Gravito-rotational Dragging (Effect)

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Abstract: Previous work on the concept of gravito-rotational acceleration (GRA) and its dynamical role is surveyed, with main emphasis on the GRA's influence exerted on a test-body in orbital motion around a rotating gravitational source (heavy central mass). Such an influence is typically interpreted as a 'gravito-rotational dragging (effect)', this phenomenon is very similar to the so-called Lense-Thirring effect (frame dragging) in the framework of general relativity theory (GRT). Some applications are considered include (i) the orbital plane dragging of artificial satellites LAGEOS, LAGEOS II and LARES; (ii) the effect of the Earth's GRA on the orbital motion of the Moon; (iii) the effect of the Sun's GRA on the orbital motion of Mercury. Also, it is shown that the conceptual existence of 'gravito-rotational dragging (effect)' and its quantitative agreement with GRT-predictions should imply, among other things, that the geometrization of gravity is unnecessary for physics.

Keywords: combined gravitational action, combined gravitational potential energy; gravito-rotational acceleration, gravito-rotational dragging (effect)

1. Introduction

As it was frequently pointed out in a series of articles [1-7] relating to the Combined Gravitational Action (CGA) as an alternative gravity theory that should regard as a refinement and generalization of Newton's one. Also in the same papers we have shown that the CGA is very capable of investigating, explaining and predicting some old and new gravitational phenomena [1-7]. This characteristic is greatly due to the coherence and simplicity of the CGA-formalism that is utterly based on the concept of combined gravitational potential energy (CGPE), which is in fact a new form of velocity-dependent gravitational potential energy defined as follows

$$U \equiv U(r,v) = -\frac{k}{r}\left(1+\frac{v^2}{w^2}\right), \tag{1}$$

where $k = GMm$; G being the Newton's gravitational constant; M and m are the masses of the gravitational source A and the moving test-body B; $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$ is the relative distance between A and B; $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ with $v_x = dx/dt$ etc, is the velocity of the test-body B relative to the inertial reference frame of source A; and w is a specific kinematical parameter having the physical dimensions of a constant velocity defined by

$$w = \begin{cases} c, \text{ if } B \text{ is in relative motion inside the vicinity of } A \\ v_{esc} = \sqrt{2GM/R}, \text{ if } B \text{ is in relative motion outside the vicinity of } A \end{cases}, \tag{2}$$

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where c is the light speed in local vacuum and v_{esc} is the escape velocity at the surface of the gravitational source A.

The CGPE (1) ,which is conceptually the foundation of CGA and considered to be simpler and more useful than the usual velocity-dependent-GPE because it contains several properties.
Among the consequences of CGA-formalism, there is the concept of gravito-rotational acceleration (GRA), $\boldsymbol{\lambda}$,[6]. Phenomenologically, GRA is very similar to the dynamic gravitational field, $\boldsymbol{\Lambda}$, [3], that is if $\boldsymbol{\Lambda}$ is mainly induced by the relative motion of the test-body in the vicinity of the principal gravitational source (heavy central mass), the GRA is intrinsically generated by any massive body in a state of rotational motion, independently of any principal gravitational source, which itself may be characterized by its proper GRA during its axial-rotation. Therefore GRA is, in fact, a combination of *gravity* and *rotation*, consequently, the rotational (heavy) central mass is, in this case, the main source of GRA, according to [6],the magnitude of GRA is defined by

$$\lambda = \kappa \frac{M}{P^2}, \quad \kappa = 4\pi^2 G/c^2 \tag{3}$$

where M and P are, respectively, the mass and rotational period of the rotating source of GRA. Curiously, Broginsky, Caves and Thorn, in their seminal paper [8] entitled ‘Laboratory experiments to test relativistic gravity’ published in 1977, they found an extra-gravitational acceleration called by them post-Newtonian gravitational acceleration (Eq.(2.1) in Ref.[8]) whose magnitude is qualitatively comparable to that of $\boldsymbol{\lambda}$.

In Ref.[6], we have investigated some consequences and applications of GRA and we have seen its importance for the compact stellar objects and Sun. Now, we focus our attention on the effect of GRA on the orbital motion of a test-body. We have repeatedly claimed, in previous articles [1-7], that CGA is not only a generalization and refinement of Newton's gravity theory but also it should be regarded as a *counterexample* to general relativity theory (GRT) since many gravitational phenomena that are traditionally attributed to GRT solely also exist in the CGA-context and as a direct result the perihelion advance of planets, angular deflection of light ray passing nearby a massive body, gravitational lensing, gravitational time dilation, de Sitter (geodetic) precession and Lense-Thirring effect (frame dragging) are not causally due to the so-called curvature of space-time but simply are due to the action of extra-gravitational terms like, *e.g.*, the couple $(\boldsymbol{\Lambda}, \mathbf{F}_{\mathrm{D}})$ and $\boldsymbol{\lambda}$. For detailed discussion, the interested reader is referred to Refs.[3-7]. At present, we are exclusively concerned with LT-effect.

2. LT-effect

Historically, in 1918, the Austrian theoretical physicists Josef Lense and Hans Thirring had predicted, in the framework of GRT, the existence of the fallowing effect: the rotation of a heavy body like Earth will drag the local inertial frame of reference around it, which will affect the orbit of a satellite [9]. Quantitatively, the magnitude of LT-effect is usually defined by the formula

$$\dot{\Omega}_{\mathrm{LT}} = \frac{2GL}{c^2 a^3 \left(1 - e^2\right)^{3/2}} \quad \text{(rad/s)}, \tag{4}$$

where L is the rotational angular momentum of the (heavy) central mass; a and e are, respectively, the semi-major axis and the mean orbital eccentricity of test-body's orbit. Like the case of the de Sitter (geodesic) precession [10], which has its analogue in the CGA-formalism, formula (20) in Ref.[4], also the LT-effect has its analogue in the framework of CGA as we shall see soon. However, according to the traditional believe, the LT-effect is interpreted as an additional space-time curvature caused by the rotation of a central mass. This phenomena has been called gravitomagnetism [11,12] for its supposed analogy with magnetism in electrodynamics.

3. Gravito-rotational Dragging (Effect)

In the CGA-context, the dragging effect is a direct and natural consequence of the influence of GRA. More precisely, this effect is a sort of a (very) small perturbation of the test-body's orbit. Phenomenologically, it occurs when the said test-body orbiting the rotating source (heavy central mass) of GRA, explicitly, during its orbital motion, the test-body undergoes the influence of source's GRA such that it should have its orbital plane *dragged* around the rotating source in the same sense as the rotation of source. We call this phenomenon 'gravito-rotational dragging (effect), GRD' and the derivation of the expression of its magnitude may be summarized as follows: after the generalization of CGPE (1) by taking into account the rotational (spinning) motion of the central mass and after performing a long algebraic calculation and manipulation, we get an expression containing many terms, among them, the following CGA-term

$$\dot{\Omega}_{\mathrm{GRD}} = \frac{4}{5}\frac{\lambda R^2}{\omega r^3}, \tag{5}$$

which having, in terms of magnitude, the physical dimensions of angular velocity, where r is the average radial distance between the orbiting test-body and the source of GRA; λ, R and ω are, respectively, the GRA's magnitude, radius and magnitude of angular velocity of the source. Furthermore, since the expression of CGPE (1) is almost similar to that of Barker and O'Connell –used in their very influential article entitled 'Derivation of the Equations of Motion of a Gyroscope from the Quantum Theory of Gravitation' published in 1970, therefore, we can adopt their method [13] to get the same expression (5). Now, let us consider the very important case, that is, when the orbit of test-body is elliptic or almost elliptic. To this end, we put $r = a(1 - e^2)^{1/2}$ and after substitution in (5), we obtain the very expected expression for the magnitude of GRD, *i.e.*, the precession of orbital plane of test-body

$$\dot{\Omega}_{\mathrm{GRD}} = \frac{4q^2}{5a\left(1 - e^2\right)^{3/2}}\frac{\lambda}{\omega}, \quad q = R/a\,. \tag{6}$$

The explicit presence of GRA's magnitude in (6), implies among other things, that the causal origin of GRD is the influence of GRA exerted on the test-body during its orbital motion around the source of GRA. Or in terms of force, the rotating central mass –as principal source of GRA– is permanently acting on the orbiting test-body of mass m a certain gravito-rotational force $\mathbf{F} = m\boldsymbol{\lambda}$ which behaves like an additional force. Thus, according to (3), the average magnitude of this force is given by $F = m\lambda$, and the magnitude of GRD (6) may be rewritten in terms of F as follows

$$\dot{\Omega}_{\text{GRD}} = \frac{4q^2}{5\,a\left(1 - e^2\right)^{3/2}} \frac{F}{m\omega} \; . \qquad (7)$$

Therefore, phenomenologically, the GRD is typically a *gravito-rotational* phenomenon, a sort of combination of *gravity* and *rotation*, and has nothing to do with the so-called curvature of space-time or gravitomagnetism as we shall see immediately.

4. Comparison between GRT-predictions and CGA-predictions

Epistemologically speaking, the best way to test any scientific theory is to compare its predictions with other alternative models. Thus, presently, our main aim is to verify the predictions of the CGA *via* the applications of the formula (6) and by comparing its results with those of GRT.

4.1. LAGEOS, LAGEOS II and LARES

Recalling, LEGEOS and LEGEOS II are two satellites orbiting the Earth to experimentally test GRT-predictions, namely, the LT-effect ,*i.e*., the rate by which the satellites' orbital plane processes caused by Earth's rotation. The basic idea of the LARES experiment is to accurately reconstruct the actual orbit from laser ranging data and to compare it with the theoretical one obtained using all the forces and perturbations acting on the satellite therefore, the LARES mission is very similar to LEGEOS and LEGEOS II.

Thus, the explicit application of the formulae (4) and (6) should give us the magnitude of LT-effect and GRD, respectively. To this end, we have from Refs.[14,15,16], the following orbital parameters of LEGEOS, LEGEOS II and LARES: semi-major axes $a_{\text{I}} = 12270\,\text{km}$, $a_{\text{II}} = 12210\,\text{km}$, $a_{\text{LARES}} = 7821\,\text{km}$; eccentricities $e_{\text{I}} = 0.004$, $e_{\text{II}} = 0.014, e_{\text{LARES}} = 0.0007$ (the subscripts I and II indicate LEGEOS and LEGEOS II, respectively). It will be convenient, as adding much to the simplicity of the subject, to consider the earth as a perfect sphere. Thus for the numerical values of the Earth's mass, mean radius, rotational period, magnitude of angular velocity and the physical constants, we take $M_{\oplus} = 5.97260 \times 10^{24}\,\text{kg}$; $R_{\oplus} = 6371\,\text{km}$; $P_{\oplus} = 24\,\text{h}$; $\omega_{\oplus} = 7.268518 \times 10^{-5}\ \text{rad}\,\text{s}^{-1}$; $G = 6.67384 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\text{s}^{-2}$ and $c = 299792458\,\text{m}\,\text{s}^{-1}$.

Firstly, let us calculate the values of two important physical quantities, namely, the magnitudes of angular momentum and GRA of the Earth. Thus, with the help of (3) and the usual formula $L = I\omega$ where $I = \frac{2}{5}MR^2$ is the moment of inertia, we get the following values: $\lambda_{\oplus} = 2.343092 \times 10^{-11}\ \text{m}\,\text{s}^{-2}$ and $L_{\oplus} = 7.048301 \times 10^{33}\ \text{kg}\,\text{m}^2\,\text{s}^{-1}$. Secondly , from the formulae (4) and (6), the theoretically predicted magnitudes of LT-effect and GRD for LAGEOS, LAGEOS II and LARES are computed and listed in Table 1.

Satellite	GRT-predictions $\dot{\Omega}_{\text{LT}}$ $\left(\text{mas yr}^{-1}\right)$	CGA-predictions $\dot{\Omega}_{\text{GRD}}$ $\left(\text{mas yr}^{-1}\right)$
LAGEOS	36.90322	36.90322
LAGEOS II	37.44993	37.45726
LARES	142.49870	142.49870

Table 1. Above, column 1 gives the satellite's name; columns 2 and 3 give, respectively, GRT-predictions and CGA-predictions of LT-effect and GRD for each satellite.
Note: mas = milli-arcsecond.

It is quite clear from Table 1, the CGA-predictions are perfectly identical to GRT-predictions. This result means, among other things, that the causal origin of LT-effect is not the (local) curvature of space-time, and again this effect has nothing to do with the absurd idea of the so-called gravitomagnetism.

4.2. Moon

Let us examine the effect of the Earth's GRA on the Moon *through* GRD. Explicitly, during its orbital motion, the Moon should have its orbit processes at a rate defined by (6). We have for the Moon's orbital parameters: semi-major axis $a = 384400\text{km}$ and eccentricity $e = 0.0549$. Thus, after substituting the needed parameter values in (6), we find for the Moon: $\dot{\Omega}_{\text{GRD}} = 1.20\mu\text{as yr}^{-1}$ (μas = micro-arcsecond). That is, the Moon's orbital plane processes at rate of $2.235\,\text{mm yr}^{-1}$. As we can notice, the GRD of the Moon's orbit caused by the Earth's GRA is negligible because, according to the formula (6), the effect decreases quickly with distance and the Earth is not rotating particularly fast. But sufficiently near-Earth satellites undergo larger orbital perturbations due to GRD.

4.3. Mercury

As Mercury is the nearest planet to the Sun, thus the GRD of Mercury's orbit due to the Sun's GRA should be, certainly, very small but relatively important with respect to the rest of planets' orbital perturbations caused by the same effect. We have the following physical and orbital parameters for the Sun and Mercury, respectively. $M_{\text{sun}} = 1.9891\times10^{30}\,\text{kg}$, sidereal rotation period at equator $P_{\text{sun}} = 25.05\,\text{d}$, $R_{\text{sun}} = 695508\,\text{km}$; semi-major axis $a = 57.92\times10^{6}\,\text{km}$ and eccentricity $e = 0.205$. Exactly like before, that is after inserting the appropriate parameter values in (6), we get for Mercury: $\dot{\Omega}_{\text{GRD}} = 59.34\,\mu\text{as yr}^{-1}$. As we can see, the magnitude of GRD on

Mercury's orbit is of the order of ‘micro arc second *per* year’ which is quite small in comparison with the magnitude of Mercury's perihelion precession $43.11\,\text{as cy}^{-1}$.

5. Discussion

The excellent agreement between GRT-predictions and CGA-predictions displaying in Table1 raises many very important epistemological and physical questions relative to the space-time and its curvature. Space-time itself is just a pure mathematical concept used to construct mathematical models. In physical reality, there is no such thing as a *substance* called space-time. This extremely important fact is often ignored when scientific theories are presented to the public. Recalling, in the geometric interpretation of gravity, a massive object curves the (local) space-time around it, causing ,*e.g.*, a test-body to follow that curvature in preference to following straight lines through space. To facilitate the comprehension, this interpretation is generally described by using the ‘rubber sheet’ analogy. However, this *pseudo* analogy is completely very far from the physical reality, *i.e.*, contrary to space-time , the ‘rubber sheet’ represents a physical reality, it is more precisely a substance/matter and consequently it is characterized by its own chemical and physical properties.

It is frequently asserted that the stress-energy tensor plays a role in GRT very similar to that of mass distribution in Newton's theory; more precisely, it tells space-time how to deform, creating what we observe as gravity. Therefore, space-time is not an inert entity. It acts on matter and can be acted upon. Accordingly, curved space-time itself behaves like a sort of matter, not merely a geometrical seat in which arise physical phenomena without specific dynamical properties.

Phenomenologically, the geometrization of gravity implies the materialization of (curved) space-time itself, and as a direct result the usual principle of causality is violated because the causal source of such *materialization* is absolutely without existence.

Furthermore, it would be quite logical to ask the following central question: Historically, in 1686-1687, Isaac Newton realized that the motion of the planets and the moon as well as that of a falling apple could be explained by his Law of Universal Gravitation, which states that any two objects attract each other with a force equal to the product of their masses divided by the square of their separation times a constant of proportionality, that is $G = 6.67384\times10^{-11}\,\text{m}^3\,\text{kg}^{-1}\text{s}^{-2}$, however since, in GRT, *gravity* is not considered as a force at all –therefore, why did GRT-formalism include the Newton's gravitational constant?

It is worthwhile to note that when we said in the present work that the GRD is typically a *gravito-rotational* phenomenon, this claim is obviously reflected by the presence of the physical quantities λ and F in the formulae (6) and (7), respectively. However, concerning GRT, there is no counterpart. Normally, the curvature as an essential parameter should be contained in the formula (4) because according to GRT, the curvature of space-time is a contributing factor in LT-effect.

6. Conclusion

In this work, we have revealed the conceptual existence of the ‘gravito-rotational dragging (effect), GRD’. This phenomenon is very similar to the so-called Lense-Thirring effect (frame dragging) in the framework of GRT. Also, we have shown that in the CGA-context, the GRD is a direct and natural consequence of the influence of GRA of the central heavy mass. More precisely, the GRD is a sort of a (very) small perturbation of the test-body's orbit. Phenomenologically, it occurs when the said test-body orbiting the rotating source (heavy central mass) of GRA, explicitly, during its orbital motion, the test-body undergoes the influence of source's GRA such that it should have its orbital plane *dragged* around the rotating source in the same sense as the rotation of source.

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