On The Factorization Theorem of Circulant Determinant

(Fermat last Theorem was proved in 1991)

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Abstract

In this paper we study the factorization theorem of circulant determinants. We prove that Fermat equation is the subset of circulant determinant and every factor of n has a Fermat equation. On Oct. 25, 1991 without using any number theory we have proved Fermat last theorem

We have defined the complex hyperbolic functions of order n with $n-1$ variables, where n is an odd number [1],

$$S_i = \frac{1}{n}\left[e^A + 2\Sigma_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos(\theta_j + (-1)^j \frac{(i-1)j\pi}{n}) \right], \tag{1}$$

$$A = \Sigma_{\alpha=1}^{\frac{n-1}{2}}(t_\alpha + t_{n-\alpha}), \ B_j = \Sigma_{\alpha=1}^{\frac{n-1}{2}}(t_\alpha + t_{n-\alpha})(-1)^{\alpha j}\cos\frac{\alpha j\pi}{n}, A + 2\Sigma_{j=1}^{\frac{n-1}{2}}B_j = 0, \tag{2}$$

$$\theta_j = -\Sigma_{\alpha=1}^{\frac{n-1}{2}}(t_\alpha - t_{n-\alpha})(-1)^{(\alpha+1)j}\sin\frac{\alpha j\pi}{n}. \tag{3}$$

(1) may be written in the matrix form

$$\begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ \vdots \\ S_n \end{pmatrix} = \frac{1}{n}\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n} \\ 1 & \cos\cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}$$

$$\times \begin{pmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \vdots \\ 2\exp\left(B_{\frac{n-1}{2}}\right)\sin\left(\theta_{\frac{n-1}{2}}\right) \end{pmatrix} \tag{4}$$

where $(n-1)/2$ is an even number. From (4) we obtain its inverse tranformation

$$\times \begin{pmatrix} e^A \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \vdots \\ \exp\left(B_{\frac{n-1}{2}}\right)\sin\left(\theta_{\frac{n-1}{2}}\right) \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & -\cos\dfrac{\pi}{n} & \cos\dfrac{2\pi}{n} & \cdots & \cos\dfrac{(n-1)\pi}{n} \\ 0 & \sin\dfrac{\pi}{n} & \sin\dfrac{2\pi}{n} & \cdots & \sin\dfrac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\sin\dfrac{(n-1)\pi}{2n} & -\sin\dfrac{(n-1)\pi}{n} & \cdots & \sin\dfrac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \\ \vdots \\ S_n \end{pmatrix} \tag{5}$$

From (5) we have

$$e^A = \Sigma_{i=1}^n S_i, \tag{6}$$

$$e^{B_j}\cos\theta_j = S_1 + \Sigma_{i=1}^{n-1}(-1)^{ij} S_{i+1}\cos\frac{ij\pi}{n},$$

$$e^{B_j}\sin\theta_j = (-1)^{j+1}\Sigma_{i=1}^{n-1}(-1)^{ij} S_{i+1}\sin\frac{ij\pi}{n}, \tag{7}$$

From (7) we have

$$\exp(2B_j) = \left[S_1 + \Sigma_{i=1}^{n-1}(-1)^{ij} S_{i+1}\cos\frac{ij\pi}{n}\right]^2 + \left[\Sigma_{i=1}^{n-1}(-1)^{ij} S_{i+1}\sin\frac{ij\pi}{n}\right]^2. \tag{8}$$

Let $n=3$. From (6) and (8) we have

$$e^A = S_1 + S_2 + S_3, \quad e^{2B_1} = S_1^2 + S_2^2 + S_3^2 - S_1 S_2 - S_1 S_3 - S_2 S_3. \tag{9}$$

(9) may be written in the form of circulant determinant

$$\exp(A + 2B_1) = \begin{vmatrix} S_1 & S_3 & S_2 \\ S_2 & S_1 & S_3 \\ S_3 & S_2 & S_1 \end{vmatrix} = 1. \tag{10}$$

Let $n = 5$. From (6) and (8) we have

$$e^A = S_1 + S_2 + S_3 + S_4 + S_5. \tag{11}$$

$$\exp(2B_1) = \left[S_1 + \Sigma_{i=1}^4 (-1)^i S_{i+1} \cos \frac{i\pi}{5} \right]^2 + \left[\Sigma_{i=1}^4 (-1)^i S_{i+1} \sin \frac{i\pi}{5} \right]^2,$$

$$\exp(2B_2) = \left[S_1 + \Sigma_{i=1}^4 S_{i+1} \cos \frac{2i\pi}{5} \right]^2 + \left[\Sigma_{i=1}^4 S_{i+1} \sin \frac{2i\pi}{5} \right]^2. \tag{12}$$

(11) and (12) may be written in the form of circulant determinant

$$\exp(A + 2B_1 + 2B_2) = \begin{vmatrix} S_1 & S_5 & S_4 & S_3 & S_2 \\ S_2 & S_1 & S_5 & S_4 & S_3 \\ S_3 & S_2 & S_1 & S_5 & S_4 \\ S_4 & S_3 & S_2 & S_1 & S_5 \\ S_5 & S_4 & S_3 & S_2 & S_1 \end{vmatrix} = 1. \tag{13}$$

In the same way we have

$$\exp(A + 2\Sigma_{j=1}^{\frac{n-1}{2}} B_j) = \begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ S_3 & S_2 & \cdots & S_4 \\ \cdots & \cdots & \cdots & \cdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix} = 1. \tag{14}$$

Theorem 1. Let $n = 3p$, where p is an odd prime. (14) can be factorized both circulant subdeterminants.

Proof. First we discuss $n = 15$. From (14) we have

$$\exp(A + 2\Sigma_{j=1}^7 B_j) = \begin{vmatrix} S_1 & S_{15} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ S_3 & S_2 & \cdots & S_4 \\ \cdots & \cdots & \cdots & \cdots \\ S_{15} & S_{14} & \cdots & S_1 \end{vmatrix} = 1. \tag{15}$$

From (6) and (8) we have

$$e^A = S_{3\cdot1} + S_{3\cdot2} + S_{3\cdot3} = \Sigma_{i=1}^{15} S_i, \tag{16}$$

$$e^{2B_5} = S_{3\cdot1}^2 + S_{3\cdot2}^2 + S_{3\cdot3}^2 - S_{3\cdot1}S_{3\cdot2} - S_{3\cdot1}S_{3\cdot3} - S_{3\cdot2}S_{3\cdot3}, \tag{17}$$

where $S_{3\cdot i} = \Sigma_{\alpha=0}^4 S_{3\alpha+i}$. From (2) we have

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$$A + 2B_5 = 3(t_3 + t_{12} + t_6 + t_9)]. \tag{18}$$

$$\exp(A + 2B_5) = \begin{vmatrix} S_{3\cdot1} & S_{3\cdot3} & S_{3\cdot2} \\ S_{3\cdot2} & S_{3\cdot1} & S_{3\cdot3} \\ S_{3\cdot3} & S_{3\cdot2} & S_{3\cdot1} \end{vmatrix} = [\exp(t_3 + t_{12} + t_6 + t_9)]^3. \tag{19}$$

From (6) and (8) we have

$$e^A = \Sigma_{i=1}^5 S_{5\cdot i} = \Sigma_{i=1}^{15} S_i, \tag{20}$$

$$\exp(2B_3) = \left[S_{5\cdot1} + \Sigma_{i=1}^4 (-1)^i S_{5\cdot i+1} \cos\frac{i\pi}{5} \right]^2 + \left[\Sigma_{i=1}^4 (-1)^i S_{5\cdot i+1} \sin\frac{i\pi}{5} \right]^2, \tag{21}$$

$$\exp(2B_6) = \left[S_{5\cdot1} + \Sigma_{i=1}^4 S_{5\cdot i+1} \cos\frac{2i\pi}{5} \right]^2 + \left[\Sigma_{i=1}^4 S_{5\cdot i+1} \sin\frac{2i\pi}{5} \right]^2, \tag{22}$$

where $S_{5\cdot i} = \Sigma_{\alpha=0}^2 S_{5\alpha+i}$. From (2) we have

$$A + 2B_3 + 2B_6 = 5(t_5 + t_{10}). \tag{23}$$

(20), (21), (22) and (23) may be written in the form of circulant determinant

$$\exp(A + 2B_3 + 2B_6) = \begin{vmatrix} S_{5\cdot1} & S_{5\cdot5} & S_{5\cdot4} & S_{5\cdot3} & S_{5\cdot2} \\ S_{5\cdot2} & S_{5\cdot1} & S_{5\cdot5} & S_{5\cdot4} & S_{5\cdot3} \\ S_{5\cdot3} & S_{5\cdot2} & S_{5\cdot1} & S_{5\cdot5} & S_{5\cdot4} \\ S_{5\cdot4} & S_{5\cdot3} & S_{5\cdot2} & S_{5\cdot1} & S_{5\cdot5} \\ S_{5\cdot5} & S_{5\cdot4} & S_{5\cdot3} & S_{5\cdot2} & S_{5\cdot1} \end{vmatrix} = [\exp(t_5 + t_{10})]^5, \tag{24}$$

(19) and (24) are the circulant subdeterminants of (15). Let $n = 3p$, where p is an odd prime.

From (14) we have

$$\exp(A + 2\Sigma_{j=1}^{\frac{3p-1}{2}} B_j) = \begin{vmatrix} S_1 & S_{3p} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ S_3 & S_2 & \cdots & S_4 \\ \cdots & \cdots & \cdots & \cdots \\ S_{3p} & S_{3p-1} & \cdots & S_1 \end{vmatrix} = 1. \tag{25}$$

From (6) and (8) we have

$$e^A = S_{3\cdot1} + S_{3\cdot2} + S_{3\cdot3} = \Sigma_{i=1}^{3p} S_i, \tag{26}$$

$$\exp(2B_p) = S_{3\cdot1}^2 + S_{3\cdot2}^2 + S_{3\cdot3}^2 - S_{3\cdot1}S_{3\cdot2} - S_{3\cdot1}S_{3\cdot3} - S_{3\cdot2}S_{3\cdot3}, \tag{27}$$

where $S_{3\cdot i} = \Sigma_{\alpha=0}^{p-1} S_{3\alpha+i}$. From (2) we have

$$A + 2B_p = 3\Sigma_{\alpha=1}^{\frac{p-1}{2}} (t_{3\alpha} + t_{3p-3\alpha}). \tag{28}$$

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(26), (27) and (28) may be written in the form of circulant determinant

$$\exp(A + 2B_p) = \begin{vmatrix} S_{3\cdot1} & S_{3\cdot3} & S_{3\cdot2} \\ S_{3\cdot2} & S_{3\cdot1} & S_{3\cdot3} \\ S_{3\cdot3} & S_{3\cdot2} & S_{3\cdot1} \end{vmatrix} = \left[\exp\left(\Sigma_{\alpha=1}^{\frac{p-1}{2}}(t_{3\alpha} + t_{3p-3\alpha}) \right) \right]^3 . \tag{29}$$

From (6) and (8) we have

$$e^A = \Sigma_{i=1}^{p} S_{p\cdot i} = \Sigma_{i=1}^{3p} S_i , \tag{30}$$

$$\exp(2B_{3j}) = \left[S_{p\cdot1} + \Sigma_{i=1}^{p-1}(-1)^{ij} S_{p\cdot i+1} \cos\frac{ij\pi}{p} \right]^2 + \left[\Sigma_{i=1}^{p-1}(-1)^{ij} S_{p\cdot i+1} \sin\frac{ij\pi}{p} \right]^2 , \tag{31}$$

where $S_{p\cdot i} = \Sigma_{\alpha=0}^{2} S_{p\alpha+i}$. From (2) we have

$$A + 2\Sigma_{i=1}^{\frac{p-1}{2}} B_{3j} = p(t_p + t_{2p}) . \tag{32}$$

（30）,（31）and (32) may be written in the form of circulant determinant

$$A + 2\Sigma_{j=1}^{\frac{p-1}{2}} B_{3j}) = \begin{vmatrix} S_{p\cdot1} & S_{p\cdot p} & \cdots & S_{p\cdot2} \\ S_{p\cdot2} & S_{p\cdot1} & \cdots & S_{p\cdot3} \\ S_{p\cdot3} & S_{p\cdot2} & \cdots & S_{p\cdot4} \\ \cdots & \cdots & \cdots & \cdots \\ S_{p\cdot p} & S_{p\cdot p-1} & \cdots & S_{p\cdot1} \end{vmatrix} = [\exp(t_p + t_{2p})]^p . \tag{33}$$

(29) and (33) are circulant subdeterminants of (25).

Theorem 2. Every factor of n has a circulant determinant.

Theorem 3. Fermat equation is the subset of circulant determinant.

Proof. Let in (25) $S_1 \neq 0$, $S_2 \neq 0$, $S_i = 0$, where $i = 3, 4, \ldots, 3p$. $S_i = 0$ are $3p - 2$

indeterminate equations with $3p - 1$ variables. $S_{3\cdot1} = S_1$, $S_{3\cdot2} = S_2$, $S_{3\cdot3} = 0$; $S_{p\cdot1} = S_1$,

$S_{p\cdot2} = S_2$, $S_{p\cdot i} = 0$, where $i = 3, 4, \ldots, p$. From (15), (19) and (24) we have Fermat equations

$$S_1^{15} + S_2^{15} = 1, \quad S_1^3 + S_2^3 = [\exp(t_3 + t_{12} + t_6 + t_9)]^3, \quad S_1^5 + S_2^5 = [\exp(t_5 + t_{10})]^5, \tag{34}$$

$S_1^3 + S_2^3$ and $S_1^5 + S_2^5$ are the factors of $S_1^{15} + S_2^{15}$. $S_1^{15} + S_2^{15}$ is a subset of (15); $S_1^3 + S_2^3$ is

subset of (19); $S_1^5 + S_2^5$ is a subset of (24). From (25), (29) and (33) we have Fermat equations

$$S_1^{3p} + S_2^{3p} = 1, \quad S_1^3 + S_2^3 = \left[\exp\left(\Sigma_{\alpha=1}^{\frac{p-1}{2}}(t_{3\alpha} + t_{3p-3\alpha}) \right) \right]^3 ,$$

$$S_1^p + S_2^p = \left[\exp(t_p + t_{2p}) \right]^p , \tag{35}$$

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$S_1^3 + S_2^3$ and $S_1^p + S_2^p$ are the factors of $S_1^{3p} + S_2^{3p}$. $S_1^{3p} + S_2^{3p}$ is a subset of (25); $S_1^3 + S_2^3$

is a subset of (29); $S_1^p + S_2^p$ is a subset of (33).

In (34) Euler proved $S_1^{15} + S_2^{15} = 1$ and $S_1^3 + S_2^3 = [\exp(t_3 + t_{12} + t_6 + t_9)]^3$.

Therefore $S_1^5 + S_2^5 = [\exp(t_5 + t_{10})]^5$ has no rational solutions.

In (35) Euler proved $S_1^{3p} + S_2^{3p} = 1$ and $S_1^3 + S_2^3 = \left[\exp\left(\Sigma_{\alpha=1}^{\frac{p-1}{2}} (t_{3\alpha} + t_{3p-3\alpha}) \right) \right]^3$.

Therefore $S_1^p + S_2^p = [\exp(t_p + t_{2p})]^p$ has no rational solutions for any prime $p > 3$. On Oct.

25, 1991 using this method we have proved the Feirmat last theorem.

Theorem 4. Every factor of n has a Fermat equation [2].

References

1. Jiang, Chun-xuan. Hypercomplex variable theory. Preprints. 1989.

2. Jiang, Chun-xuan. Fermat last theorem has been proved. (Chinese. English summary). Qian Kexue, 2(1992) 17-20. Preprints (English),. December, 1991.

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Dear Dr. Jiang, Chun-xuan,

I was happy to receive news from you. You will be pleased to know that I have accepted your article for publication in Algebras Grous and Geometries. The sole change has been the removal of the last line of the test and of referencs [3.4.5]. This is due to the fact that these references are not published and would damage your paper. Instead of this mention in the current paper, I encourage you to write a condensed article on Fermat theorem for AGG.

You may be interested to know that I have invited Prof. Quing-ming Cheng of the Institute of Mathematics of Fudan Universit, Shangai, to become an Editor for AGG.

I am working at Vol. III of Elements of Hadronic Mechanics (see enclosed flier) in which I shall review your results published in the Haronic Journal regarding the total number of electrons predicted by the number theory in the hadronic structure. I shall send you a complimentary copy

of the volume with the review of your work when printed.

Wishing you the best, I remain

Yours, Truly

Prof. Ruggero Maria Santilli
Editor in Chief
Algebras Groups and Geometries

Appendix

Assume $S_1 \neq 0$, $S_2 \neq 0$, $S_3 = S_4 = \cdots = S_n = 0$. From (6) and (8) we obtain

$$e^A = S_1 + S_2, \quad e^{2B_j} = S_1^2 + S_2^2 + (-1)^j 2S_1 S_2 \cos \frac{j\pi}{n} \tag{36}$$

From (2) and (36) we obtain the Fermat's equations [2]

$$n = 3 \quad \exp(A + 2B_1) = S_1^3 + S_2^3 = 1.$$

$$n = 5 \quad \exp(A + 2B_1 + 2B_2) = S_1^5 + S_2^5 = 1.$$

$$n = p \quad \exp(A + 2\Sigma_{j=1}^{\frac{p-1}{2}} B_j) = S_1^p + S_2^p = 1.$$

$$n = 15 \quad \exp(A + 2\Sigma_{j=1}^{7} B_j) = S_1^{15} + S_2^{15} = 1.$$

$$\exp(A + 2B_5) = S_1^3 + S_2^3 = \left[\exp \Sigma_{i=1}^{2}(t_{3\alpha} + t_{15-3\alpha}) \right]^3$$

$$\exp(A + 2\Sigma_{j=1}^{2} B_{3j}) = S_1^5 + S_2^5 = \left[\exp(t_5 + t_{10}) \right]^5$$

$$n = 21 \quad \exp(A + 2\Sigma_{j=1}^{10} B_j) = S_1^{21} + S_2^{21} = 1.$$

$$\exp(A + 2\Sigma_{j=1}^{3} B_{3j}) = S_1^7 + S_2^7 = \left[\exp(t_7 + t_{14}) \right]^7$$

$$\exp(A + 2B_7) = S_1^3 + S_2^3 = \left[\exp \Sigma_{\alpha=1}^{3}(t_{3\alpha} + t_{21-3\alpha}) \right]^3$$

$$n = 3p \quad \exp(A + 2\Sigma_{j=1}^{\frac{3p-1}{2}} B_j) = S_1^{3p} + S_2^{3p} = 1.$$

$$\exp(A + 2B_p) = S_1^3 + S_2^3 = \left[\exp \Sigma_{\alpha=1}^{\frac{p-1}{2}}(t_{3\alpha} + t_{3p-3\alpha}) \right]^3$$

$$\exp(A + 2\Sigma_{j=1}^{\frac{p-1}{2}} B_{3j}) = S_1^p + S_2^p = \left[\exp(t_p + t_{2p}) \right]^p$$

$$n = 35 \quad \exp(A + 2\Sigma_{j=1}^{17} B_j) = S_1^{35} + S_2^{35} = 1.$$

$$\exp(A + 2\sum_{j=1}^{2} B_{7j}) = S_1^5 + S_2^5 = \left[\exp(\Sigma_{\alpha=1}^{3}(t_{5\alpha} + t_{35-5\alpha})) \right]^5$$

$$\exp(A + 2\Sigma_{j=1}^{3}B_{5j}) = S_1^7 + S_2^7 = \left[\exp\Sigma_{\alpha=1}^{2}(t_{7\alpha} + t_{35-7\alpha})\right]^7$$

$$n = 5p \quad \exp(A + 2\Sigma_{j=1}^{\frac{5p-1}{2}}B_j) = S_1^{5p} + S_2^{5p} = 1.$$

$$\exp(A + 2\Sigma_{j=1}^{2}B_{pj}) = S_1^5 + S_2^5 = \left[\exp\Sigma_{\alpha=1}^{\frac{p-1}{2}}(t_{5\alpha} + t_{5p-5\alpha})\right]^5$$

$$\exp(A + 2\Sigma_{j=1}^{\frac{p-1}{2}}B_{5j}) = S_1^p + S_2^p = \left[\exp\Sigma_{\alpha=1}^{2}(t_{p\alpha} + t_{5p-p\alpha})\right]^p$$

$$n = 7p \quad \exp(A + 2\Sigma_{j=1}^{\frac{7p-1}{2}}B_j) = S_1^{7p} + S_2^{7p} = 1.$$

$$\exp(A + 2\Sigma_{j=1}^{3}B_{pj}) = S_1^7 + S_2^7 = \left[\exp\Sigma_{\alpha=1}^{\frac{p-1}{2}}(t_{7\alpha} + t_{7p-7\alpha})\right]^7$$

$$\exp(A + 2\Sigma_{j=1}^{\frac{p-1}{2}}B_{7j}) = S_1^p + S_2^p = \left[\exp\Sigma_{\alpha=1}^{3}(t_{p\alpha} + t_{7p-p\alpha})\right]^p$$

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1. Jiang, Chun-xuan. Hypercomplex variable theory. Preprints. 1989.

2. Jiang, Chun-xuan. Fermat last theorem has been proved. (Chineses. English summary). Qian Kexue, 2(1992) 17-20. Preprints (English), December, 1991.

3. Jiang, Chun-xuan. More than 300 years ago Fermat last theorem was proved (Chineses, English summary). Qian Kiexue, 6(1992) 18-20.

4. Jiang, Chun-xuan. Fermat proof for Fermat last theorem. Preprints (English), March, 1992.

5. Jiang, Chun-xuan. Factorization theorem for Fermat equation. Preprints (English). May, 1992

Note. We found out a new method for proving Fermat last theorem in 1991. We proved Fermat last theorem at one stroke for all prime exponents $p > 3$. This proof is too simple for one to believe, but one can understand it. Let one know the important result, we gave out about 600 preprints in 1991-1992. There are my preprints in the west universities and journals. It the same time both papers were published in Chinese. As yet, no one disprove this proof. Anyone can not deny it. It is a simple and marvelous proof. We sent dept of math (Princeton University) a preprint on Jan. 15, 1992. They surely read it. Andrew, Wiles claims the second proof of FLT after two years. We believe that the experts of mathematical history will write the course of the proof of FTI, because many mathmaticans in the West read my preprints in 1991-1992.