Divide the Beal's Conjecture into Several Parts to Prove the Beal's Conjecture

Zhang Tianshu

Zhanjiang city, Guangdong province, China

Email: chinazhangtianshu@126.com

Introduction: The Beal's Conjecture was discovered by Andrew Beal in 1993. Later the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society. Yet it is still an unproved as well as un-negated conjecture hitherto.

AMS subject classification: 11D××, 00A05.

Abstract

In this article, we first classify A, B and C according to their respective odevity, and thereby get rid of two kinds from $A^X+B^Y=C^Z$. Then, affirmed the existence of $A^X+B^Y=C^Z$ in which case A, B and C have at least a common prime factor by certain of concrete examples. After that, proved $A^X+B^Y\neq C^Z$ in which case A, B and C have not any common prime factor by the mathematical induction with the aid of the symmetric law of positive odd numbers after divide the inequality in four. Finally, we proved that the Beal's conjecture does hold water via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

Keywords: Beal's conjecture, indefinite equation, inequality, odevity, mathematical induction, symmetric law of positive odd numbers.

The Proof

The Beal's Conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

We consider the limits of values of aforesaid A, B, C, X, Y and Z as given requirements for hinder concerned indefinite equations and inequalities.

First we classify A, B and C according to their respective odevity, and thereby remove following two kinds from $A^X+B^Y=C^Z$.

1. If A, B and C all are positive odd numbers, then A^X+B^Y is an even number, yet C^Z is an odd number, so there is only $A^X+B^Y\neq C^Z$ under these circumstances according to an odd number ≠ an even number.

2. If any two of A, B and C are positive even numbers, yet another is a positive odd number, then C^Z is an odd number when A^X+B^Y is an even number, yet C^Z is an even number when A^X+B^Y is an odd number, so there is only $A^X+B^Y\neq C^Z$ under these circumstances according to an odd number ≠ an even number.

Thus, we merely continue to have two kinds of $A^X+B^Y=C^Z$ under the given requirements plus following each set of qualifications.

1. A, B and C all are positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number.

For indefinite equation $A^X+B^Y=C^Z$ satisfying aforementioned either set of qualifications, in fact, it has many sets of solution with A, B and C which are positive integers. Let us instance two concrete equalities respectively to explain the proposition as follows.

When A, B and C all are positive even numbers, if let A=B=C=2 and X=Y ≥3, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $2^X+2^X=2^{X+1}$. Obviously indefinite equation $A^X+B^Y=C^Z$ at the here has a set of solution with A, B and C which are positive integers 2, 2 and 2, and A, B and C have common prime factor 2. In addition, if let A=B=162, C=54, X=Y=3 and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $162^3+162^3=54^4$. So indefinite equation $A^X+B^Y=C^Z$ at the here has a set of solution with A, B and C which are positive integers 162, 162 and 54, and A, B and C have common prime factors 2 and 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3 and Z=5, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $3^3+6^3=3^5$. So indefinite equation $A^X+B^Y= C^Z$ at the here has a set of solution with A, B and C which are positive integers 3, 6 and 3, and A, B and C have common prime factor 3. In addition, if let A=B=7, C=98, X=6, Y=7 and Z=3, then indefinite equation $A^X+B^Y=C^Z$ is changed into equality $7^6+7^7=98^3$. So indefinite equation $A^X+B^Y=C^Z$ at the here has a set of solution with A, B and C which are positive integers 7, 7 and 98, and A, B and C have common prime factor 7.

Consequently, indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either set of qualifications is able to hold water, but A, B and C must have at least a common prime factor.

By now, if we can prove that there is only $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor, then the conjecture is tenable definitely.

Since A, B and C have common prime factor 2 when A, B and C all are positive even numbers, so these circumstances that A, B and C have not a common prime factor can only occur in which case A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not a common prime factor, then any two of them have not a common prime factor either, because in case any two have a common prime factor, namely A^X+B^Y, C^Z-A^X or C^Z-B^Y has a common prime factor, yet C^Z, B^Y or A^X has not the common prime factor, then it will directly lead up to $A^X+B^Y\neq C^Z$, $C^Z-A^X\neq B^Y$ or $C^Z-B^Y\neq A^X$ according to the unique factorization theorem of natural number.

Unquestionably, let following two inequalities add together to replace $A^X+B^Y\neq C^Z$ under the given requirements plus the set of qualifications that A, B and C are two positive odd numbers and a positive even number without a common prime factor, this is possible completely.

1. $A^X+B^Y\neq 2^Z G^Z$ under the given requirements plus the set of qualifications that A and B are two positive odd numbers, G is a positive integer, and A, B and 2G have not a common prime factor.

2. $A^X+2^YD^Y \neq C^Z$ under the given requirements plus the set of qualifications that A and C are two positive odd numbers, D is a positive integer, and A, C and 2D have not a common prime factor.

For aforesaid $A^X+B^Y \neq 2^ZG^Z$, when G=1, it is exactly $A^X+B^Y \neq 2^Z$. When G>1: if G is an odd number, then the inequality changes not, namely it is still $A^X+B^Y \neq 2^ZG^Z$; if G is an even number, then the inequality is expressed by $A^X+B^Y \neq 2^W$ or $A^X+B^Y \neq 2^WH^Z$, where H is an odd number ≥3, and W is an integer > Z.

Without doubt, $A^X+B^Y \neq 2^W$ can represent $A^X+B^Y \neq 2^Z$, and $A^X+B^Y \neq 2^WH^Z$ can represent $A^X+B^Y \neq 2^ZG^Z$, where H is an odd numbers ≥3, and W is an integer ≥3. So $A^X+B^Y \neq 2^ZG^Z$ is expressed by two inequalities as follows.

(1) $A^X+B^Y \neq 2^W$, where A and B are positive odd numbers without a common prime factor, and X, Y and W are integers ≥3.

(2) $A^X+B^Y \neq 2^WH^Z$, where A, B and H are positive odd numbers without a common prime factor, X, Y, Z and W are integers ≥3, and H ≥3.

Again come around to aforementioned $A^X+2^YD^Y \neq C^Z$ to say, when D=1, it is exactly $A^X+2^Y \neq C^Z$. When D>1: if D is an odd number, then the inequality changes not, namely it is still $A^X+2^YD^Y \neq C^Z$; if D is an even number, then the inequality is expressed by $A^X+2^W \neq C^Z$ or $A^X+2^WR^Y \neq C^Z$, where R is an odd number ≥3, and W is an integer >Y.

Without doubt, $A^X+2^W \neq C^Z$ can represent $A^X+2^Y \neq C^Z$, and $A^X+2^WR^Y \neq C^Z$

can represent $A^X+2^YD^Y \neq C^Z$, where R is an odd number ≥3, and W is an integer ≥3. So $A^X+2^YD^Y \neq C^Z$ is expressed by two inequalities as follows.

(3) $A^X+2^W \neq C^Z$, where A and C are positive odd numbers without a common prime factor, and X, W and Z are integers ≥3.

(4) $A^X+2^WR^Y \neq C^Z$, where A, R and C are positive odd numbers without a common prime factor, X, Y, Z and W are integers ≥3, and R ≥3.

We regard the limits of values of A, B, C, H, R, X, Y, Z and W in above-listed four inequalities plus their co-prime relation in each of inequalities as known requirements, thereinafter.

Thus it can be seen, the proof of $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor is changed to prove the existence of above-listed four inequalities under the known requirements. Such being the case, we shall first prove $A^X+B^Y \neq 2^W$ and $A^X+B^Y \neq 2^WH^Z$. For this purpose, we must expound beforehand circumstances and terminologies relating to the proof.

First let us divide all positive odd numbers into two kinds, i.e. Φ and Ω. Namely the form of Φ is 1+4n, and the form of Ω is 3+4n, where n ≥ 0. Odd numbers of Φ and Ω respectively arrange orderly as the follows.

Φ: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61…1+4n …

Ω: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63…3+4n …

Besides, we use symbol Φ as well to denote one of Φ, and likewise use symbol Ω to denote one of Ω in the sequence of non-concrete odd numbers and at the expression concerning the symmetry of odd numbers. Then, let us list positive odd numbers from small to large plus $2^W H^Z$ among them below, where H is an odd number≥1, and W and Z are integers≥ 3. Also label the belongingness of each of odd numbers.

1^W∈Φ, 3∈Ω; 5∈Φ, 7∈Ω, (2^3), 9∈Φ, 11∈Ω, 13∈Φ, 15∈Ω, (2^4), 17∈Φ, 19∈Ω, 21∈Φ, 23∈Ω, 25∈Φ, 3^3∈Ω, 29∈Φ, 31∈Ω, (2^5), 33∈Φ, 35∈Ω, 37∈Φ, 39∈Ω, 41∈Φ, 43∈Ω, 45∈Φ, 47∈Ω, 49∈Φ, 51∈Ω, 53∈Φ, 55∈Ω, 57∈Φ, 59∈Ω, 61∈Φ, 63∈Ω, (2^6), 65∈Φ, 67∈Ω, 69∈Φ, 71∈Ω, 73∈Φ, 75∈Ω, 77∈Φ, 79∈Ω, 3^4∈Φ, 83∈Ω, 85∈Φ, 87∈Ω, 89∈Φ, 91∈Ω, 93∈Φ, 95∈Ω, 97∈Φ, 99∈Ω, 101∈Φ, 103∈Ω, 105∈Φ, 107∈Ω, 109∈Φ, 111∈Ω, 113∈Φ, 115∈Ω, 117∈Φ, 119∈Ω, 121∈Φ, 123∈Ω, 5^3∈Φ, 127∈Ω, (2^7), 129∈Φ, 131∈Ω, 133∈Φ, 135∈Ω, 137∈Φ, 139∈Ω, 141∈Φ, 143∈Ω, 145∈Φ, 147∈Ω, 149∈Φ, 151∈Ω, 153∈Φ, 155∈Ω, 157∈Φ, 159∈Ω, 161∈Φ, 163∈Ω, 165∈Φ, 167∈Ω, 169∈Φ, 171∈Ω, 173∈Φ, 175∈Ω, 177∈Φ, 179∈Ω, 181∈Φ, 183∈Ω, 185∈Φ, 187∈Ω, 189∈Φ, 191∈Ω, 193∈Φ, 195∈Ω, 197∈Φ, 199∈Ω, 201∈Φ, 203∈Ω, 205∈Φ, 207∈Ω, 209∈Φ, 211∈Ω, 213∈Φ, 215∈Ω, ($2^3×3^3$), 217∈Φ, 219∈Ω, 221∈Φ, 223∈Ω, 225∈Φ, 227∈Ω, 229∈Φ, 231∈Ω, 233∈Φ, 235∈Ω, 237∈Φ, 239∈Ω, 241∈Φ, 3^5∈Ω, 245∈Φ, 247∈Ω, 249∈Φ, 251∈Ω, 253∈Φ, 255∈Ω, (2^8), 257∈Φ, 259∈Ω, 261∈Φ, 263∈Ω, 265∈Φ, 267∈Ω, 269∈Φ, 271∈Ω …

Thus it can be seen, the permutation of positive odd numbers from small to large has infinitely many cycles of Φ plus Ω, to wit Φ, Ω; Φ, Ω; Φ, Ω;

Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω ⋯

Next, list orderly many kinds of odd numbers which have a common odd number as the base number, and label the belongingness of each of them.

$1^1 \in \Phi$, $3^1 \in \Omega$, $5^1 \in \Phi$, $7^1 \in \Omega$, $9^1 \in \Phi$, $11^1 \in \Omega$,

$1^2 \in \Phi$, $3^2 \in \Phi$, $5^2 \in \Phi$, $7^2 \in \Phi$, $9^2 \in \Phi$, $11^2 \in \Phi$,

$1^3 \in \Phi$, $3^3 \in \Omega$, $5^3 \in \Phi$, $7^3 \in \Omega$, $9^3 \in \Phi$, $11^3 \in \Omega$,

$1^4 \in \Phi$, $3^4 \in \Phi$, $5^4 \in \Phi$, $7^4 \in \Phi$, $9^4 \in \Phi$, $11^4 \in \Phi$,

$1^5 \in \Phi$, $3^5 \in \Omega$, $5^5 \in \Phi$, $7^5 \in \Omega$, $9^5 \in \Phi$, $11^5 \in \Omega$,

$1^6 \in \Phi$, $3^6 \in \Phi$, $5^6 \in \Phi$, $7^6 \in \Phi$, $9^6 \in \Phi$, $11^6 \in \Phi$,

… … … … … …

$13^1 \in \Phi$, $15^1 \in \Omega$, $17^1 \in \Phi$, $19^1 \in \Omega$, $21^1 \in \Phi$, $23^1 \in \Omega$ …

$13^2 \in \Phi$, $15^2 \in \Phi$, $17^2 \in \Phi$, $19^2 \in \Phi$, $21^2 \in \Phi$, $23^2 \in \Phi$ …

$13^3 \in \Phi$, $15^3 \in \Omega$, $17^3 \in \Phi$, $19^3 \in \Omega$, $21^3 \in \Phi$, $23^3 \in \Omega$ …

$13^4 \in \Phi$, $15^4 \in \Phi$, $17^4 \in \Phi$, $19^4 \in \Phi$, $21^4 \in \Phi$, $23^4 \in \Phi$ …

$13^5 \in \Phi$, $15^5 \in \Omega$, $17^5 \in \Phi$, $19^5 \in \Omega$, $21^5 \in \Phi$, $23^5 \in \Omega$ …

$13^6 \in \Phi$, $15^6 \in \Phi$, $17^6 \in \Phi$, $19^6 \in \Phi$, $21^6 \in \Phi$, $23^6 \in \Phi$ …

… … … … … …

From above-listed many kinds of odd numbers, we are not difficult to see that odd numbers whereby each of Φ to act as a base number belong still within Φ; odd numbers which every even power of Ω forms belong within Φ; and odd numbers which every odd power of Ω forms belong within Ω, i.e. $\Phi^X \in \Phi$, $\Omega^{2n} \in \Phi$ and $\Omega^{2n-1} \in \Omega$, where $X \geq 1$ and $n \geq 1$.

In other words, odd numbers whose exponents are even numbers belong within Φ, and odd numbers which odd powers of Φ form belong within Φ; yet odd numbers which odd powers of Ω form belong within **Ω.**

Moreover two adjacent odd numbers which have an identical exponent or a common odd number as base number except for 1 are an even number apart. But also, such even numbers are getting greater and greater along with two exponents or two base numbers are getting greater and greater.

Altogether, odd numbers of odd exponents plus even exponents are exactly all odd numbers of Φ plus **Ω.** Yet odd numbers whose exponents ≥ 3 are merely a part of all odd numbers, and this part is dispersed among all odd numbers, thus the part odd numbers conform to the symmetric law of positive odd numbers that we shall define soon hereinafter.

We put even numbers $2^{W-1}H^{Z}$ among the sequence of positive odd numbers, and regard each of $2^{W-1}H^{Z}$ as a symmetric center of odd numbers concerned, where H is an odd number≥1, W≥3 and Z≥3. Then odd numbers on the left side of $2^{W-1}H^{Z}$ and odd numbers near $2^{W-1}H^{Z}$ on the right side of $2^{W-1}H^{Z}$ are one-to-one bilateral symmetries at the number axis or in the sequence of natural numbers. For example, if we regard 2^{W-1} as a symmetric center, then $2^{W-1}-1\in\Omega$ and $2^{W-1}+1\in\Phi$, $2^{W-1}-3\in\Phi$ and $2^{W-1}+3\in\Omega$, $2^{W-1}-5\in\Omega$ and $2^{W-1}+5\in\Phi$, $2^{W-1}-7\in\Phi$ and $2^{W-1}+7\in\Omega$ etc are one-to-one bilateral symmetry respectively.

We regard one-to-one bilateral symmetries between odd numbers of Φ and odd numbers of Ω for symmetric center $2^{W-1}H^{Z}$ as the symmetric law of positive odd numbers at the number axis or in the sequence of natural numbers.

The symmetric law of positive odd numbers indicates that for any symmetric center $2^{W-1}H^{Z}$, it can only symmetrize one of Φ and one of Ω, yet can not symmetrize two of Φ or two of Ω.

After regard one of $2^{W-1}H^{Z}$ as a symmetric center, from this $2^{W-1}H^{Z}$ to start out, both there are finitely many cycles of Ω plus Φ leftwards until Ω=3 with Φ=1, and infinitely many cycles of Φ plus Ω rightwards.

According to the symmetric law of positive odd numbers, two distances from a symmetric center to bilateral symmetric Φ and Ω are possessed of equal-long two segments at the number axis or identical two odd differences in the sequence of natural numbers.

Consequently, the sum of every pair of bilateral symmetric odd numbers Φ and Ω is equal to the double of the even number as the symmetric center. Yet over the left, a sum of two non-symmetric odd numbers is unequal to the double of the even number as the symmetric center absolutely. In other words, after regard a certain $2^{W-1}H^{Z}$ as a symmetric center, not just can only symmetrize Φ and Ω, but also this $2^{W}H^{Z}$ as the sum of two odd numbers can only be obtained from the addition of bilateral symmetric Φ and Ω. Please, you pay attention to the conclusion, because it is considered as an important basis that concerns the proof.

Before do the proof, it is necessary to define two terminologies, namely for an integer, if its exponent is greater than or equal to 3, then we term the integer an integer of the greater exponent; if its exponent is equal to 1 or 2, then we term the integer an integer of the smaller exponent.

Pursuant to preceding basic concepts and the conclusion about the double of $2^{W-1}H^{Z}$, we set to prove aforesaid four inequalities, one by one.

Firstly, Prove $A^{X}+B^{Y}\neq 2^{W}$ under the known requirements.

Let us regard 2^{W-1} as a symmetric center of odd numbers concerned to prove $A^{X}+B^{Y}\neq 2^{W}$ under the known requirements by the mathematical induction with the aid of the symmetric law of positive odd numbers.

(1) When W-1=2, bilateral symmetric odd numbers on two sides of symmetric center 2^{2} are listed as follows.

1^{3}, 3, (2^{2}), 5, 7

Obviously there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{2}. So we get $A^{X}+B^{Y}\neq 2^{3}$ under the known requirements.

When W-1=3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of symmetric center 2^{W-1} are listed as follows.

1^{6}, 3, 5, 7, (2^{3}), 9, 11, 13, 15, (2^{4}), 17, 19, 21, 23, 25, 3^{3}, 29, 31, (2^{5}), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^{6}), 65, 67, 69, 71, 73, 75, 77, 79, 3^{4}, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^{3}, 127

From above-listed odd numbers plus 2^{W-1}, we are not difficult to see that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{W-1}, where W-1=3, 4, 5 and 6. So there are $A^X+B^Y\neq 2^4$, $A^X+B^Y\neq 2^5$, $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$ under the known requirements.

(2) When W-1=K with K≥ 6, suppose that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^K. That is to say, suppose $A^X+B^Y\neq 2^{K+1}$ under the known requirements.

(3) When W-1=K+1, prove that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}. In other words, it needs us to prove $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

Proof * We have known that odd numbers whereby 2^{W-1} including 2^K plus 2^{K+1} to act as a symmetric center conform to the symmetric law of positive odd numbers. Let us now list odd numbers whereby 2^{K+1} including 2^K to act as a symmetric center as follows.

1^{K+1}, 3, 5, 7, …Φ, Ω, Φ, Ω, (2^K),Φ, Ω, Φ, Ω, …Φ, Ω, Φ, Ω, …Φ, Ω, Φ, Ω, (2^{K+1}),Φ, Ω, Φ, Ω,…Φ, Ω, Φ, Ω, …Φ, Ω, Φ, Ω, Φ, Ω, Φ, Ω,…Φ, Ω, Φ, Ω.

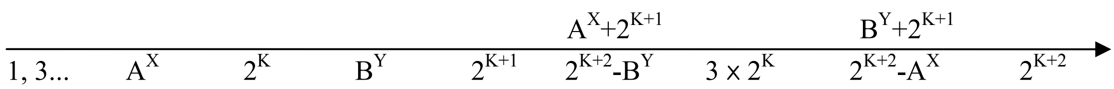
Since every pair of bilateral symmetric odd numbers for symmetric center 2^{W-1} is only Φ and Ω, then two differences from 2^{W-1} to bilateral symmetric Φ and Ω are an identical odd number actually.

In reality, all odd numbers whereby 2^K to act as a symmetric center are exactly odd numbers on the left side of symmetric center 2^{K+1}. Thus, for odd numbers whereby 2^{K+1} to act as a symmetric center, their a half retains still original places after move symmetric center 2^K to 2^{K+1}, and the half lies on the left side of 2^{K+1}, while another half is formed from 2^{K+1} plus each of odd numbers whereby 2^K to act as the symmetric center, and the half lies on the right side of 2^{K+1}.

Suppose that A^X and B^Y are any pair of bilateral symmetric odd numbers for symmetric center 2^K, then there is $A^X+B^Y=2^{K+1}$ according to the preceding conclusion about the double of $2^{W-1}H^Z$.

Since there are not two odd numbers of the greater exponent on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^K according to second step of the mathematical induction, thus let tentatively A^X as an odd number of the greater exponent, and B^Y as an odd number of the smaller exponent, i.e. let $X\geq3$, and $Y=1$ or 2.

By now, let B^Y plus 2^{K+1} makes B^Y+2^{K+1}. Please, see a simple illustration at the number axis as the follows.



Since there is only $A^X+B^Y\neq2^{K+1}$ under the known requirements according to second step of the mathematical induction, then there is inevitably $A^X+B^Y=2^{K+1}$ under the known requirements except for Y and Y=1 or 2, and further has $B^Y+2^{K+1}=A^X+2B^Y=2^{K+2}-A^X$. Evidently A^X and $2^{K+2}-A^X$ are a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1} due to $A^X+(2^{K+2}-A^X)=2^{K+2}$. Namely A^X and A^X+2B^Y in the case are bilateral symmetric odd numbers for symmetric center 2^{K+1}. Thus, it has $A^X+(2^{K+2}-A^X)=A^X+(A^X+2B^Y)=2^{K+2}$ under the known requirements except for Y and Y=1 or 2. Of course, A^X and A^X+2B^Y in the case are still a pair of bilateral symmetric Φ and Ω for symmetric center 2^{K+1}.

But then, there is only $A^X+B^Y\neq 2^{K+1}$ under the known requirements, thus it has $A^X+ [A^X+2B^Y] =2[A^X+B^Y] \neq 2^{K+2}$.

In any case, A^X+2B^Y is a positive odd number, so let $A^X+2B^Y=D^E$, where E expresses the greatest common divisor of exponents of distinct prime divisors of D, and D is a positive odd number, so we get $A^X+ [A^X+2B^Y] =A^X+D^E\neq 2^{K+2}$ under the known requirements.

That is to say, no matter what positive integer which E equals and no matter what positive odd number which D equals from $A^X+2B^Y=D^E$ under the known requirements, there is $A^X+D^E\neq 2^{K+2}$ invariably. Namely A^X and D^E in which case $A^X+2B^Y=D^E$ under the known requirements are not bilateral symmetric two odd numbers for symmetric center 2^{K+1}.

Whereas A^X and D^E in which case $A^X+2B^Y=D^E$ under the known requirements except for Y and Y=1 or 2 are indeed a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}, and from this get $A^X+(A^X+2B^Y)=2^{K+2}$ according to the preceding conclusion reached. Such being the case, provided slightly change the evaluation of any letter of

A^X+2B^Y, then it at once is not original that A^X+2B^Y under the known requirements except for Y and Y=1 or 2, of course now it lies not on the place of the symmetry of A^X either. Namely A^X and A^X+2B^Y under the known requirements are not bilateral symmetric odd numbers for symmetric center 2^{K+1} because the value of Y has changed from Y=1 or 2 to $Y\geq 3$, so there is $A^X+[A^X+2B^Y]=A^X+D^E\neq 2^{K+2}$ under the known requirements according to the preceding conclusion about the double of $2^{W-1}H^Z$. In addition, A^X was supposed as any positive odd number of the greater exponent on the left side of symmetric center 2^{K+1}, and there is $A^X+B^Y=2^{K+1}$ under the known requirements except for Y and Y=1 or 2, therefore, it has $A^X+2B^Y=2^{K+1}+B^Y$. Thus it can be seen, A^X+2B^Y i.e. D^E lies on the right side of symmetric center 2^{K+1}.

For inequality $A^X+D^E\neq 2^{K+2}$, let us substitute D therein by B, since B and D can express identical any positive odd number, and substitute Y for E, where $E\geq 3$, since $Y\geq 3$.

Consequently, we obtain $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

In the proof, if B^Y is an odd number of the greater exponent, then A^X is surely an odd number of the smaller exponent, and a conclusion concluded on the premise is one and the same with $A^X+B^Y\neq 2^{K+2}$ under the known requirements too.

If A^X and B^Y are a pair of bilateral symmetric odd numbers of the smaller exponents for symmetric center 2^K, then whether A^X and A^X+2B^Y, or B^Y

and B^Y+2A^X are still a pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}. But, no matter what positive odd number which A^X+2B^Y or B^Y+2A^X equal, it can not let the pair of bilateral symmetric odd numbers turn into two odd numbers of the greater exponents, since A^X or B^Y in the pair is not an odd number of the greater exponent originally.

To sum up, we have proven that when W-1=K+1 with K≥6, there is only $A^X+B^Y\neq 2^{K+2}$ under the known requirements. In other words, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center 2^{K+1}.

Apply the preceding way of doing, we can continue to prove that when W-1=K+2, K+3…up to every integer >K+3, there are merely $A^X+B^Y\neq 2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$ … up to $A^X+B^Y\neq 2^W$ under the known requirements.

Secondly, Let us successively prove $A^X+B^Y\neq 2^W H^Z$ under the known requirements, and point out H ≥3 at the here emphatically.

From now on, we set about the proof of $A^X+B^Y\neq 2^W H^Z$ under the known requirements by the mathematical induction with the aid of the symmetric law of positive odd numbers for the second time.

(1) When H=1, $2^{W-1}H^Z$ to wit 2^{W-1}, we have proven $A^X+B^Y\neq 2^W$ under the known requirements in the preceding section. Namely there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center 2^{W-1}.

(2) When H=J, and J is an odd number ≥1, $2^{W-1}H^Z$ to wit $2^{W-1}J^Z$, suppose

$A^X+B^Y \neq 2^W J^Z$ under the known requirements. Namely suppose that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}J^Z$.

(3) When H=K with K=J+2, $2^{W-1}H^Z$ to wit $2^{W-1}K^Z$, prove $A^X+B^Y \neq 2^W K^Z$ under the known requirements. Namely prove that there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetry center $2^{W-1}K^Z$.

Proof * Since after regard $2^{W-1}H^Z$ as a symmetric center, the sum of every pair of bilateral symmetric odd numbers is equal to $2^W H^Z$, yet a sum of any two odd numbers of no symmetry is unequal to $2^W H^Z$ absolutely.

In addition, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}J^Z$. Namely there is only $A^X+B^Y \neq 2^W J^Z$ under the known requirements according to step 2 of the mathematical induction.

Such being the case, let us suppose that A^X and B^Y are any pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}J^Z$, also tentatively let Y ≥3 and X=1 or 2, then there is surely $A^X+B^Y=2^W J^Z$.

On the other, after regard $2^{W-1}K^Z$ as a symmetric center, B^Y and $2^W K^Z-B^Y$ are a pair of bilateral symmetric odd numbers due to $B^Y+(2^W K^Z-B^Y)=2^W K^Z$ according to the preceding conclusion about the double of $2^{W-1}H^Z$.

By now, let A^X plus $2^W(K^Z-J^Z)$ makes $A^X+2^W(K^Z-J^Z)$, also $A^X+2^W(K^Z-J^Z) = A^X+2^W K^Z-2^W J^Z=2^W K^Z-(2^W J^Z-A^X) =2^W K^Z-B^Y$ under the known requirements except for X and X=1 or 2, due to $A^X+B^Y=2^WJ^Z$ in the case.

Now that there is $A^X+2^W(K^Z-J^Z)=2^WK^Z-B^Y$ under the known requirements except for X and X=1 or 2; in addition B^Y and $2^WK^Z-B^Y$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, then B^Y and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, thus we get $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ under the known requirements except for X and X=1 or 2.

Of course, B^Y and $A^X+2^W(K^Z-J^Z)$ in the case are still a pair of bilateral symmetric Φ and Ω for symmetric center $2^{W-1}K^Z$.

From $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W(K^Z-J^Z)$ and preceding supposed $A^X+B^Y\neq 2^WJ^Z$ under the known requirements, we get $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^WK^Z-2^WJ^Z\neq 2^WK^Z$ under the known requirements. Thus it can be seen, B^Y and $A^X+2^W(K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ because the sum of B^Y plus $A^X+2^W(K^Z-J^Z)$ is not equal to 2^WK^Z.

It is obvious that $A^X+2^W(K^Z-J^Z)$ in the aforesaid two cases expresses two disparate odd numbers, due to $X\geq 3$ in one and X=1 or 2 in another.

From $A^X+2^W(K^Z-J^Z)=2^WK^Z-(2^WJ^Z-A^X)$ and $2^WJ^Z-A^X\neq B^Y$ under the known requirements, we get $A^X+2^W(K^Z-J^Z)\neq 2^WK^Z-B^Y$.

In any case, $A^X+2^W(K^Z-J^Z)$ is a positive odd number, thus let $A^X+2^W(K^Z-J^Z)=F^V$, where V expresses the greatest common divisor of exponents of distinct prime divisors of F, and F is a positive odd number, so there is $F^V\neq$

$2^W K^Z - B^Y$ due to $A^X+2^W(K^Z-J^Z) \neq 2^W K^Z - B^Y$ under the known requirements. Namely there is $B^Y+F^V \neq 2^W K^Z$ under the known requirements.

Since B^Y and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, and $B^Y+[A^X+2^W(K^Z-J^Z)] = 2^W K^Z$ under the known requirements except for X and X=1 or 2 according to the conclusion reached previously. Such being the case, provided slightly change the evaluation of any letter of $A^X+2^W(K^Z-J^Z)$, then it at once is not original that $A^X+2^W(K^Z-J^Z)$ under the known requirements except for X and X=1 or 2, of course now it lies not on the place of the symmetry of B^Y either. Namely B^Y and $A^X+2^W(K^Z-J^Z)$ under the known requirements are not bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ because the value of X has changed from X=1 or 2 to X≥3, therefore there is $B^Y+[A^X+2^W(K^Z-J^Z)] \neq 2^W K^Z$ under the known requirements according to the preceding conclusion about the double of $2^{W-1}H^Z$. Namely there is $B^Y+F^V \neq 2^W K^Z$ under the known requirements, due to $A^X+2^W(K^Z-J^Z) = F^V$.

For inequality $B^Y+F^V \neq 2^W K^Z$, let us substitute F therein by A, since A and F can express identical any positive odd number, and substitute X for V, where V≥3, since X≥3.

Consequently, we obtain $A^X+B^Y \neq 2^W K^Z$ under the known requirements.

In the proof, if A^X is an odd number of the greater exponent, then B^Y is surely an odd number of the smaller exponent, and a conclusion concluded on the premise is one and the same with $A^X+B^Y \neq 2^W K^Z$ under the known requirements too.

If A^X and B^Y are a pair of bilateral symmetric odd numbers of the smaller exponents for symmetric center $2^{W-1}J^Z$, then whether B^Y and $A^X+2^W(K^Z-J^Z)$, or A^X and $B^Y+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$ too. But, no matter what positive odd number which $A^X+2^W(K^Z-J^Z)$ or $B^Y+2^W(K^Z-J^Z)$ equal, it can not let the pair of bilateral symmetric odd numbers turn into two odd numbers of the greater exponents, since B^Y or A^X in the pair is not an odd number of the greater exponent originally.

To sum up, we have proven $A^X+B^Y\neq 2^WK^Z$ with K=J+2 under the known requirements. Namely when H=J+2, there are not two odd numbers of the greater exponents on two places of every pair of bilateral symmetric odd numbers for symmetric center $2^{W-1}(J+2)^Z$.

Apply the above-mentioned way of doing, we can continue to prove that when H=J+4, J+6… up to every odd number > J+6, there are merely $A^X+B^Y\neq 2^W(J+4)^Z$, $A^X+B^Y\neq 2^W(J+6)^Z$… up to $A^X+B^Y\neq 2^WH^Z$ under the known requirements, and point out H ≥3 at the here emphatically.

Thirdly, we proceed to prove $A^X+2^W\neq C^Z$ under the known requirements.

Proof* Since we have proven $A^X+B^Y\neq 2^W$ under the known requirements, thereby it can affirm $E^P+C^Z\neq 2^M$, where E and C are positive odd numbers without a common prime factor, P, Z and M are integers, P, Z≥3, and M≥4 therein. Since E and C have not a common prime factor, so it has $E^P\neq C^Z$ in accordance with the unique factorization theorem of natural number, then we let $C^Z > E^P$.

From $2^M = 2^{M-1} + 2^{M-1}$ and $E^P + C^Z \neq 2^M$, we conclude $E^P + C^Z > 2^{M-1} + 2^{M-1}$ or $E^P + C^Z < 2^{M-1} + 2^{M-1}$. Namely there is $C^Z - 2^{M-1} > 2^{M-1} - E^P$ or $C^Z - 2^{M-1} < 2^{M-1} - E^P$.

Besides, $A^X + E^P \neq 2^{M-1}$ exists objectively too according to proven $A^X + B^Y \neq 2^W$ under the known requirements, where A and E are positive odd numbers without a common prime factor, and X, P and M-1 are integers ≥3.

So we conclude $2^{M-1} - E^P > A^X$ or $2^{M-1} - E^P < A^X$ from $A^X + E^P \neq 2^{M-1}$.

Then, there is $C^Z - 2^{M-1} > 2^{M-1} - E^P > A^X$ or $C^Z - 2^{M-1} < 2^{M-1} - E^P < A^X$.

Therefore, there is $C^Z - 2^{M-1} > A^X$ or $C^Z - 2^{M-1} < A^X$.

In a word, there is $C^Z - 2^{M-1} \neq A^X$, i.e. $A^X + 2^{M-1} \neq C^Z$.

For inequality $A^X + 2^{M-1} \neq C^Z$, let us substitute M-1 therein by W, since M-1 and W can express identical any integer ≥3.

Consequently, there is only $A^X + 2^W \neq C^Z$ under the known requirements.

Fourthly, let us last prove $A^X + 2^W R^Y \neq C^Z$ under the known requirements, and point out R≥3 at the here emphatically.

Proof* Since we have proven $A^X + B^Y \neq 2^W H^Z$ under the known requirements, of course there is $F^S + C^Z \neq 2^N R^Y$ too, where F, C and R are positive odd numbers without a common prime factor, S, Z, N and Y are integers, and S, Z, Y ≥3, N ≥4, also R ≥3.

Since F and C have not a common prime factor, so there is $F^S \neq C^Z$ in accordance with the unique factorization theorem of natural number, then we let $C^Z > F^S$.

From $2^N R^Y = 2^{N-1} R^Y + 2^{N-1} R^Y$ and $F^S + C^Z \neq 2^N R^Y$, we get $F^S + C^Z > 2^{N-1} R^Y + 2^{N-1} R^Y$ or $F^S + C^Z < 2^{N-1} R^Y + 2^{N-1} R^Y$.

That is to say, there is $C^Z - 2^{N-1} R^Y > 2^{N-1} R^Y - F^S$ or $C^Z - 2^{N-1} R^Y < 2^{N-1} R^Y - F^S$.

In addition, there is $A^X + F^S \neq 2^{N-1} R^Y$ according to proven $A^X + B^Y \neq 2^W H^Z$ under the known requirements, where A, F and R are positive odd numbers without a common prime factor, X, S, N-1 and Y are integers ≥ 3, and $R \geq 3$.

So, there is $2^{N-1} R^Y - F^S > A^X$ or $2^{N-1} R^Y - F^S < A^X$ from $A^X + F^S \neq 2^{N-1} R^Y$.

Thus, there is $C^Z - 2^{N-1} R^Y > 2^{N-1} R^Y - F^S > A^X$ or $C^Z - 2^{N-1} R^Y < 2^{N-1} R^Y - F^S < A^X$.

Therefore, there is $C^Z - 2^{N-1} R^Y > A^X$ or $C^Z - 2^{N-1} R^Y < A^X$.

In a word, there is $C^Z - 2^{N-1} R^Y \neq A^X$, i.e. $A^X + 2^{N-1} R^Y \neq C^Z$.

For inequality $A^X + 2^{N-1} R^Y \neq C^Z$, let us substitute N-1 therein by W, since N-1 and W can express identical any integer ≥ 3.

Consequently, there is only $A^X + 2^W R^Y \neq C^Z$ under the known requirements, and point out $R \geq 3$ at the here emphatically.

To sun up, we have proven every kind of $A^X + B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor.

In addition, in the beginning of this article, we have proven by concrete examples that $A^X + B^Y = C^Z$ under the given requirements plus the qualification that A, B and C have at least a common prime factor has many sets of solution with A, B and C which are positive integers.

Last, via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements, we have reached inevitably such a conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the given requirements is that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal's conjecture does hold water.

PS. If this proof can be accepted by the world of mathematics, then the Beal's conjecture is tenable, so let X=Y=Z, next indefinite equation $A^X+B^Y=C^Z$ is transformed into $A^X+B^X=C^X$. In addition, divide three terms of $A^X+B^X=C^X$ by maximal common factor of the three terms, then you will get a set of positive integer's solution of A, B and C without a common prime factor. Obviously this conclusion is in contradiction with proven the Beal's conjecture as the true, so we have proved Fermat's Last Theorem as easy as the pie extra.