Martin Erik Horn: **Sandwich Products and Reflections**,
PhyDid B – Paper DD 17.7, DPG spring conference 2015 at Wuppertal.
URL: http://phydid.physik.fu-berlin.de/index.php/phydid-b/article/view/642

5. Overview: Reflection formulas

Reflection at a point (represented by scalar ℓ):

Scalars:	$k_{ref} = \ell\, k\, \ell^{-1}$	{1}
Vectors:	$\mathbf{r}_{ref} = -\ell\, \mathbf{r}\, \ell^{-1}$	{2}
Bivectors:	$\mathbf{A}_{ref} = \ell\, \mathbf{A}\, \ell^{-1}$	{3}
Trivectors:	$\mathbf{V}_{ref} = -\ell\, \mathbf{V}\, \ell^{-1}$	{4}
Quadvectors:	$\boldsymbol{Q}_{ref} = \ell\, \boldsymbol{Q}\, \ell^{-1}$	{5}
Pentavectors:	$\boldsymbol{P}_{ref} = -\ell\, \boldsymbol{P}\, \ell^{-1}$	{6}
Hexavectors:	$\boldsymbol{H}_{ref} = \ell\, \boldsymbol{H}\, \ell^{-1}$	{7}
Septavectors:	$\boldsymbol{S}_{ref} = -\ell\, \boldsymbol{S}\, \ell^{-1}$	{8}

Reflection at an axis (represented by vector **n**):

Scalars:	$k_{ref} = \mathbf{n}\, k\, \mathbf{n}^{-1}$	{9}
Vectors:	$\mathbf{r}_{ref} = \mathbf{n}\, \mathbf{r}\, \mathbf{n}^{-1}$	{10}
Bivectors:	$\mathbf{A}_{ref} = \mathbf{n}\, \mathbf{A}\, \mathbf{n}^{-1}$	{11}
Trivectors:	$\mathbf{V}_{ref} = \mathbf{n}\, \mathbf{V}\, \mathbf{n}^{-1}$	{12}
Quadvectors:	$\boldsymbol{Q}_{ref} = \mathbf{n}\, \boldsymbol{Q}\, \mathbf{n}^{-1}$	{13}
Pentavectors:	$\boldsymbol{P}_{ref} = \mathbf{n}\, \boldsymbol{P}\, \mathbf{n}^{-1}$	{14}
Hexavectors:	$\boldsymbol{H}_{ref} = \mathbf{n}\, \boldsymbol{H}\, \mathbf{n}^{-1}$	{15}
Septavectors:	$\boldsymbol{S}_{ref} = \mathbf{n}\, \boldsymbol{S}\, \mathbf{n}^{-1}$	{16}

Reflection at a plane (represented by bivector **N**):

Scalars:	$k_{ref} = \mathbf{N}\, k\, \mathbf{N}^{-1}$	{17}
Vectors:	$\mathbf{r}_{ref} = -\mathbf{N}\, \mathbf{r}\, \mathbf{N}^{-1}$	{18}
Bivectors:	$\mathbf{A}_{ref} = \mathbf{N}\, \mathbf{A}\, \mathbf{N}^{-1}$	{19}
Trivectors:	$\mathbf{V}_{ref} = -\mathbf{N}\, \mathbf{V}\, \mathbf{N}^{-1}$	{20}
Quadvectors:	$\boldsymbol{Q}_{ref} = \mathbf{N}\, \boldsymbol{Q}\, \mathbf{N}^{-1}$	{21}
Pentavectors:	$\boldsymbol{P}_{ref} = -\mathbf{N}\, \boldsymbol{P}\, \mathbf{N}^{-1}$	{22}
Hexavectors:	$\boldsymbol{H}_{ref} = \mathbf{N}\, \boldsymbol{H}\, \mathbf{N}^{-1}$	{23}
Septavectors:	$\boldsymbol{S}_{ref} = -\mathbf{N}\, \boldsymbol{S}\, \mathbf{N}^{-1}$	{24}

Reflection at a 3d space or reduced spacetime
(represented by trivector **T**):

Scalars:	$k_{ref} = \mathbf{T}\, k\, \mathbf{T}^{-1}$	{25}
Vectors:	$\mathbf{r}_{ref} = \mathbf{T}\, \mathbf{r}\, \mathbf{T}^{-1}$	{26}
Bivectors:	$\mathbf{A}_{ref} = \mathbf{T}\, \mathbf{A}\, \mathbf{T}^{-1}$	{27}
Trivectors:	$\mathbf{V}_{ref} = \mathbf{T}\, \mathbf{V}\, \mathbf{T}^{-1}$	{28}
Quadvectors:	$\boldsymbol{Q}_{ref} = \mathbf{T}\, \boldsymbol{Q}\, \mathbf{T}^{-1}$	{29}
Pentavectors:	$\boldsymbol{P}_{ref} = \mathbf{T}\, \boldsymbol{P}\, \mathbf{T}^{-1}$	{30}
Hexavectors:	$\boldsymbol{H}_{ref} = \mathbf{T}\, \boldsymbol{H}\, \mathbf{T}^{-1}$	{31}
Septavectors:	$\boldsymbol{S}_{ref} = \mathbf{T}\, \boldsymbol{S}\, \mathbf{T}^{-1}$	{32}

Reflection at a 4d hyperspace or spacetime
(represented by quadvector **Q**):

Scalars:	$k_{ref} = \mathsf{\mathbf{Q}}\, k\, \mathsf{\mathbf{Q}}^{-1}$	{33}
Vectors:	$\mathbf{r}_{ref} = -\mathsf{\mathbf{Q}}\, \mathbf{r}\, \mathsf{\mathbf{Q}}^{-1}$	{34}
Bivectors:	$\mathbf{A}_{ref} = \mathsf{\mathbf{Q}}\, \mathbf{A}\, \mathsf{\mathbf{Q}}^{-1}$	{35}
Trivectors:	$\mathbf{V}_{ref} = -\mathsf{\mathbf{Q}}\, \mathbf{V}\, \mathsf{\mathbf{Q}}^{-1}$	{36}
Quadvectors:	$\boldsymbol{Q}_{ref} = \mathsf{\mathbf{Q}}\, \boldsymbol{Q}\, \mathsf{\mathbf{Q}}^{-1}$	{37}
Pentavectors:	$\boldsymbol{P}_{ref} = -\mathsf{\mathbf{Q}}\, \boldsymbol{P}\, \mathsf{\mathbf{Q}}^{-1}$	{38}
Hexavectors:	$\boldsymbol{H}_{ref} = \mathsf{\mathbf{Q}}\, \boldsymbol{H}\, \mathsf{\mathbf{Q}}^{-1}$	{39}
Septavectors:	$\boldsymbol{S}_{ref} = -\mathsf{\mathbf{Q}}\, \boldsymbol{S}\, \mathsf{\mathbf{Q}}^{-1}$	{40}

Reflection at a 5d hyperspace, hyperspacetime or spacetimevelocity (represented by pentavector **P**):

Scalars:	$k_{ref} = \mathsf{\mathbf{P}}\, k\, \mathsf{\mathbf{P}}^{-1}$	{41}
Vectors:	$\mathbf{r}_{ref} = \mathsf{\mathbf{P}}\, \mathbf{r}\, \mathsf{\mathbf{P}}^{-1}$	{42}
Bivectors:	$\mathbf{A}_{ref} = \mathsf{\mathbf{P}}\, \mathbf{A}\, \mathsf{\mathbf{P}}^{-1}$	{43}
Trivectors:	$\mathbf{V}_{ref} = \mathsf{\mathbf{P}}\, \mathbf{V}\, \mathsf{\mathbf{P}}^{-1}$	{44}
Quadvectors:	$\boldsymbol{Q}_{ref} = \mathsf{\mathbf{P}}\, \boldsymbol{Q}\, \mathsf{\mathbf{P}}^{-1}$	{45}
Pentavectors:	$\boldsymbol{P}_{ref} = \mathsf{\mathbf{P}}\, \boldsymbol{P}\, \mathsf{\mathbf{P}}^{-1}$	{46}
Hexavectors:	$\boldsymbol{H}_{ref} = \mathsf{\mathbf{P}}\, \boldsymbol{H}\, \mathsf{\mathbf{P}}^{-1}$	{47}
Septavectors:	$\boldsymbol{S}_{ref} = \mathsf{\mathbf{P}}\, \boldsymbol{S}\, \mathsf{\mathbf{P}}^{-1}$	{48}

Reflection at a 6d hyperspace or hyperspacetime
(represented by hexavector **H**):

Scalars:	$k_{ref} = \mathsf{\mathbf{H}}\, k\, \mathsf{\mathbf{H}}^{-1}$	{49}
Vectors:	$\mathbf{r}_{ref} = -\mathsf{\mathbf{H}}\, \mathbf{r}\, \mathsf{\mathbf{H}}^{-1}$	{50}
Bivectors:	$\mathbf{A}_{ref} = \mathsf{\mathbf{H}}\, \mathbf{A}\, \mathsf{\mathbf{H}}^{-1}$	{51}
Trivectors:	$\mathbf{V}_{ref} = -\mathsf{\mathbf{H}}\, \mathbf{V}\, \mathsf{\mathbf{H}}^{-1}$	{52}
Quadvectors:	$\boldsymbol{Q}_{ref} = \mathsf{\mathbf{H}}\, \boldsymbol{Q}\, \mathsf{\mathbf{H}}^{-1}$	{53}
Pentavectors:	$\boldsymbol{P}_{ref} = -\mathsf{\mathbf{H}}\, \boldsymbol{P}\, \mathsf{\mathbf{H}}^{-1}$	{54}
Hexavectors:	$\boldsymbol{H}_{ref} = \mathsf{\mathbf{H}}\, \boldsymbol{H}\, \mathsf{\mathbf{H}}^{-1}$	{55}
Septavectors:	$\boldsymbol{S}_{ref} = -\mathsf{\mathbf{H}}\, \boldsymbol{S}\, \mathsf{\mathbf{H}}^{-1}$	{56}

Reflection at a 7d hyperspace or hyperspacetime
(represented by septavector **S**):

Scalars:	$k_{ref} = \mathsf{\mathbf{S}}\, k\, \mathsf{\mathbf{S}}^{-1}$	{57}
Vectors:	$\mathbf{r}_{ref} = \mathsf{\mathbf{S}}\, \mathbf{r}\, \mathsf{\mathbf{S}}^{-1}$	{58}
Bivectors:	$\mathbf{A}_{ref} = \mathsf{\mathbf{S}}\, \mathbf{A}\, \mathsf{\mathbf{S}}^{-1}$	{59}
Trivectors:	$\mathbf{V}_{ref} = \mathsf{\mathbf{S}}\, \mathbf{V}\, \mathsf{\mathbf{S}}^{-1}$	{60}
Quadvectors:	$\boldsymbol{Q}_{ref} = \mathsf{\mathbf{S}}\, \boldsymbol{Q}\, \mathsf{\mathbf{S}}^{-1}$	{61}
Pentavectors:	$\boldsymbol{P}_{ref} = \mathsf{\mathbf{S}}\, \boldsymbol{P}\, \mathsf{\mathbf{S}}^{-1}$	{62}
Hexavectors:	$\boldsymbol{H}_{ref} = \mathsf{\mathbf{S}}\, \boldsymbol{H}\, \mathsf{\mathbf{S}}^{-1}$	{63}
Septavectors:	$\boldsymbol{S}_{ref} = \mathsf{\mathbf{S}}\, \boldsymbol{S}\, \mathsf{\mathbf{S}}^{-1}$	{64}

Similar equations with the same sandwich product structure can be found for reflections at higher-dimensional hyperspaces and hyper-spacetimes.