The generalized Stokes theorem

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Abstract

When applied to a quaternionic manifold, the generalized Stokes theorem can provide an elucidating space-progression model in which elementary objects float on top of symmetry centers that act as their living domain. The paper indicates that integration depends on the ordering of the involved parameter spaces. In this way, it elucidates the origin of the electric charges and color charges of elementary particles.

1 Introduction

This paper uses the fact that separable Hilbert spaces can only cope with number systems that are division rings. We use the most elaborate version of these division rings and that is the quaternionic number system. Quaternionic number systems exist in multiple versions, that differ in the way they are ordered. Ordering influences the arithmetic properties of the number system and it appears that it influences the behavior of quaternionic functions under integration. Another important fact is that every infinite dimensional separable Hilbert system owns a companion Gelfand triple, which is a non-separable Hilbert space. We will use these Hilbert spaces as structured storage media for discrete quaternionic data and for quaternionic manifolds. We use the reverse bra-ket method in order to relate operators and their eigenspaces to pairs of functions and their parameter spaces. Subspaces act as Hilbert space domains in relation to which manifolds are defined.

The existence of Hilbert spaces are a corollary of the existence of a deeper foundation, which is an orthomodular lattice. The set of closed subspaces of a separable Hilbert space has the relational structure of an orthomodular lattice. This deeper foundation does not use number systems. Thus notions, such as space and progression do not exist at this level. The number systems are introduced by extending the deeper foundation to a separable Hilbert space. This extension enforces the restriction to number systems that are division rings. With other words, space and progression must be expressible by elements of a division ring.

2 Without discontinuities

The generalized Stokes theorem is in fact a combination of two versions. One is the using the divergence part of the exterior derivative $d\omega$. It is also known as the generalized divergence theorem. The other version uses the curl part of the exterior derivative. For quaternionic manifolds the two versions can be combined. The theorem can be applied when everywhere in Ω the derivative $\mathrm{d}\omega$ exists and when everywhere in $\partial\Omega$ the manifold ω is continuous and integrable.

Without discontinuities in the manifold ω the generalized Stokes theorem is represented by a simple formula [1].

$$\int_{\Omega} d\omega = \int_{\partial\Omega} \omega \left(= \oint_{\partial\Omega} \omega \right) \tag{1}$$

The domain Ω is encapsulated by a boundary $\partial\Omega$.

$$\Omega \subset \partial\Omega \tag{2}$$

In this paper, the manifolds ω and $d\omega$ represent quaternionic fields $\mathfrak{F}$ and $\mathrm{d}\mathfrak{F}$, while inside $\partial\Omega$ the manifold ω represents the quaternionic boundary of the quaternionic field $\mathfrak{F}$.

$d\omega$ is the exterior derivative of ω. The theorem exists in the form of a divergence based version and in the form of a curl based version [2].

2.1 Domains and parameter spaces

The quaternionic ***domain*** Ω is supposed to be defined as part of the ***domain*** $\mathfrak{R}$ of a ***reference operator*** $\mathfrak{R}$ that resides in the non-separable quaternionic Hilbert space $\mathcal{H}$. The reverse bra-ket method [4] relates the eigenspace $\{q\}$ of reference operator $\mathfrak{R}$ to a flat quaternionic ***function*** $\mathfrak{R}(q)$. The target of function $\mathfrak{R}(q)$ is its own ***parameter space*** $\{q\}$. Here we explicitly use the same symbol $\mathfrak{R}$ for all directly related objects.

$$\mathfrak{R} = |q\rangle\mathfrak{R}(q)\langle q| = |q\rangle q\langle q| \tag{1}$$

The domain $\mathfrak{R}$ is spanned by the eigenvectors $\{|q\rangle\}$ of operator $\mathfrak{R}$.

The reverse bra-ket method also relates the eigenspace $\mathfrak{R}$ to an equivalent eigenspace $\mathcal{R}$ of a reference operator $\mathcal{R}$, which resides in the infinite dimensional separable Hilbert space $\mathfrak{H}$. Both eigenspaces are related to the same version of the quaternionic number system. However, the second eigenspace $\mathcal{R}$ only uses rational quaternions q_i.

$$\mathcal{R} = |q_i\rangle\mathfrak{R}(q_i)\langle q_i| = |q_i\rangle q_i\langle q_i| \tag{2}$$

Quaternionic number systems can be ordered in several ways. Operator $\mathcal{R}$ corresponds with one of these orderings. $\mathcal{R}$ is supposed to be ***Cartesian-ordered***. $\mathcal{R}$ is a normal operator and its eigenspace is countable. It means that the set of eigenvectors of $\mathcal{R}$ can be enumerated by the separate eigenvalues of $\mathcal{R}$. The eigenspace is the Cartesian product of four partially ordered sets in which the set, which represents the real part takes a special role. The eigenspace of the Hermitian part $\mathcal{R}_0 = ½(\mathcal{R} + \mathcal{R}^\dagger)$ of normal operator $\mathcal{R}$ can be used to enumerate a division of $\mathfrak{H}$ into a countable number of disjunctive subspaces, which are spanned by eigenvectors of $\mathcal{R}$. Cartesian ordering means partial ordering of the eigenvalues of $\mathcal{R}_0$ and additional ordering of the eigenvalues of the anti-Hermitian operator $\mathfrak{R} = ½(\mathfrak{R} - \mathfrak{R}^\dagger)$ by selecting a Cartesian coordinate system. Eight mutually independent Cartesian coordinate systems exist. $\mathcal{R}_0 = (\mathcal{R} + \mathcal{R}^\dagger)/2$ is a self-adjoint operator. The ordered eigenvalues of $\mathcal{R}_0$ can be interpreted as ***progression values***. The eigenvalues of $\boldsymbol{\mathcal{R}}$ can be interpreted as ***spatial values.*** This differs from the physical notions of time and space. Here we are talking about a mathematical test model.

In this way, parameter spaces as well as domains correspond to closed subspaces of the Hilbert spaces. The domain subspaces are subspaces of the domains of the corresponding reference operators. The parameter spaces are ordered by a selected coordinate system. The Ω domain is represented by a part of the eigenspace of reference operator $\mathfrak{R}$. The flat quaternionic function $\mathfrak{R}(q)$ defines the parameter space $\mathfrak{R}$. It installs an ordering by selecting a Cartesian coordinate system for the eigenspace of its anti-Hermitian part $\mathfrak{R} = ½(\mathfrak{R} - \mathfrak{R}^\dagger)$. Several mutually independent selections are possible. The chosen selection attaches a corresponding symmetry flavor to this parameter space. In the mathematical test model, this symmetry flavor will become the reference symmetry flavor. Thus, the symmetry flavor of parameter space $\mathfrak{R}^{⓪}$ may be distinguished by its superscript ^{⓪}.

The manifold ω is also defined as the continuum eigenspace of a dedicated normal operator ω which is related to domain Ω and to parameter space $\mathfrak{R}^{⓪}$ via function $\mathfrak{F}$. Within this parameter space $\mathfrak{F}$ may have discontinuities, but these must be excluded from the domain over which integration takes place. This exclusion will be treated below.

2.2 Interpreting the exterior derivative

In this section we assume that the quaternionic manifold ω is represented by the target of a quaternionic function $\mathfrak{F}(q)$. Function $\mathfrak{F}(q)$ has a flat parameter space $\mathfrak{R}$.

$\mathfrak{R}$ is a flat quaternionic manifold, which is represented by the target of function $\mathfrak{R}(q) = q$.

We presume that the exterior derivative $\mathrm{d}\mathfrak{F}$ of $\mathfrak{F}$ can be interpreted by the following equations:

$$\mathrm{d}\mathfrak{F} = \sum_{\mu=0}^{3} e^{\mu} \frac{\partial \mathfrak{F}}{\partial x_\mu} dx_\mu = \sum_{\mu=0}^{3} e^{\mu} dx_\mu \sum_{\nu=0}^{3} e^{\nu} \frac{\partial \mathfrak{F}_\nu}{\partial x_\mu} = e^{\mu\nu} D_\mu \mathfrak{F}_\nu \tag{1}$$

$$D_\mu = dx_\mu \frac{\partial}{\partial x_\mu} \tag{2}$$

Thus $\mathrm{d}\mathfrak{F}$ is represented by a tensor. This differs from the partial differentials that appear in partial differential equations.

$$\mathfrak{G} = \sum_{\varsigma=0}^{3} e^{\varsigma}\mathfrak{G}_{\varsigma} = e^{\varsigma}\mathfrak{G}_{\varsigma} \tag{3}$$

$$= \nabla\mathfrak{F} = \sum_{\mu=0}^{3} e^{\mu}\frac{\partial\mathfrak{F}}{\partial x_{\mu}} = \sum_{\mu=0}^{3} e^{\mu}\sum_{\nu=0}^{3} e^{\nu}\frac{\partial\mathfrak{F}_{\nu}}{\partial x_{\mu}} = e^{\mu}e^{\nu}\partial_{\mu}\mathfrak{F}_{\nu} = e^{\mu\nu}\partial_{\mu}\mathfrak{F}_{\nu}$$

In the right parts of the above formulas, the summation rules for subscripts and superscripts are applied.

We use the fact that quaternions can be considered as a combination of a real scalar and an imaginary vector. Further, we apply the fact that quaternionic partial differential operators act as multipliers.

$$\mathfrak{F} = \mathfrak{F}_0 + \mathbf{\mathfrak{F}} \tag{4}$$

$$\mathfrak{G} = \nabla\mathfrak{F} = \mathfrak{G}_0 + \mathbf{\mathfrak{G}} = (\nabla_0 + \boldsymbol{\nabla})(\mathfrak{F}_0 + \mathbf{\mathfrak{F}}) \tag{5}$$

$$\mathfrak{G}_0 = \nabla_0\mathfrak{F}_0 - \langle\boldsymbol{\nabla}, \mathbf{\mathfrak{F}}\rangle \tag{6}$$

$$\mathbf{\mathfrak{G}} = \nabla_0\mathbf{\mathfrak{F}} + \boldsymbol{\nabla}\mathfrak{F}_0 \pm \boldsymbol{\nabla} \times \mathbf{\mathfrak{F}} \tag{7}$$

For some fields, some parts of $\mathbf{\mathfrak{G}}$ may get special symbols. This is applied in Maxwell-like equations.

$$\mathbf{\mathfrak{E}} = -\nabla_0\mathbf{\mathfrak{F}} - \boldsymbol{\nabla}\mathfrak{F}_0 \tag{8}$$

$$\mathbf{\mathfrak{B}} = \boldsymbol{\nabla} \times \mathbf{\mathfrak{F}} \tag{9}$$

In the integrals below some terms of $\nabla\mathfrak{F}$ are combined.

$$\boldsymbol{\nabla}\mathbf{\mathfrak{F}} = -\langle\boldsymbol{\nabla}, \mathbf{\mathfrak{F}}\rangle \pm \boldsymbol{\nabla} \times \mathbf{\mathfrak{F}} \tag{10}$$

$$\boldsymbol{\nabla}\mathfrak{F} = \boldsymbol{\nabla}\mathfrak{F}_0 - \langle\boldsymbol{\nabla}, \mathbf{\mathfrak{F}}\rangle \pm \boldsymbol{\nabla} \times \mathbf{\mathfrak{F}} \tag{11}$$

$$\nabla_0 \mathfrak{F} = \nabla_0 \mathfrak{F}_0 + \nabla_0 \mathfrak{F} \tag{12}$$

The $\pm$ sign indicates the fact that combinations of quaternionic parameter spaces and quaternionic functions exist in versions that differ in the handedness of their external vector product.

It must be noticed that

$$\mathrm{d}\mathfrak{F} \neq \sum_{\mu=0}^{3} e^{\mu} \mathfrak{G}_{\mu} dx_{\mu} \tag{13}$$

This is the reason that the generalized Stokes integral uses the wedge product $dx \wedge dy \wedge dz \wedge d\tau$. These wedge products are merely a warning that a tensor is active. It is not a clear exposure of the mechanism.

Some partial differential operators do not mix scalar and vector parts of functions. These are:

∇_0

$\nabla_0 \nabla_0$

$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle$

These operators can be combined in additions as well as in products and do not require to treat them as tensors. Two particular operators are:

$\nabla \nabla^* = \nabla^* \nabla = \nabla_0 \nabla_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle$

$\mathfrak{D} = -\nabla_0 \nabla_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle$

The last one is the quaternionic version of d'Alembert's operator. The first one can be split into ∇ and ∇^*. The second one cannot be split into quaternionic differential operators.

The field $\mathfrak{F}$ is considered to be regular in spatial regions where the field $\mathfrak{F}$ obeys

$$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \mathfrak{F} = 0 \tag{14}$$

Similar considerations hold for regions where:

$$\nabla \nabla^* \mathfrak{F} = (\nabla_0 \nabla_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle) \mathfrak{F} = 0 \tag{15}$$

$$\mathfrak{D} \mathfrak{F} = (-\nabla_0 \nabla_0 + \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle) \mathfrak{F} = 0 \tag{16}$$

2.3 A special boundary between the real part and the imaginary part of the domain

In the special case that is investigated here, the generalized Stokes theorem constructs a rim $\mathfrak{F}(\boldsymbol{x},\tau)$ between the past history of the field $[\mathfrak{F}(\boldsymbol{x},t)]_{t<\tau}$ and the future $[\mathfrak{F}(\boldsymbol{x},t)]_{t>\tau}$ of that field. It means that the boundary $\mathfrak{F}(\boldsymbol{x},\tau)$ of field $[\mathfrak{F}(\boldsymbol{x},t)]_{t<\tau}$ represents a universe wide static status quo of that field.

More specifically, the form of the generalized Stokes theorem for the sketched situation runs as:

$$\int_{t=0}^{\tau}\iiint_{V}\mathrm{d}\mathfrak{F}\,(x)=\int_{t=0}^{\tau}\left(\iiint_{V}\nabla\mathfrak{F}(x)\;dx\wedge dy\wedge dz\right)\wedge d\tau=\left[\iiint_{V}\mathfrak{F}\,(\boldsymbol{x})d\boldsymbol{x}\right]_{t=\tau} \tag{1}$$

$$x=\boldsymbol{x}+\tau \tag{2}$$

Here $[\mathfrak{F}(\boldsymbol{x},t)]_{t=\tau}$ represents the static status quo of a quaternionic field at instance τ. V represents the spatial part of the quaternionic domain of $\mathfrak{F}$, but it may represent only a restricted part of that parameter space. This last situation corresponds to the usual form of the divergence theorem.

As mentioned above great care must be taken by interpreting $\mathrm{d}\mathfrak{F}(x)=\nabla\mathfrak{F}(x)\;dx\wedge dy\wedge dz\wedge d\tau$.

The split that has been selected, sets a category of operators apart that are all Cartesian-ordered in the same way as operator $\mathcal{R}$ is. It enables a space-progression model in which progression steps in the separable Hilbert space $\mathfrak{H}$ and flows in its non-separable companion $\mathcal{H}$. Via the reverse bra-ket method the Cartesian-ordering of $\mathcal{R}$ can be transferred to $\mathfrak{R}$.

2.3.1 Interpretation of the selected encapsulation

The boundary $\partial\Omega$ is selected between the real part and the imaginary part of domain $\mathfrak{R}$. But it also excludes part of the real part. That part is the range of the real part from τ to infinity. τ is interpreted as the current progression value.

The future $\mathfrak{R}-\Omega$ is kept on the outside of the boundary $\partial\Omega$. As a consequence, the mechanisms that generate new data, operate on the rim $\partial\Omega$ between past Ω and future $\mathfrak{R}-\Omega$.

This split of quaternionic space results in a space-progression model that is to a large extent similar to the way that physical theories describe their space time models. However, the physical theories apply a spacetime model that has a Minkowski signature. The quaternionic model, which is represented here, is strictly Euclidean.

The paper does not claim that this quaternionic space-progression model reflects the structure and the habits of physical reality. The quaternionic space-progression model is merely promoted as a mathematical test model.

What happens in the mathematical test model is an ongoing process that embeds the subsequent static status quo's of the separable Hilbert space into the Gelfand triple.

Controlling mechanisms act as a function of progression τ in a stochastic and step-wise fashion in the realm of the separable Hilbert space. The results of their actions are stored in eigenspaces of

corresponding stochastic operators that reside in the separable Hilbert space. These stochastic operators differ from the kind of operators that are handled by the reverse bra-ket method.

At the same instance this part of the separable Hilbert space is embedded into its companion Gelfand triple. The controlling mechanisms will provide all generated data with a ***progression stamp*** τ. This progression stamp reflects the state of a model wide clock tick. The whole model, including its physical fields will proceed with these progression steps. However, in the Gelfand triple this progression can be considered to flow.

At the defined rim, any forecasting will be considered as mathematical cheating. Thus, at the rim, the uncertainty principle does not work for the progression part of the parameter spaces. Differential equations that offer advanced as well as retarded solutions must reinterpret the advanced solutions and turn them in retarded solutions, which in that case represent another kind of object. If the original object represents a particle, then the reversed particle is the anti-particle.

As a consequence of the construct, the history, which is stored-free from any uncertainty-in the already processed part of the eigenspaces of the physical operators, is no longer touched. Future is unknown or at least it is inaccessible.

2.3.2 Integrals over regular spatial domains

If in a spatial domain, function $\mathfrak{F}$ obeys the homogeneous equation

$$\boldsymbol{\nabla}\boldsymbol{\nabla}\mathfrak{F} = 0 \tag{1}$$

then the function $\mathfrak{F}$ and the corresponding field $\mathfrak{F}$ is considered to be regular in that domain. For functions $\mathfrak{F}$ that are regular in spatial domain V hold:

$$\iiint_V \boldsymbol{\nabla}\mathfrak{F} = \oiint_S \boldsymbol{n}\mathfrak{F} \tag{2}$$

$$\iiint_V \boldsymbol{\nabla}\mathfrak{F}_0 = \oiint_S \boldsymbol{n}\mathfrak{F}_0 \tag{3}$$

$$\iiint_V \boldsymbol{\nabla}, \boldsymbol{\mathfrak{F}} = \oiint_S \boldsymbol{n}, \boldsymbol{\mathfrak{F}} \tag{4}$$

$$\iiint_V \boldsymbol{\nabla} \times \boldsymbol{\mathfrak{F}} = \oiint_S \boldsymbol{n} \times \boldsymbol{\mathfrak{F}} \tag{5}$$

2.3.3 Integrating irregular functions

We can use the gradient of the inverse of the spatial distance $|\boldsymbol{q} - \boldsymbol{c}|$.

$$\boldsymbol{\nabla}\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|}=-\frac{\boldsymbol{q}-\boldsymbol{c}}{|\boldsymbol{q}-\boldsymbol{c}|^{\boldsymbol{3}}} \tag{1}$$

The divergence of this gradient is a Dirac delta function.

$$\delta(\boldsymbol{q}-\boldsymbol{c})=-\frac{1}{4\pi}\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|}\rangle=-\frac{1}{4\pi}\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{2}$$

This means that:

$$\phi(\boldsymbol{c})=\iiint_V\phi(\boldsymbol{q})\delta(\boldsymbol{q}-\boldsymbol{c})=-\frac{1}{4\pi}\iiint_V\phi(\boldsymbol{q})\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{3}$$

As alternative, we can also use the Green's function $G(\boldsymbol{q})$ of the partial differential equation.

$$\phi(\boldsymbol{c})=\iiint_V\phi(\boldsymbol{q})G(\boldsymbol{q}-\boldsymbol{c}) \tag{4}$$

For the Laplacian $\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle$ this obviously means:

$$\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle\mathfrak{F}=\phi(\boldsymbol{q}) \tag{5}$$

$$G(\boldsymbol{q}-\boldsymbol{c})=\frac{1}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{6}$$

However, when added to the Green's function, every solution f of the homogeneous equation

$$\langle\boldsymbol{\nabla},\boldsymbol{\nabla}\rangle f=0 \tag{7}$$

is also a solution of the Laplace equation.

$$\phi(\boldsymbol{c})=\iiint_V\frac{\phi(\boldsymbol{q})}{|\boldsymbol{q}-\boldsymbol{c}|} \tag{8}$$

Function $\phi(\boldsymbol{c})$ can be interpreted as the potential that is raised by charge distribution $\phi(\boldsymbol{q})$.

These facts also hold for the full quaternionic integration. But there we have two equations that can determine regularity.

In pure spherical conditions the Laplacian reduces to:

$$\langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \mathfrak{F}(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \mathfrak{F}(r)}{\partial r} \right) \tag{9}$$

For the following test function $\mathfrak{F}(r)$ this means [3]:

$$\mathfrak{F}(r) = \frac{Q}{4\pi} \frac{ERF\left({}^{r}/_{\sigma\sqrt{2}}\right)}{r} \tag{10}$$

$$\rho(r) = \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \mathfrak{F}(r) = \frac{Q}{\left(\sigma\sqrt{2\pi}\right)^3} \exp\left(- \frac{r^2}{2\sigma^2} \right) \tag{11}$$

Thus, for a Gaussian location distribution $\rho(r)$ of point-like artifacts the corresponding contribution to field $\mathfrak{F}(r)$ equals an error function divided by its argument. At first sight this may look in contradiction with equations (4) − (8), but here the distribution of artifacts extends over the boundary of domain V.

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \frac{ERF(r)}{r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(- ERF(r) + r \frac{2}{\sqrt{\pi}} \exp(-r^2) \right)$$

$$= \frac{1}{r^2} \left(- \frac{2}{\sqrt{\pi}} \exp(-r^2) + \frac{2}{\sqrt{\pi}} \exp(-r^2) - 2r \frac{2}{\sqrt{\pi}} \exp(-r^2) \right) = \frac{4}{\sqrt{\pi}} \exp(-r^2)$$

3 The detailed generalized Stokes theorem

We separate all point-like discontinuities from the domain Ω by encapsulating them in an extra boundary. Symmetry centers represent spherically ordered parameter spaces in regions H_n^x that float on a background parameter space $\mathfrak{R}$. The boundaries $\partial\mathrm{H}_n^x$ separate the regions H_n^x from the domain Ω. The regions H_n^x are platforms for local discontinuities in basic fields [2]. These fields are continuous in domain $\Omega - \mathrm{H}$.

$$H = \bigcup_n \mathrm{H}_n^x \tag{1}$$

The symmetry centers $\mathfrak{S}_n^x$ are encapsulated in regions H_n^x and the encapsulating boundary $\partial\mathrm{H}_n^x$ is not part of the disconnected boundary which encapsulates all continuous parts of the quaternionic manifold ω that exist in the quaternionic model.

$$\int_{\Omega-\mathrm{H}} d\omega = \int_{\partial\Omega\cup\partial\mathrm{H}} \omega = \int_{\partial\Omega} \omega - \sum_n \int_{\partial\mathrm{H}_n^x} \omega \tag{2}$$

If we take the unit normal to point outward on all of the boundary, this reverses the direction of the normal on $\partial\mathrm{H}_n^x$, which negates the integral. Thus, in this formula, the contributions of boundaries $\{\partial\mathrm{H}_n^x\}$ are subtracted from the contributions of boundary $\partial\Omega$. This means that $\partial\Omega$ also surrounds the regions $\{\mathrm{H}_n^x\}$.

Domain Ω corresponds to part of the reference parameter space $\mathfrak{R}^{⓪}$. As mentioned before the symmetry centers $\{\mathfrak{S}_n^x\}$ represent encapsulated regions $\{\mathrm{H}_n^x\}$ that float on parameter space $\mathfrak{R}^{⓪}$.

The geometric center of symmetry center $\mathfrak{S}_n^x$ is represented by a floating location on parameter space $\mathfrak{R}^{⓪}$.

The relation between the ***subspace*** S_Ω that corresponds to the domain Ω and the ***subspace*** $S_\mathfrak{R}$ that corresponds to the parameter space $\mathfrak{R}^{⓪}$ is given by:

$$\underbrace{\Omega}_{S_\Omega} \subset \underbrace{\mathfrak{R}^{⓪}}_{S_\mathfrak{R}} \tag{3}$$

Similarly:

$$\underbrace{\mathrm{H}_n^x}_{S_{\mathrm{H}_n^x}} \subset \underbrace{\mathfrak{S}_n^x}_{S_{\mathfrak{S}_n^x}} \tag{4}$$

4 Symmetry centers as floating parameter spaces

If we tolerate discontinuities, then these artifacts must be encapsulated by boundaries $\partial \mathrm{H}_n^x$ and in that way they are separated from the main domain Ω.

In that case the model may apply different parameter spaces, which have their own private symmetry flavor [5]. A separable quaternionic Hilbert space can cope with coexisting parameter spaces and these spaces are served by dedicated operators. The reverse bra-ket method relates the parameter space to a corresponding operator. For example [4]:

Let $\{q_i\}$ be the set of ***rational*** quaternions in a selected quaternionic number system and let $\{|q_i\rangle\}$ be the set of corresponding base vectors. They are eigenvectors of a normal operator $\mathcal{R}$.

Here we enumerate the eigenvalues and the base vectors with the same index i. This shows how the reverse bra-ket method works.

$$\mathcal{R} \equiv |q_i\rangle q_i \langle q_i| \tag{1}$$

For all bra's $\langle x|$ and ket's $|y\rangle$ hold:

$$\langle x|\mathcal{R}\, y\rangle = \sum_i \langle x|q_i\rangle q_i \langle q_i|y\rangle \tag{2}$$

$\mathcal{R}_0 = \left(\mathcal{R} + \mathcal{R}^\dagger\right)/2$ is a self-adjoint and thus Hermitian operator. Its eigenvalues can be used to arrange the order of the eigenvectors by enumerating them with the eigenvalues. The ordered eigenvalues can be interpreted as ***progression values***.

$\boldsymbol{\mathcal{R}} = \left(\mathcal{R} - \mathcal{R}^\dagger\right)/2$ is the corresponding anti-Hermitian operator.

We will use the same symbol for the operator $\mathcal{R}$, for the eigenspace $\{q_i\}$ and for the defined parameter space. $\boldsymbol{\mathcal{R}}$ is supposed to be ordered by using a selected Cartesian coordinate system. Eight mutually independent selections are possible. Together with the ordering of the real part $\mathcal{R}_0$, the Cartesian ordering of the imaginary part $\boldsymbol{\mathcal{R}}$ determines the symmetry flavor of the eigenspace of $\mathcal{R}$.

We define a category of anti-Hermitian operators $\{\mathfrak{S}_n^x\}$ that have no Hermitian part and that are distinguished by the way that their eigenspace is ordered by applying a polar coordinate system. We call them symmetry centers $\mathfrak{S}_n^x$. A polar ordering always start with a selected Cartesian ordering. The geometric center of the eigenspace of the symmetry center floats on a background parameter space of the normal reference operator $\mathcal{R}$, whose eigenspace defines a full quaternionic parameter space. The eigenspace of the symmetry center $\mathfrak{S}_n^x$ acts as a three dimensional spatial parameter space. The super script ^{x} refers to the symmetry flavor of $\mathfrak{S}_n^x$. The subscript $_n$enumerates the symmetry centers. Sometimes we omit the subscript.

$$\mathfrak{S}^x = |\mathfrak{s}_i^x\rangle \mathfrak{s}_i^x \langle \mathfrak{s}_i^x| \tag{4}$$

$$\mathfrak{S}^{x\dagger} = -\mathfrak{S}^x \tag{5}$$

In the companion Gelfand triple of an infinite dimensional separable Hilbert space the reverse bra-ket method can define continuum parameter spaces and relate them to corresponding operators. In this way the countable parameter space $\mathcal{R}$ relates to a continuum parameter space $\mathfrak{R}$.

The quaternionic field $\mathfrak{F}$ can also be represented by a dedicated operator. Here we use a parameter space $\mathfrak{R}$ that is spanned by a full quaternionic number system.

For all bra's $\langle x|$ and ket's $|y\rangle$ hold:

$$\langle x|\mathfrak{R}\, y\rangle = \int_q \langle x|q\rangle q\langle q|y\rangle \; dq \tag{6}$$

$$\langle x|\mathfrak{F}\, y\rangle = \int_q \langle x|q\rangle \mathfrak{F}(q)\langle q|y\rangle \; dq \tag{7}$$

Here, we use the symbol $\mathfrak{F}$ for the field, the function and the operator. However, another parameter space R would deliver another function F for the same field $\mathfrak{F}$. So, what determines the field $\mathfrak{F}$ is stored in the eigenspace $\mathfrak{F}$ of operator $\mathfrak{F}$ and can be coupled to different pairs of functions and parameter spaces.

4.1 Symmetry flavor of the symmetry center

The symmetry center $\mathfrak{S}_n^x$ is characterized by a private symmetry flavor. That symmetry flavor relates to the Cartesian ordering of this parameter space. When the orientation of the coordinate axes is fixed, then eight independent Cartesian orderings are possible [5]. We use the Cartesian ordering of $\mathfrak{R}^{⓪}$ as the reference for the orientation of the axes. $\mathfrak{R}^{⓪}$ has the same Cartesian ordering as $\mathcal{R}^{⓪}$ has.

$$\int_{\Omega-\mathrm{H}} d\omega = \int_{\partial\Omega} \omega - \sum_n \int_{\partial \mathrm{H}_n^x} \omega \tag{1}$$

In this formula the boundaries $\partial\Omega$ and $\partial\mathrm{H}_n^x$ are subtracted. This subtraction is affected by the ordering of the domains Ω and H_n^x.

This can best be comprehended when the encapsulation $\partial\mathrm{H}_n^x$ is performed by a cubic space form that is aligned along the Cartesian axes. Now the six sides of the cube contribute different to the effects of the encapsulation when the ordering differs from the Cartesian ordering of the reference parameter space $\mathfrak{R}^{⓪}$. Each discrepant axis ordering corresponds to one third of the surface of the cube. This effect is represented by the symmetry related charge and the color charge of the symmetry center [5]. It is easily related to the algorithm which is introduced for the computation of the symmetry related charge. Also the relation to the color charge will be clear.

The symmetry related charge and the color charge of symmetry center $\mathfrak{S}_n^x$ are supposed to be located at the geometric center of the symmetry center. A Green's function together with these

charges can represent the local defining function $\varphi^x(q)$ of the contribution φ^x to the symmetry related field $\mathfrak{U}^x$ within and beyond the realm of the floating region H_n^x.

Nothing else than the discrepancy of the ordering of symmetry center $\mathfrak{S}_n^x$ with respect to the ordering of the parameter spaces $\mathcal{R}^{⓪}$and $\mathfrak{R}^{⓪}$ causes the existence of the symmetry related charge, which is related to the symmetry center. Anything that resides on this symmetry center will inherit that symmetry related charge.

4.2 Single symmetry center

H_n^x is a spatial domain. The regions H_n^x that are combined in H are excluded from domain Ω. The Stokes theorem does not hold for the separate regions H_n^x. Instead, the difference between the integrals defines a potential. In case of isotropic symmetry flavor of the symmetry center $\mathfrak{S}_n^x$ holds:

$$Q_n^x = |\boldsymbol{q} - \boldsymbol{c}_n^x| \left\{ \int_{\mathrm{H}_n^x} d\omega - \int_{\partial \mathrm{H}_n^x} \omega \right\} \tag{1}$$

$\boldsymbol{c}_n^x$ is the geometric center of symmetry center $\mathfrak{S}_n^x$. Q_n^x is the symmetry related charge. This corresponds to the symmetry related potential $\varphi_n^x(q)$ that exists at the outskirts of the encapsulation.

$$\varphi_n^x(\boldsymbol{q}) = \frac{Q_n^x}{|\boldsymbol{q} - \boldsymbol{c}_n^x|} = \int_{\mathrm{H}_n^x} d\omega - \int_{\partial \mathrm{H}_n^x} \omega \tag{2}$$

The potential $\varphi_n^x(\boldsymbol{q} - \mathbf{c}_n^x)$ contributes to the symmetry related field $\mathfrak{U}^x$.

4.3 Bounded center

A locally a spatially connected union $H_⊎$ of encapsulations H_n^x is defined by:

$$H_⊎ = \bigcup_{n=1}^{N^x} \mathrm{H}_n^x \tag{1}$$

$H_⊎$ encapsulates multiple symmetry centers. In case that $H_⊎$ exists, we consider the objects that reside within that encapsulation $\partial H_⊎$ as bounded by the symmetry related charges.

$$\phi^x(\boldsymbol{q}) = \sum_{n=1}^{N^x} \frac{Q_n^x}{|\boldsymbol{q} - \boldsymbol{c}_n^x|} \tag{2}$$

At large enough distance from this bounded center, all charges can be considered to be merged in a single charge with symmetry related potential function $\phi(q)$:

$$\phi(q) = \frac{\sum_{n=1}^{N} Q_n^x}{|\boldsymbol{q} - \boldsymbol{r}|} \tag{3}$$

$$\boldsymbol{r} = \frac{1}{N}\sum_{n=1}^{N} \boldsymbol{c}_n \tag{4}$$

4.4 Grouped artifacts

Next we consider grouped artifacts that cause discontinuities in a bounded center. The concerned field need not be the symmetry related field. Since we do no longer focus on symmetry related charges, we will omit the superscript ^{x}.

We consider the case that the locations of the artifacts form a coherent swarm $\{\boldsymbol{c}_n\}$ that can be characterized by a continuous location density distribution $\rho(\boldsymbol{q})$.

$$\chi(\boldsymbol{q}) = \sum_{n=0}^{N} \iiint_V \rho(\boldsymbol{q})\, Q_n\, \delta(\boldsymbol{q} - \boldsymbol{c}_n) = -\frac{1}{4\pi}\sum_{n=0}^{N} \iiint_V \rho(\boldsymbol{q}) Q_n \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \frac{1}{|\boldsymbol{q} - \boldsymbol{c}|} \rangle \tag{1}$$

If we use the spherical symmetric Gaussian location distribution of artifacts $\rho(r)$ that was introduced earlier as test function,

$$\rho(r) = \langle \boldsymbol{\nabla}, \boldsymbol{\nabla} \rangle \mathfrak{F}(r) = -\frac{Q}{\left(\sigma\sqrt{2\pi}\right)^3} \exp\left(-\frac{r^2}{2\sigma^2}\right) \tag{2}$$

then a potential in the form of

$$\chi(r) = -\frac{Q}{4\pi} \frac{ERF\left({}^{r}/_{\sigma\sqrt{2}}\right)}{r} \tag{3}$$

results.

At somewhat larger distances the potential behaves like a single charge potential.

$$\chi(r) \approx \frac{-Q}{4\pi r} \tag{4}$$

This gives an idea of what happens when a mechanism that acts within the realm of a symmetry center produces a coherent swarm of artifacts that will be embedded into a field that gets deformed by these artifacts.

Despite the fact that it is constituted from a myriad of singular contributions, the potential in equation (3) is a continuous function and its gradient at the center point equals zero! Thus the corresponding deformation has a "wide-spread" binding effect.

4.5 Discrepant regions

The symmetry centers correspond to point-like discontinuities. However, also large connected regions of $\mathfrak{R}^{⓪}$ may exist that disrupt the continuity of the manifold. For example a region that is surrounded by a boundary where the deformation is so strong that information contained in ω cannot pass the boundary of this region. These regions must also be separated from domain Ω. In this way these regions will correspond to ***cavities*** in the domain Ω. ***The information contained in the manifold cannot pass the surface of the cavity.*** The cavities act as information holes. Within the cavity the manifold can be considered to be non-existent. Within that region it has no defining function.

4.6 The embedding field

Apart from the symmetry related fields $\mathfrak{A}^x$ that are raised by the charges of the symmetry centers, at least one other fields exists. That field is the embedding field $\mathfrak{C}$. The embedding field is not directly affected by the symmetry related charges of the symmetry centers. It is indirectly affected, because the symmetry related fields affect the location of the symmetry centers that house the objects that can deform the embedding field. The embedding field is affected by the embedding of artifacts that are picked by a dedicated controlling mechanism that uses a symmetry center $\mathfrak{S}_n^x$. as a resource. After selection of the location of the artifact, the controlling mechanism embeds this artifact into the embedding continuum $\mathfrak{C}$. This continuum is represented by the continuum eigenspace of operator $\mathfrak{C}$. Thus, in this way, the previously discussed domain Ω also applies to the field $\mathfrak{C}$.

Each of these mechanisms operates in a cyclic and stochastic fashion. The embedding events occur in the direct neighborhood of the geometric center of the symmetry center. The result is a recurrently regenerated coherent location swarm that also represent a stochastic hopping path. The swarm is centered around the geometric center of the symmetry center. Hopping means that the controlling mechanism generates at the utmost one embedding location per progression step. This means that the hopping object can be considered as a point-like artifact. At the embedding instant the artifact actually resides at the location that is represented by an element of the location swarm. Thus, the swarm represents the spatial map of a set of potential detection locations. The swarm is generated within the symmetry center $\mathfrak{S}_n^x$ and is encapsulated by $\partial \mathrm{H}_n^x$. The actions of the mechanisms deform the field $\mathfrak{C}$ inside the floating regions H_n^x. The deformation of $\mathfrak{C}$ reaches beyond the region H_n^x.

In this way, the mechanism creates an elementary object, which is able to deform the embedding field $\mathfrak{C}$ and inherits the symmetry related charge from the symmetry center. The deformation represents the gravitation potential of the elementary object that owns the swarm. This is treated in more detail in reference [6].

4.7 Acceleration of the symmetry center

The fields $\mathfrak{A}$ and $\mathfrak{C}$ may accelerate the location of the symmetry center on which an elementary object resides. This occurs via the interaction of these fields with the contributions that the symmetry center and the recurrently embedded elementary object add to these fields.

The symmetry center and with it the residing elementary object float over the background parameter space $\mathfrak{R}$. This means that these items also float over the fields $\mathfrak{A}$ and $\mathfrak{C}$.

4.7.1 The symmetry related field

The symmetry related charge Q_n^x of the symmetry center $\mathfrak{S}_n^x$ contributes the local scalar potential φ_{n_0} to the symmetry related field $\mathfrak{A}$.

$$\varphi_{n_0}(\boldsymbol{q}) = \frac{Q_n^x}{|\boldsymbol{q} - \boldsymbol{c}_n^x|} \tag{1}$$

On the other hand

$$\boldsymbol{E}_n(\boldsymbol{q}) = \boldsymbol{\nabla}\varphi_{n_0} = \frac{Q_n^x(\boldsymbol{q} - \boldsymbol{c}_n^x)}{|\boldsymbol{q} - \boldsymbol{c}_n^x|^3} \tag{2}$$

Another symmetry center $\mathfrak{S}_m^x$ contributes potential φ_{m_0} to the symmetry related field $\mathfrak{A}$. The force $\boldsymbol{F}_{nm}$ between the two symmetry centers equals:

$$\boldsymbol{F}_{nm} = \boldsymbol{E}_n Q_m^x = \frac{Q_n^x Q_m^x(\boldsymbol{c}_n^x - \boldsymbol{c}_m^x)}{|\boldsymbol{c}_n^x - \boldsymbol{c}_m^x|^3} = -\boldsymbol{F}_{mn} = -\boldsymbol{E}_m Q_n^x \tag{3}$$

This need not correspond to an actual acceleration. On the other hand, if the movement of the symmetry center $\mathfrak{S}_n^x$ is uniform with speed $\boldsymbol{v}_n$, then the scalar potential φ_{n_0} corresponds to a vector potential $\boldsymbol{\varphi}_n = \varphi_{n_0}\boldsymbol{v}_n$. If the symmetry center actually accelerates, then this goes together with a field $\boldsymbol{E}_n = \dot{\boldsymbol{\varphi}}_n = \varphi_{n_0}\dot{\boldsymbol{v}}_n$ that represents the corresponding change of field $\mathfrak{A}$. Thus. If the two forces $\boldsymbol{F}_{nm}$ and $\boldsymbol{F}_{mn}$ do not hold each other in equilibrium, then the field $\mathfrak{A}$ will change dynamically.

4.7.2 The embedding field

The location swarms that are generated by dedicated controlling mechanisms produce a local potential that also can accelerate the symmetry center on which the location swarm resides. We analyze the situation by assuming that the swarm is represented by a Gaussian location distribution. Thus, we use the corresponding artifact as a test particle. The corresponding local potential that contributes to field $\mathfrak{C}$ equals

$$\chi_n(r) = -\frac{Q_n}{4\pi}\frac{ERF\left({}^{r}/_{\sigma\sqrt{2}}\right)}{r} \tag{1}$$

Here Q_n represents the strength of the local potential. At somewhat larger distances the potential behaves as a single "charge" potential.

$$\chi_n(\boldsymbol{q}) \approx \frac{-Q_n}{4\pi|\boldsymbol{q}-\boldsymbol{c}_n^x|} \tag{2}$$

This virtual "charge" is located at the center of the symmetry center $\mathfrak{S}_n^x$. The scalar potential $\chi_n(\boldsymbol{q})$ adds to the embedding field $\mathfrak{C}$. The result is that $\mathfrak{C}$ gets deformed.

The local scalar potential $\chi_n(\boldsymbol{q})$ corresponds to a derived field $\boldsymbol{\mathcal{E}}_n(\boldsymbol{q})$.

$$\boldsymbol{\mathcal{E}}_n(\boldsymbol{q}) = \boldsymbol{\nabla}\chi_n = -\frac{Q_n(\boldsymbol{q}-\boldsymbol{c}_n^x)}{|\boldsymbol{q}-\boldsymbol{c}_n^x|^3} \tag{3}$$

Another symmetry center $\mathfrak{S}_m^x$ contributes potential $\chi_m(\boldsymbol{q})$ to the embedding field $\mathfrak{C}$. The force $\boldsymbol{F}_{nm}$ between the two symmetry centers equals:

$$\boldsymbol{F}_{nm} = \boldsymbol{\mathcal{E}}_n Q_m = -\frac{Q_n Q_m(\boldsymbol{c}_n^x-\boldsymbol{c}_m^x)}{|\boldsymbol{c}_n^x-\boldsymbol{c}_m^x|^3} = -\boldsymbol{F}_{mn} = -\boldsymbol{\mathcal{E}}_m Q_n \tag{4}$$

This need not correspond to an actual acceleration.

If the platform $\mathfrak{S}_m^x$ on which the swarm resides moves with uniform speed $\boldsymbol{v}$, then the local potential corresponds to a local vector potential.

$$\boldsymbol{\chi}_n = \chi_n \boldsymbol{v} \tag{5}$$

If this platform accelerates, then this goes together with an extra contribution to field $\boldsymbol{\mathcal{E}}_n$ that counteracts the acceleration.

$$\boldsymbol{\mathcal{E}}_n = \dot{\boldsymbol{\chi}}_n = \chi_n \dot{\boldsymbol{v}} \tag{6}$$

This effect is known as ***inertia***.

4.8 Path of the symmetry center

The symmetry center $\mathfrak{S}_n^x$ that conforms to encapsulated region H_n^x, keeps its private symmetry flavor. At the passage through the boundary the symmetry flavor of the background parameter space

$\mathfrak{R}^{⓪}$ flips from history to future. As a consequence the symmetry related charge of the symmetry center will flip.

However, the passage of the symmetry center through the rim may also be interpreted as the annihilation of the historic symmetry center and the creation of a new symmetry center with a reverse symmetry flavor that will extend its live in the future.

The passage of the symmetry centers through the rim goes together with annihilation and creation phenomena for the objects that reside on these platforms. Thus, this passage is related to the annihilation and creation of elementary objects.

In the quaternionic space-progression model the existence of symmetry centers is independent of progression. With other words the number of symmetry centers is a model constant. The passage through the rim does not influence this number. Only the characteristics of the combination of the symmetry center and the background parameter space are affected by the passage.

4.8.1 Path integral

In this investigation we ignore the action of the symmetry related potential.

Elementary objects reside on an individual symmetry center. A dedicated mechanism controls its recurrent generation and embeds the object into the embedding field. The path of the symmetry center is the averaged path of the embedded object. The embedded object is hopping along the elements of the generated location swarm. The landing locations of the hops are generated by the controlling mechanism in a stochastic fashion, but such that at first approximation the swarm as a whole can be considered to be moving as one unit. This is possible when the swarm is characterized by a continuous location density distribution, which owns a displacement generator. That is the case when the location density distribution owns a Fourier transform. This fact enables the description of the path of the swarm by a “path integral”. The hopping of the embedded object can be described by a sequence of factors that after multiplication represent the whole path. Each factor represents three sub-factors.

The first sub-factor represents the jump from configuration space to momentum space. This sub-factor is given by the inner product of the Hilbert vector that represents the current location and the Hilbert vector that represents the momentum of the swarm. This second Hilbert vector is assumed to be constant during the generation of the location swarm.

The second sub-factor represents the effect of the hop in momentum space.

The third sub-factor represents the jump back from momentum space to configuration space.

In the sequence of factors the third sub-factor of the current term compensates the effect of the first sub-factor of next term. Their product equals unity.

What results is a sequence of factors that are very close to unity and that represent the effects of the hops in momentum space. Due to the fact that the momentum is considered to be constant the logarithms of the terms can be taken and added in an overall sum. In this way, the multiplication is equal to the sum of the logarithms of the factors.

This summation approaches what is known as the “path integral”. In our interpretation it is not an integral, but instead it is a finite summation. In more detail the procedure can be described as follows.

We suppose that momentum $\boldsymbol{p}_n$ is constant during the particle generation cycle in which the controlling mechanism produces the swarm $\{a_i\}$. Every hop gives a contribution to the path. These contributions can be divided into three steps per contributing hop:

1. Change to Fourier space. This involves as sub-factor the inner product $\langle a_i|p_n\rangle$.
2. Evolve during an infinitesimal progression step into the future.
    a. Multiply with the corresponding displacement generator $\boldsymbol{p}_n$.
    b. The generated step in configuration space is $(\boldsymbol{a}_{i+1} - \boldsymbol{a}_i)$.
    c. The action contribution factor in Fourier space is $\langle \boldsymbol{p}_n, \boldsymbol{a}_{i+1} - \boldsymbol{a}_i\rangle$.
3. Change back to configuration space. This involves as sub-factor the inner product $\langle p_n|a_{i+1}\rangle$
4. The combined term contributes a factor $\langle a_i|p_n\rangle\exp(\langle \boldsymbol{p}_n, \boldsymbol{a}_{i+1} - \boldsymbol{a}_i\rangle)\langle p_n|a_{i+1}\rangle$.
5. Two subsequent steps give:

$$\langle a_i|p_n\rangle\exp(\langle \boldsymbol{p}_n, \boldsymbol{a}_{i+1} - \boldsymbol{a}_i\rangle)\langle p_n|a_{i+1}\rangle\langle a_{i+1}|p_n\rangle\exp(\langle \boldsymbol{p}_n, \boldsymbol{a}_{i+1} - \boldsymbol{a}_i\rangle)\langle p_n|a_{i+2}\rangle \tag{1}$$

$$= \langle a_i|p_n\rangle\exp(\langle \boldsymbol{p}_n, \boldsymbol{a}_{i+2} - \boldsymbol{a}_i\rangle)\langle p_n|a_{i+2}\rangle$$

The terms in the middle turn into unity. The other terms also join.

Over a full particle generation cycle with N steps this results in:

$$\prod_{i=1}^{N-1}\langle a_i|p_n\rangle\exp(\langle \boldsymbol{p}_n, \boldsymbol{a}_{i+1} - \boldsymbol{a}_i\rangle)\langle p_n|a_{i+1}\rangle \tag{2}$$

$$= \langle a_1|p_n\rangle\exp(\langle \boldsymbol{p}_n, \boldsymbol{a}_N - \boldsymbol{a}_1\rangle)\langle p_n|a_N\rangle = \langle a_1|p_n\rangle \exp\left(\sum_{i=2}^{N}\langle \boldsymbol{p}_n, \boldsymbol{a}_{i+1} - \boldsymbol{a}_i\rangle\right)\langle p_n|a_N\rangle$$

$$= \langle a_1|p_n\rangle \exp(L)\,\langle p_n|a_N\rangle$$

$$L\,d\tau = \sum_{i=2}^{N-1}\langle \boldsymbol{p}_n, \boldsymbol{a}_{i+1} - \boldsymbol{a}_i\rangle = \langle \boldsymbol{p}_n, d\boldsymbol{q}\rangle \tag{3}$$

$$L = \langle \boldsymbol{p}_n, \dot{\boldsymbol{q}}\rangle \tag{4}$$

L is known as the Lagrangian.

Equation (4) holds for the special condition in which $\boldsymbol{p}_n$ is constant. If $\boldsymbol{p}_n$ is not constant, then the Hamiltonian H varies with location. In the next equations we ignore subscript $_n$.

$$\frac{\partial H}{\partial q_i} = -\dot{p}_i \tag{5}$$

$$\frac{\partial H}{\partial p_i} = \dot{q}_i \tag{6}$$

$$\frac{\partial L}{\partial q_i} = \dot{p} \tag{7}$$

$$\frac{\partial L}{\partial \dot{q}_i} = p_i \tag{8}$$

$$\frac{\partial H}{\partial \tau} = -\frac{\partial H}{\partial \tau} \tag{9}$$

$$\frac{d}{d\tau}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i} \tag{10}$$

$$H + L = \sum_{i=1}^{3} \dot{q}_i p_i \tag{11}$$

Here we used proper time τ rather than coordinate time t.

The effect of the hopping path is that the geometric center of the symmetry center is moved over a small resulting distance $\boldsymbol{a}_N - \boldsymbol{a}_1$. Together with "charge" $N\ Q_n$ this move determines the next version of momentum $\boldsymbol{p}_n$.

The result is that both the symmetry related fields $\mathfrak{A}^x$ and the embedding field $\mathfrak{C}$ influence the location of the geometric center of the symmetry center $\mathfrak{S}_n^x$.

4.9 At the start of progression

At progression value $\tau = 0$, the mechanisms that generate the artifacts, which cause discontinuities in the embedding manifold $\mathfrak{C}$ have not yet done any work. It means that this manifold was flat and its defining function equaled its parameter space at instance $\tau = 0$.

The model offers the possibility that the domain Ω expands as a function of τ. In that case it is possible that domain Ω covers a growing amount of symmetry centers.

5 Discussion

This paper only considers the divergence based version of the generalized Stokes theorem. The consequences for the curl based version are not investigated. From fluid dynamics it is known that artifacts that are embedded in a fluid may suffer from the vorticity of the embedding field [2].

This paper does not investigate the consequences of polar ordering. It probably relates to the spin properties of elementary objects. In that case the polar ordering of symmetry centers regulates the distinction between fermions and bosons. The half integer spin particles may use ordering of the azimuth, where the integer spin particles use the ordering of the polar angle. However, this does not explain the difference in behavior between these categories.

The concept of exterior derivative is carefully crafted by skillful mathematicians, such that it becomes independent of the selection of parameter spaces. However, in a situation like this in which several parameter space float on top of a background parameter space, the selection of the ordering of the parameter spaces does matter. The symmetry flavors of the coupled parameter spaces determine the values of the integrals that account for the contributions of the artifacts. It is represented by the symmetry related charges of these artifacts [6]. These symmetry related charges are supposed to be located at the geometric centers of the symmetry centers.

As happens so often, physical reality reveals facts (such as the symmetry related charges) that cannot easily be discovered by skilled mathematicians. The standard model contains a short list of electric charges that correspond to the symmetry related charges. The standard model does not give an explanation for the existence of this short list. Here it becomes clear that the electric charge and the color charge are a properties of connected parameter spaces and not a property of the objects that use these parameter spaces. Instead, these objects inherit the charge properties from the platform on which they reside.

Both the symmetry related fields and the embedding continuum affect the geometric location of the symmetry center. They do that in different ways.

If electric charges are properties of the connection between spaces, then the fields to which these charges contribute implement the forces between these connections. No extra objects are needed to implement these forces!

It is sensible to expect that depending on the type of their "charges" all basic fields are capable of attracting or repelling the spaces on which these "charges" reside. This behavior is described by the differential and integral equations that are obeyed by the considered field.

This model is no more and no less than a mathematical test case. The paper does not pretend that physical reality behaves like this model. But the methods used and the results obtained in this paper might learn more about how physical reality can be structured and how it can behave.

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