function half_plane_less(R0,R1,imsize)
  n_vector=[-(R1[2]-R0[2]),R1[1]-R0[1]]
  B=zeros(imsize)
  for jjj=1:imsize[2]
    for iii=1:imsize[1]
      x_vector=[iii-R0[1],jjj-R0[2]]
      value=n_vector'*x_vector
      if value[1,1]>0.
        B[iii,jjj]=1.0
      else
        B[iii,jjj]=0.
      end
    end
  end
  copy(B)
end
```

The following is the test program.



(a) (b)

Figure 2: (a) Full plane. (b) Half plane created from 2 Points R_0 and R_1.

```
imsize=(600,500)
B0=ones(imsize)
my_view_flip(B0)
RR0=[100,100]
RR1=[400,100]
imsize=(600,500)
B1=half_plane(RR0,RR1,imsize)
my_view_flip(B1)
```

Figure(2) shows full plane and half plane which is created from 2 points R0 and R1.

III. FAN REGION

The two half can create a fan region. Assume we have 3 points. R_0,R_1 can define a half plane $H(R_0, R_1)$, $H(R_1, R_0)$ and $HL(R_0, R_1)$, $HL(R_1, R_0)$ as before. It is same to the points R_1, R_2. One half palne and one half plane less can create a fan region . See, Figure(3).

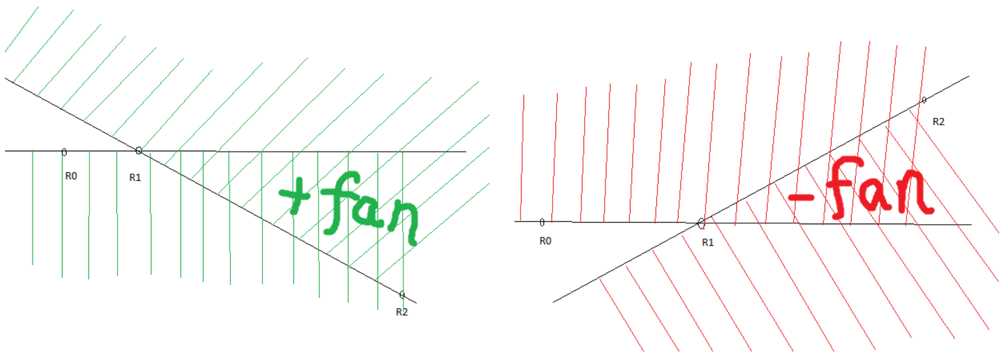
Details of definition is following

$$F(i, j) = HL(R_1, R_0) \cap H(R_1, R_2)(+1) \qquad if\ v \geq 0$$

$$F(i, j) = HL(R_0, R_1) \cap H(R_2, R_1)(-1) \qquad if\ v < 0$$

Where

$$v \equiv N \cdot X$$



(a) (b)

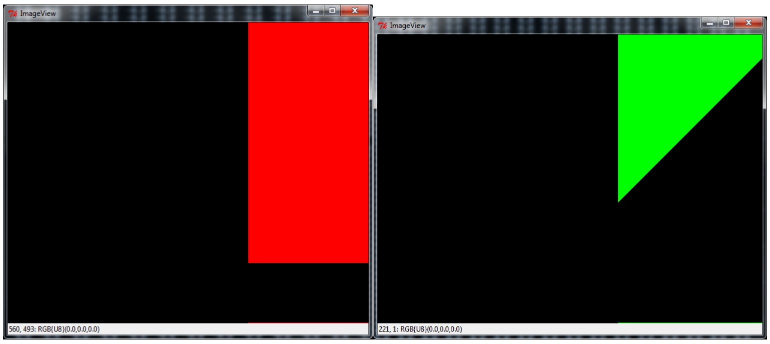
Figure 3: (a) positive fan with Green color, (b) negitive fan with red color.

$$X \equiv X_2 - X_1$$

$$N = [-R_1[2] - R_0[2], R_1[1] - R_0[1]]$$

F is all points inside the fan region. The above formaul means if R_2 inside the half plane $H(R_0, R_1)$ the fan's value is negitive. If R_2 is not inside the half plane $H(R_0, R_1)$ the fan's value is negitive . The Julia source code is following,

```
# calculate the fan image:
function fanregion(R0,R1,R2,imsize)
  n_vector=[-(R1[2]-R0[2]),R1[1]-R0[1]]
  x_vector=R2-R1
  value=n_vector'*x_vector
  if value[1,1]>=0.
    B=half_plane(R0,R1,imsize).*half_plane_less(R2,R1,imsize)
    B*=-1.0
  else
    B=half_plane_less(R1,R0,imsize).*half_plane(R1,R2,imsize)
    B*=+1.0
  end
  copy(B)
end
```



(a) (b)

Figure 4: (a) A negative fan region created from R_0,R_1 and R_2. (b) A positve fan region created from points R_1, R_2 and R_3.

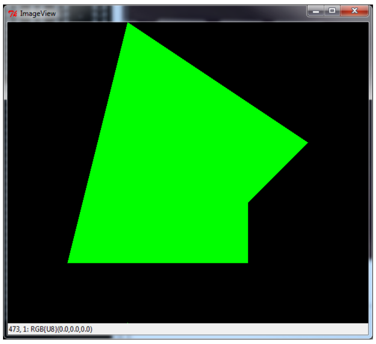
The following is the program with 4 points to test the fan region. We create two fan regions.

```
RR0=[100,100]
RR1=[400,100]
RR2=[400,200]
RR3=[500,300]
RR4=[200,500]
imsize=(600,500)
B2=fanregion(RR0,RR1,RR2,imsize)
my_view_flip(B2)
B3=fanregion(RR1,RR2,RR3,imsize)
my_view_flip(B3)
```

IV. POLYGON

Poygon can be build from all fan region, some is positive some is negative. Assume we have 5 points R_0,R_1,R_2,R_3,R_4

$$Polygon = F(R_0, R_1, R_2) + F(R_1, R_2, R_3) + F(R_2, R_3, R_4) + F(R_3, R_4, R_0) + F(R_4, R_0, R_1) + 1$$



(a)

Figure 5: (a) The polygon created from 5 points.

The Julia program code is following,

```
polygon=ones(imsize)+
  fanregion(RR0,RR1,RR2,imsize)+
  fanregion(RR1,RR2,RR3,imsize)+
  fanregion(RR2,RR3,RR4,imsize)+
  fanregion(RR3,RR4,RR0,imsize)+
  fanregion(RR4,RR0,RR1,imsize)
my_view_flip(polygon)
```

The general function to created a polygon is given as following.

```
function my_polygon(points,image_size)
  imsize=size(points)
  R0=points[:,end]
  R1=points[:,1]
  R2=points[:,2]
  poli=fanregion(R0,R1,R2,image_size)
  my_view(poli)
  for iii=1:imsize[2]-1
    R0=points[:,iii]
```

```
    iii_1=iii+1
    R1=points[:,iii_1]
    iii_2=iii+2
    if iii_2>imsize[2]
      iii_2-=imsize[2]
    end
    R2[1]=points[1,iii_2]
    R2[2]=points[2,iii_2]
    fan=fanregion(R0,R1,R2,image_size)
    poli+=fan
    my_view(poli)
  end
  poli+=1.
  my_view(poli)
  return poli
end
```

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V. IMAGE VIEWER

In order to view the image, we have flowing functions. We have flipped the image along for y coordinates, so y coordinates directed above.

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```
function flip_y(B)
  imsize=size(B)
  D=copy(B)
  size_y=imsize[2]
  half_size=Int(floor(size_y/2))
  for jjj=1:half_size
    for iii=1:imsize[1]
      D[iii,size_y-jjj]=B[iii,jjj]
      D[iii,jjj]=B[iii,size_y-jjj]
```

```
    end
  end
  D
end
function my_view_flip(B)
  #only the size of B is
  D=[RGBU8(0,0,0) for iii=1:size(B)[1], jjj=1:size(B)[2]]
  for jjj=1:size(B)[2]
    for iii=1:size(B)[1]
      if B[iii,jjj]>0.5
        D[iii,jjj]=RGBU8(0,1,0)
      elseif B[iii,jjj]<-0.5
        D[iii,jjj]=RGBU8(1,0,0)
      else
        D[iii,jjj]=RGBU8(0,0,0)
      end
    end
  end
  D=flip_y(D)   imgc = copyproperties(img, D)   view(imgc) end
function my_view(B)
  D=[RGBU8(B[iii,jjj],B[iii,jjj],B[iii,jjj]) for iii=1:size(B)[1], jjj=1:size(B)[2]]
  imgc = copyproperties(img, D)
  view(imgc)
  imgc
end
```

VI. TEST IMAGE

```
RR0=[100,100]
RR1=[400,100]
```

```
RR2=[400,200]
RR3=[500,300]
RR4=[200,500]
imsize=(600,500)
polygon2=my_polygon(points,imsize)
```

When the program my_polygon(points,imsize) runs, it shows how a polygon is created from the fan regons, see Figure(6).

VII. CONCLUSION

Introduced a method to draw all points inside the polygon. The polygon divided as may fan region. Each time a fan is drawn. A fan is created by two half planes. The half plane is created through 2 points. This method is suitable parallel calculation for example GPU Cuda/OpenCL calculations.

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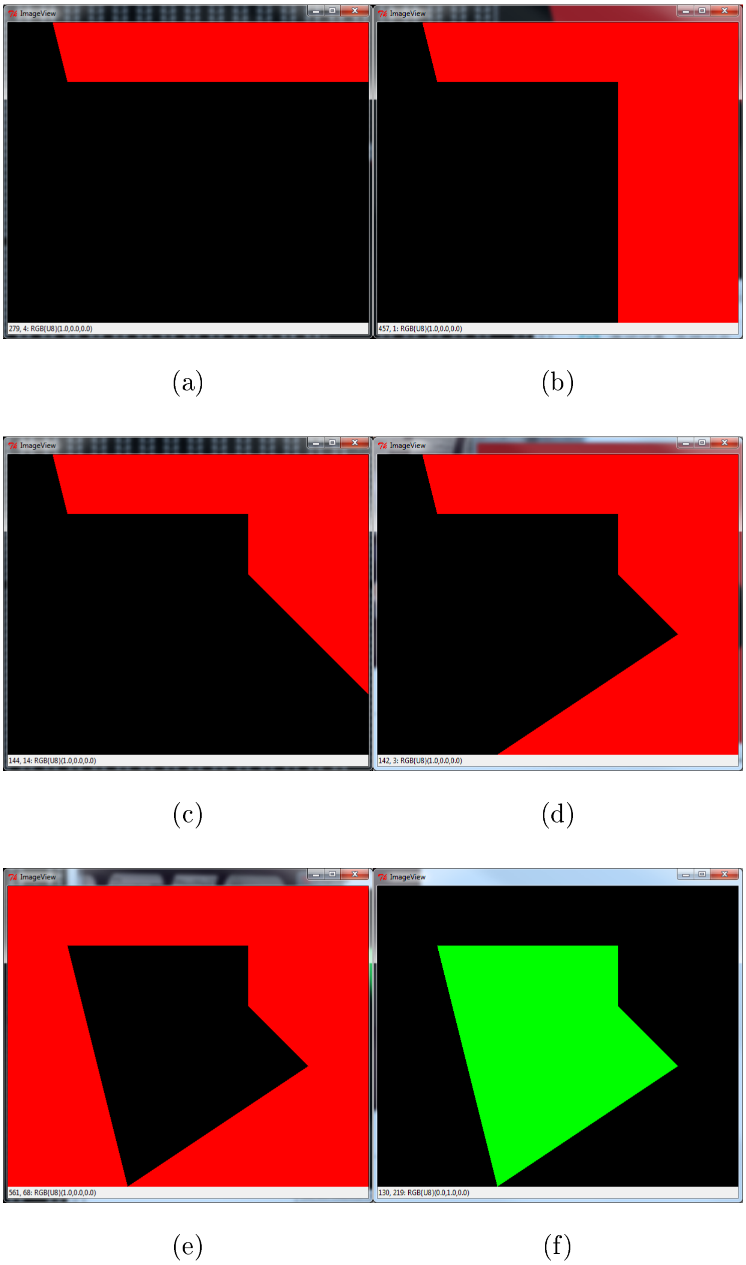

Figure 6: (a) The polygon created from 5 points.

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