Appendix one

1) A logical calculus for signification:

Definitions:

1) The primitives are products of variables both logical and fuzzy;
2) The fuzzy variables can not outnumber the logical ones;
3) For a product of n sub-variables k of which are fuzzy variables and n-k logical variables the number of operations to make a term vanish through negation as a minimum of 1 and a maximum of n-k, when the negations are made for only one variable in its negation;
4) Logical values 1 1...1, for 0 0...0 and negation of a fuzzy variable means something else is true about the logical variable;
5) Each fuzzy variable is linked to a specific logical variable.

Let us consider variables as {a, b, c, d,..,z}, and on this set let us define an operation of connotative product and connotative union. For the sake of completeness let us consider an universe in which the semantic value of a variable stays in the closest interval $0 \leq x \leq 1$. Further more we define each variable a, b, c, …., z as products of what we denote as fundamental not reducible variables.

In general if we define on this set an operation of connotative product the result is not interpretable and may not make any sense.

For instance if we have $x \equiv abc$, if any of those variables is not true in conventional denotative logic for a or b or $c \equiv 0$, the variable x will also have value zero. This is not the case in the calculus we are introducing.

In case for $x \equiv a \wedge b \wedge c$ and any one of these three variables has truth value 0, but semantic value α, the connotative value of x becomes $x \equiv a \times b \times \alpha \equiv 1\ a \wedge b \wedge \alpha$.

This definition that encompasses truth value and fuzzy value in which any fuzzy value is taken as a truth variable with value $0 \leq \alpha \leq 1$, this for any product of the variables we have that $a \wedge b \wedge c \equiv b \wedge c \wedge a$, that is the operation is commutative. On the other hand $a \wedge b \wedge c \equiv (a \wedge b) \wedge c \equiv a \wedge (b \wedge c)$, that is the operation is associative. Further more, the set includes values 0 and 1. Finally $a \wedge 1/a = 1$, that is, every element of the set as a reciprocal. Therefore our algebra structure includes a multiplicative group, with the particularity of each element having a double connotation: 0 or 1, truth value and $0 < \alpha < 1$ for connotative or semantic degree of possession of some predicate.

Lemma: if a multiplicative variable, one or more of its components have a logical truth value zero, the total value of the product does not become zero, but rather the product of the elements which have a logical value equal to 1. Demonstration. xyzv for z=0 e v=0 becomes xy.zv=xy because the value of zv make this two variables vanish from the product, but not implying that their fuzzy value vanishes.

Therefor xyzv=xy.(zv)=xy in our logical approach. As a matter of fact this means only that values of x and y are independent in their fuzzy values from z and v. In this circumstances xyzv=xy(zv)=xy *q.e.d.*

This fact makes a change in the fuzzy value of xyz that now is only xy and can only be processed together with couples (a,b) of other variables and the coupling of a,b and a,b,c,d is not allowed. We have handed this way the analysis of the problems raised by connotative logical product.

2) Connotative logical negation

Let us consider $\overline{x\,y\,z}$ the corresponding truth table of Wittgenstein is

$x\,y\,z$	$\overline{x\,y\,z}$
1 1 1 1	0 0 0 0
0 0 0 0	1 1 1 1

$\overline{x\,y\,z\,v}$	$\overline{xy}\cdot(zv)$
zv	$\overline{zv}$

Annihilation operation

This operation consists in the reduction f a variable without affecting the remaining terms of a product. For a product variable with n elements in the product this is obtained multiplying each a variable we wish to annihilate by its reciprocal. As multiplication by the reciprocal produces value 1, the values of the other components of the product is not affected once it is multiplied by 1 When there remains only one variable and the others variables of the product have already vanished there is nothing to be preserved and therefore the operation produces now value 0.

3) Sheffer stroke

The logical function defined in its normal disjunctive form by $ss = \overline{x} \cdot \overline{y}$, we will have $\overline{x} \cdot \overline{y}$, remarking that $\overline{a} \oplus a \vee 1$, in which ⊕ means exclusive or defined by the truth table:

1	1	0	0
0	1	0	1
1	0	1	1

In addition modulo 2, 1⊕1=0, because the rest of the addition is zero and 1+0=1 because the rest of the division by 2 is 1.

Furthermore note that $\overline{x} \cdot \overline{y} \equiv (x \oplus 1) \cdot (y \oplus 1) \equiv x \cdot y \oplus x \oplus 1 \equiv \overline{x}$. Note that $x \cdot y \oplus x \oplus y$ is the expression for inclusive or and the addition of modulo 2 of 1 generates its complement function which is represented by $\overline{x} \cdot \overline{y}$.

Proceeding to our aim of defining a connotative sheffer stroke we have:

$$sheffer\ stroke \equiv \overline{abcd} \cdot \overline{abcd} \cdot \overline{defg} \equiv (abcd) \cdot (defg) \oplus (abcd) \oplus (defg) \oplus 1$$

Here we have again the problem of negation, having generated a 0 value and with it we cannot construct any further functions, contrarily in the denotative logics.

4) Connotative inclusive or

$$f\,(xuy) \equiv xy \vee x\overline{y} \vee \overline{x}\,y \equiv xy \oplus x\,(y \oplus 1) \oplus y\,(x \oplus 1) \equiv xy \oplus xy \oplus x \oplus xy \oplus y \equiv xy \oplus x \oplus y$$
$$(abcd)(efgh) \oplus abcd \oplus efgh \ .$$

Note that in connotative logics the function becomes xy or x or y due to the annihilation operation.

5) Implication

$$xy \oplus \overline{x}\,y \oplus \overline{x}\,\overline{y} \equiv xy \oplus xy \oplus y \oplus (x \oplus 1)(y \oplus 1) \equiv xy \oplus xy \oplus y \oplus xy \oplus x \oplus y \oplus 1 \equiv xy \oplus x \oplus 1$$
$$(abcd)(efgh) \oplus (efgh) \oplus 1$$

In connotative logics it becomes $xy \vee x \vee y$

If we consider the normal disjunctive form of implication we have a first mean term xy which poses no problem and two further mean terms $\overline{x}\, y \wedge \overline{x}\, \overline{y}$ in which annihilation operation as made disappear the first term of the normal disjunctive form or the first and the second term, what renders the function indeterminate, except for the first mean term, rendering it an implication in the strict sense, only for xy

6)Equivalence

$$x \equiv y \equiv xy \vee \overline{x}\,\overline{n} \equiv xy \oplus xy \oplus xy \oplus x \oplus y \oplus 1 \equiv x \oplus y \oplus 1$$

$$(a\, b\, c_{z1}\, d_{z2}) \oplus (e\, f\; g_{z3}\, h_{z4}) \oplus 1$$

From a connotative view point the mean term
$\overline{x}\,\overline{y}$ disappears and only xy remains.

7)Exclusive Or

By definition exclusive or function is $x\,\overline{y} \vee \overline{x}\, y$ and in connotative logics $x \vee y$.

7) Pierce´s Function

$$x\,\overline{y}\,\overline{x}\; y\,\overline{x}\,\overline{y} \equiv x(y \oplus 1)\, y(x \oplus 1) \oplus (x \oplus 1)(y \oplus 1) \equiv xy \oplus x \oplus xy \oplus y \oplus xy \oplus x \oplus y \oplus 1 \equiv xy \oplus 1$$

Therefore: $(a\, b\, c_{z1}\, d_{z2})(e\; f\; g_{z3}\, h_{z4}) \oplus 1$

Circumstances of time and place and other determinants outside of psychological ones are considered as invariants not needing a specification of their change with the evolution of a situation. Another issue concerns fuzzy values which must be within the close interval $0 \leq v \leq 1$.

For the sake of simplicity in calculation we make this interval correspond to (0,100). As we do not have any scale we adopt a logic of differential thresholds. So in a situation in which the value is x the threshold is obtained adding Δx and we have:

$\frac{\Delta x}{x} = k$ and integrating this expression from 0 to ∞, we have:

$v = \int_0^\infty x + \frac{\Delta x}{x} \delta x = \log x + \Delta x + c$ and

$x + \frac{\Delta x}{x} = k$ in which k is a constant along the different values of log x.

Note that an analogous treatment can be done creating a syntactic structure analog to grammar relationship in formal or common language. This can reduce the lenghts of the formal treatment by telescoping or on the contrary unfolding formal expressions.

From a connotative view point $x\bar{y} \vee \bar{x} y$ becomes $x \vee y$ and the term $\bar{x}\bar{y}$ disappears

8) DeMorgan´s Theorem:

$$(\overline{xy} \oplus \bar{x} \oplus \bar{y}) \equiv \bar{x}\bar{y}$$

Note that: $xy + x + y + 1 \equiv (x \oplus 1)(y \oplus 1) \equiv xy \oplus x \oplus y \oplus 1$, *q. e. d.*

If we consider the Morgan case from the viewpoint of our annihilation operation what is stated in the De Morgan theorem becomes

$$\bar{x} \vee \bar{y} \equiv \bar{x}\bar{y}$$

Now by definition of annihilation $\bar{x}\bar{y} \equiv 0$.

This way in the normal disjunctive form of Hilbert and Ackermann the only functions for which the De Morgan theorem holds true are those which do not include $\bar{x}\bar{y}$ as a mean term.

8.1) Negation of a function

Negation of a function is equivalent to its complement.

If we have a function defined in terms of its normal disjunctive form its negation is formed by the disjunction of minterms not present in the initial function and only those.

$$x \vee y \equiv xy \oplus x\bar{y} \oplus \bar{x} y$$

complement

$$f'(x, y) \equiv x \oplus y$$

These are 2 examples which confirm the truth of the theorem. Further confirmation is left as an exercise to the reader.

Consider now that what is negated in a denotative function is the truth value. Nevertheless in a connotative function denial if it is complete annihilates any information except that the truth value becomes 0, in case it was 1, or 1 if it was 0.

In a connotative denial every trace of a previous assertion leaves no trace of it.

This generates a problem because if a 0 has been generated we have no information about what has to be generated to reconstitute the term that has been denied.

So if A is connotative denied, 0 follows. But if 0 is connotatively denied, the result is indeterminate.