Magnetohydrodynamic Equations Solutions

Abstract

The simplest solution is usually the best solution---Albert Einstein

The system of magnetohydrodynamic (MHD) equations has been solved analytically in this paper. The author applied the technique used in solving the Navier-Stokes equations and applied a new law, the law of definite ratio for MHD. This law states that in MHD, the other terms of the system of equations divide the gravity term in a definite ratio, and each term utilizes gravity to function. The sum of the terms of the ratio is always unity. It is shown that without gravity forces on earth, there would be no magnetohydrodynamics on earth as is known. The equations in the system of equations were added to produce a single equation which was then integrated. Ratios were used to split-up this single equation into sub-equations which were readily integrated, and even, the non-linear sub-equations were readily integrated. Twenty-seven sub-equations were integrated. The linear part of the relation obtained from the integration of the linear part of the equation satisfied the linear part of the equation; and the relation from the integration of the non-linear part satisfied the non-linear part of the equation. The solutions revealed the role of each term in magnetohydrodynamics. In particular, the gravity term is the indispensable term in magnetohydrodynamics. The solutions of the MHD equations were compared with the solutions of the N-S equations, and there were similarities and dissimilarities.

Solutions of the Magnetohydrodynamic Equations

This system consists of four equations and one is to solve for $V_x, V_y, V_z, B_x, B_y, B, P(x)$

Magnetohydrodynamic Equations

1. $$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0 \text{ <-- continuity equation}$$

2. $$\overbrace{\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}}^{\text{Navier–Stokes}} = \overbrace{-\frac{\partial p}{\partial x}+\frac{1}{\mu}(\nabla\times B)\times B+\rho g_x}^{\text{Lorentz force}}$$

3. $$\rho\frac{\partial B}{\partial t}=\nabla\times(V\times B)+\eta\nabla^2 B$$
$$\rho\frac{\partial B}{\partial t}=\nabla\times(V\times B)+\eta\left(\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+\frac{\partial^2 B}{\partial z^2}\right)$$
(η = magnetic diffusivity)

4. $$\nabla\bullet B=0$$
$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0$$

Step 1:

1. If ρ is constant: (for incompressible fluid)
$$\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}=0 \text{ <-- continuity equation}$$

2. $$\overbrace{\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z}}^{\text{Navier - Stokes}} = \overbrace{-\frac{\partial p}{\partial x}+\frac{1}{\mu}(\nabla\times B)\times B+\rho g_x}^{\text{Lorentz force}}$$
$$\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z} = -\frac{\partial p}{\partial x}+\frac{1}{\mu}\left(B_z\left(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}\right)-B_y\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right)\right)+\rho g_x$$
$$\boxed{\rho\frac{\partial V_x}{\partial t}+\rho V_x\frac{\partial V_x}{\partial x}+\rho V_y\frac{\partial V_x}{\partial y}+\rho V_z\frac{\partial V_x}{\partial z} = -\frac{\partial p}{\partial x}+\frac{1}{\mu}\left(B_z\frac{\partial B_x}{\partial z}-B_z\frac{\partial B_z}{\partial x}-B_y\frac{\partial B_y}{\partial x}+B_y\frac{\partial B_x}{\partial y}\right)+\rho g_x}$$

3. $$\rho\frac{\partial B}{\partial t}=\nabla\times(V\times B)+\eta\nabla^2 B$$
$$\rho\frac{\partial B}{\partial t}=\frac{\partial}{\partial y}(V_xB_y-V_yB_x)-\frac{\partial}{\partial z}(V_zB_x-V_xB_z)+\eta\left(\frac{\partial^2 B}{\partial x^2}+\frac{\partial^2 B}{\partial y^2}+\frac{\partial^2 B}{\partial z^2}\right)$$
$$\boxed{\rho\frac{\partial B}{\partial t}=\frac{\partial}{\partial y}V_xB_y-\frac{\partial}{\partial y}V_yB_x-\frac{\partial}{\partial z}V_zB_x+\frac{\partial}{\partial z}V_xB_z+\eta\frac{\partial^2 B_x}{\partial x^2}+\eta\frac{\partial^2 B_x}{\partial y^2}+\eta\frac{\partial^2 B_x}{\partial z^2}}$$

4. $$\boxed{\nabla\bullet B=0 \qquad \frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}=0}$$

Step 2:
After the "vector juggling" one obtains the following system of equations which one will solve.

$$\begin{cases} 1.\ \dfrac{\partial V_x}{\partial x}+\dfrac{\partial V_y}{\partial y}+\dfrac{\partial V_z}{\partial z}=0 \\ 2.\ \rho\dfrac{\partial V_x}{\partial t}+\rho V_x\dfrac{\partial V_x}{\partial x}+\rho V_y\dfrac{\partial V_x}{\partial y}+\rho V_z\dfrac{\partial V_x}{\partial z}+\dfrac{\partial p}{\partial x}-\dfrac{1}{\mu}B_z\dfrac{\partial B_x}{\partial z}+\dfrac{1}{\mu}B_z\dfrac{\partial B_z}{\partial x}+\dfrac{1}{\mu}B_y\dfrac{\partial B_y}{\partial x}-\dfrac{1}{\mu}B_y\dfrac{\partial B_x}{\partial y}=\rho g_x \\ 3. \\ \dfrac{\rho\partial B_x}{\partial t}-V_x\dfrac{\partial B_y}{\partial y}-B_y\dfrac{\partial V_x}{\partial y}+V_y\dfrac{\partial B_x}{\partial y}+B_x\dfrac{\partial V_y}{\partial y}+V_z\dfrac{\partial B_x}{\partial z}+B_x\dfrac{\partial V_z}{\partial z}-V_x\dfrac{\partial B_z}{\partial z}-B_z\dfrac{\partial V_x}{\partial z}-\dfrac{\eta\partial^2 B_x}{\partial x^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial y^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial z^2}=0 \\ 4.\ \dfrac{\partial B_x}{\partial x}+\dfrac{\partial B_y}{\partial y}+\dfrac{\partial B_z}{\partial z}=0 \end{cases}$$

At a glance, and from the experience gained in solving the Navier-Stokes equations, one can identify equation (2) as the driver equation, since it contains the gravity term, and the gravity term is the subject of the equation. However, since the system of equations is to be solved simultaneously and there is only a single "driver", the gravity term, all the terms in the system of equations will be placed in the driver equation, Equation 2. As suggested by Albert Einstein, Friedrich Nietzsche, and Pablo Picasso, one will think like a child at the next step.

Step 3: Thinking like a ninth grader, one will apply the following axiom:

If $a=b$ and $c=d$, then $a+c=b+d$; and therefore, add the left sides and add the right sides of the above equations . That is, $(1)+(2)+(3)+(4)=\rho g_x$

$$\begin{cases} \dfrac{\partial V_x}{\partial x}+\dfrac{\partial V_y}{\partial y}+\dfrac{\partial V_z}{\partial z}+\rho\dfrac{\partial V_x}{\partial t}+\rho V_x\dfrac{\partial V_x}{\partial x}+\rho V_y\dfrac{\partial V_x}{\partial y}+\rho V_z\dfrac{\partial V_x}{\partial z}+\dfrac{\partial p}{\partial x}-\dfrac{1}{\mu}B_z\dfrac{\partial B_x}{\partial z}+\dfrac{1}{\mu}B_z\dfrac{\partial B_z}{\partial x}+\dfrac{1}{\mu}B_y\dfrac{\partial B_y}{\partial x}- \\ \dfrac{1}{\mu}B_y\dfrac{\partial B_x}{\partial y}+\dfrac{\rho\partial B_x}{\partial t}-V_x\dfrac{\partial B_y}{\partial y}-B_y\dfrac{\partial V_x}{\partial y}+V_y\dfrac{\partial B_x}{\partial y}+B_x\dfrac{\partial V_y}{\partial y}+V_z\dfrac{\partial B_x}{\partial z}+B_x\dfrac{\partial V_z}{\partial z}-V_x\dfrac{\partial B_z}{\partial z}-B_z\dfrac{\partial V_x}{\partial z}- \\ \dfrac{\eta\partial^2 B_x}{\partial x^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial y^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial z^2}+\dfrac{\partial B_x}{\partial x}+\dfrac{\partial B_y}{\partial y}+\dfrac{\partial B_z}{\partial z}=\rho g_x \qquad \text{(Three lines per equation)} \end{cases}$$

Step 4: Writing all the linear terms first

$$\begin{cases} \dfrac{\partial V_x}{\partial x}+\dfrac{\partial V_y}{\partial y}+\dfrac{\partial V_z}{\partial z}+\rho\dfrac{\partial V_x}{\partial t}+\dfrac{\partial p}{\partial x}+\dfrac{\rho\partial B_x}{\partial t}-\dfrac{\eta\partial^2 B_x}{\partial x^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial y^2}-\dfrac{\eta\partial^2 B_x}{\eta\partial z^2}+\dfrac{\partial B_x}{\partial x}+\dfrac{\partial B_y}{\partial y}+\dfrac{\partial B_z}{\partial z} \\ +\rho V_x\dfrac{\partial V_x}{\partial x}+\rho V_y\dfrac{\partial V_x}{\partial y}+\rho V_z\dfrac{\partial V_x}{\partial z}-\dfrac{1}{\mu}B_z\dfrac{\partial B_x}{\partial z}+\dfrac{1}{\mu}B_z\dfrac{\partial B_z}{\partial x}+\dfrac{1}{\mu}B_y\dfrac{\partial B_y}{\partial x}-\dfrac{1}{\mu}B_y\dfrac{\partial B_x}{\partial y}-V_x\dfrac{\partial B_y}{\partial y}-B_y\dfrac{\partial V_x}{\partial y} \\ +V_y\dfrac{\partial B_x}{\partial y}+B_x\dfrac{\partial V_y}{\partial y}+V_z\dfrac{\partial B_x}{\partial z}+B_x\dfrac{\partial V_z}{\partial z}-V_x\dfrac{\partial B_z}{\partial z}-B_z\dfrac{\partial V_x}{\partial z}=\rho g_x \qquad \text{(Three lines per equation)} \end{cases}$$

(Since all the terms are now in the same driver equation, let ρg_x "drive them" simultaneously.)

Step 5: Solve the above 28-term equation using the ratio method. (27 ratio terms)

The ratio terms to be used are respectively the following: (Sum of the ratio terms = 1)

$\beta_1,\ \beta_2,\ \beta_3,\ a\ ,b,\ c,\ d,\ f,\ m,\ q,\ r,\ s,\ \omega_1,\ \omega_2,\ \omega_3,\omega_4,\omega_5,\ \omega_6,\ \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\lambda_7,\ \lambda_8,\ \lambda_9$

1. $\dfrac{\partial V_x}{\partial x}=\beta_1\rho g_x$ $\dfrac{dV_x}{dx}=\beta_1\rho g_x$ $V_x=\beta_1\rho g_x x+C_{16}$	2. $\dfrac{\partial V_y}{\partial y}=\beta_2\rho g_x$ $\dfrac{dV_y}{dy}=\beta_2\rho g_x$ $V_y=\beta_2\rho g_x y+C_{17}$	3. $\dfrac{\partial V_z}{\partial z}=\beta_3\rho g_x$ $\dfrac{dV_z}{dz}=\beta_3\rho g_x$ $V_z=\beta_3\rho g_x z+C_{18}$	4. $\rho\dfrac{\partial V_x}{\partial t}=a\rho g_x$ $\dfrac{\partial V_x}{\partial t}=ag_x$ $V_x=ag_x t+C_1$

5.	6.	7.
$\frac{\partial p}{\partial x} = b\rho g_x$ $\frac{dp}{dx} = b\rho g_x$ $P(x) = b\rho g_x x + C$	$\rho \frac{\partial B_x}{\partial t} = c\rho g_x$ $\frac{\partial B_x}{\partial t} = cg_x$ $\frac{dB_x}{dt} = cg_x$ $B_x = cg_x t + C_{1b}$	$-\eta \frac{\partial^2 B_x}{\partial x^2} = d\rho g_x$ $\frac{d^2 B_x}{dx^2} = -\frac{d\rho g_x}{\eta}$ $\frac{dB_x}{dx} = -\frac{d\rho g_x x}{\eta} + C_2$ $B_x = -\frac{d\rho g_x x^2}{2\eta} + C_2 x + C_3$

8.	9.	10.
$-\eta \frac{\partial^2 B_x}{\partial y^2} = f\rho g_x$ $\frac{d^2 B_x}{dy^2} = -\frac{f\rho g_x}{\eta}$ $\frac{dB_x}{dy} = -\frac{f\rho g_x y}{\eta} + C_4$ $B_x = -\frac{f\rho g_x y^2}{2\eta} + C_4 y + C_5$	$-\eta \frac{\partial^2 B_x}{\partial z^2} = m\rho g_x$ $\frac{d^2 B_x}{dz^2} = -\frac{m\rho g_x}{\eta}$ $\frac{dB_x}{dz} = -\frac{m\rho g_x z}{\eta} + C_6$ $B_x = -\frac{m\rho g_x z^2}{2\eta} + C_6 x + C_7$	$\frac{\partial B_x}{\partial x} = q\rho g_x$ $\frac{dB_x}{dx} = q\rho g_x$ $B_x = q\rho g_x x + C_{19}$

11.	12.	13.	14.
$\frac{\partial B_y}{\partial y} = r\rho g_x$ $\frac{dB_y}{dy} = r\rho g_x$ $B_y = r\rho g_x y + C_{20}$	$\frac{\partial B_z}{\partial z} = s\rho g_x$ $\frac{dB_z}{dz} = s\rho g_x$ $B_z = s\rho g_x z + C_{21}$	$\rho V_x \frac{\partial V_x}{\partial x} = \omega_1 \rho g_x$ $V_x \frac{dV_x}{dx} = \omega_1 g_x$ $V_x dV_x = \omega_1 g_x dx$ $\frac{V_x^2}{2} = \omega_1 g_x x$ $V_x^2 = 2\omega_1 g_x x$ $V_x = \pm\sqrt{2\omega_1 g_x x} + C_2$	$\rho V_y \frac{\partial V_x}{\partial y} = \omega_2 \rho g_x$ $V_y dV_x = \omega_2 g_x dy$ $V_y V_x = \omega_2 g_x y + \psi_y(V_y)$ $V_x = \frac{\omega_2 g_x y}{V_y} + \frac{\psi_y(V_y)}{V_y}$ $V_y \neq 0$

15.	16	17.
$\rho V_z \frac{\partial V_x}{\partial z} = \omega_3 \rho g_x$ $V_z \frac{dV_x}{dz} = \omega_3 g_x$ $V_z dV_x = \omega_3 g_x dz$ $V_z V_x = \omega_3 g_x z + \psi_z(V_z)$ $V_x = \frac{\omega_3 g_x z}{V_z} + \frac{\psi_z(V_z)}{V_z}$ $V_z \neq 0$	$B_z \frac{\partial B_x}{\partial z} = -\omega_4 \mu\rho g_x$ $B_z dB_x = -\omega_4 \mu\rho g_x dz$ $B_z B_x = -\omega_4 \mu\rho g_x z + \psi_z(B_z)$ $B_x = -\frac{\omega_4 \mu\rho g_x z}{B_z} + \frac{\psi_z(B_z)}{B_z}$ $B_z \neq 0$	$B_z \frac{\partial B_z}{\partial x} = \omega_5 \mu\rho g_x$ $B_z \frac{dB_z}{dx} = \omega_5 \mu\rho g_x$ $B_z dB_z = \omega_5 \mu\rho g_x dx$ $\frac{B_z^2}{2} = \omega_5 \mu\rho g_x x$ $B_z^2 = 2\omega_5 \mu\rho g_x x$ $B_z = \pm\sqrt{2\omega_5 \mu\rho g_x x} + C$

18.	19.	20
$B_y \frac{\partial B_y}{\partial x} = \omega_6 \mu \rho g_x$ $B_y \frac{dB_y}{dx} = \omega_6 \mu \rho g_x$ $B_y dB_y = \omega_6 \mu \rho g_x dx$ $\frac{B_y^{\ 2}}{2} = \omega_6 \mu \rho g_x x$ $B_y^{\ 2} = 2\omega_6 \mu \rho g_x x$ $B_y = \pm\sqrt{2\omega_6 \mu \rho g_x x} + C$	$-\frac{1}{\mu} B_y \frac{\partial B_x}{\partial y} = \lambda_1 \rho g_x$ $B_y \frac{dB_x}{dy} = -\lambda_1 \mu \rho g_x$ $B_y dB_x = -\lambda_1 \mu \rho g_x dy$ $B_y B_x = -\lambda_1 \mu \rho g_x y + \psi_y(B_y)$ $B_x = -\frac{\lambda_1 \mu \rho g_x y}{B_y} + \frac{\psi_y(B_y)}{B_y}$ $B_y \neq 0$	$-V_x \frac{\partial B_y}{\partial y} = \lambda_2 \rho g_x$ $V_x \frac{dB_y}{dy} = -\lambda_2 \rho g_x$ $V_x dB_y = -\lambda_2 \rho g_x dy$ $V_x B_y = -\lambda_2 \rho g_x y + \psi_x(V_x)$ $B_y = \frac{-\lambda_2 \rho g_x y}{V_x} + \frac{\psi_x(V_x)}{V_x}$ $V_x \neq 0$

21.	22.	23.
$-B_y \frac{\partial V_x}{\partial y} = \lambda_3 \rho g_x$ $B_y \frac{dV_x}{dy} = -\lambda_3 \rho g_x$ $B_y dV_x = -\lambda_3 \rho g_x dy$ $B_y V_x = -\lambda_3 \rho g_x y + \psi_y(B_y)$ $V_x = -\frac{\lambda_3 \rho g_x y}{B_y} + \frac{\psi_y(B_y)}{B_y}$ $B_y \neq 0$	$V_y \frac{\partial B_x}{\partial y} = \lambda_4 \rho g_x$ $V_y \frac{dB_x}{dy} = \lambda_4 \rho g_x$ $V_y dB_x = \lambda_4 \rho g_x dy$ $V_y B_x = \lambda_4 \rho g_x y + \psi_y(V_y)$ $B_x = \frac{\lambda_4 \rho g_x y}{V_y} + \frac{\psi_y(V_y)}{V_y}$ $V_y \neq 0$	$B_x \frac{\partial V_y}{\partial y} = \lambda_5 \rho g_x$ $B_x \frac{dV_y}{dy} = \lambda_5 \rho g_x$ $B_x dV_y = \lambda_5 \rho g_x dy$ $B_x V_y = \lambda_5 \rho g_x y + \psi_x(B_x)$ $V_y = \frac{\lambda_5 \rho g_x y}{B_x} + \frac{\psi_x(B_x)}{B_x}$ $B_x \neq 0$

24.	25.	26
$V_z \frac{\partial B_x}{\partial z} = \lambda_6 \rho g_x$ $V_z \frac{dB_x}{dz} = \lambda_6 \rho g_x$ $V_z dB_x = \lambda_6 \rho g_x dz$ $V_z B_x = \lambda_6 \rho g_x z + \psi_z(V_z)$ $B_x = \frac{\lambda_6 \rho g_x z}{V_z} + \frac{\psi_z(V_z)}{V_z}$ $V_z \neq 0$	$B_x \frac{\partial V_z}{\partial z} = \lambda_7 \rho g_x$ $B_x \frac{dV_z}{dz} = \lambda_7 \rho g_x$ $B_x dV_z = \lambda_7 \rho g_x dz$ $B_x V_z = \lambda_7 \rho g_x z + \psi_x(B_x)$ $V_z = \frac{\lambda_7 \rho g_x z}{B_x} + \frac{\psi_x(B_x)}{B_x}$ $B_x \neq 0$	$-V_x \frac{\partial B_z}{\partial z} = \lambda_8 \rho g_x$ $V_x \frac{dB_z}{dz} = -\lambda_8 \rho g_x$ $V_x dB_z = -\lambda_8 \rho g_x dz$ $V_x B_z = -\lambda_8 \rho g_x z + \psi_x(V_x)$ $B_z = -\frac{\lambda_8 \rho g_x z}{V_x} + \frac{\psi_x(V_x)}{V_x}$ $V_x \neq 0$

27.
$-B_z \frac{\partial V_x}{\partial z} = \lambda_9 \rho g_x$ $B_z \frac{dV_x}{dz} = -\lambda_9 \rho g_x$ $B_z dV_x = -\lambda_9 \rho g_x dz$ $B_z V_x = -\lambda_9 \rho g_x z + \psi_z(B_z)$ $V_x = -\frac{\lambda_9 \rho g_x z}{B_z} + \frac{\psi_z(B_z)}{B_z}$ $B_z \neq 0$

Step 6: One collects the integrals of the sub-equations, above, for V_x, V_y, V_z, B_x, B_y, B_z, $P(x)$

$V_x(x,y,z,t) =$ (sum of integrals from sub-equations #1, #4,#13,#14,#15,#21,#27)

$$\beta_1 \rho g_x x + a g_x t \pm \sqrt{2\omega_1 g_x x} + \frac{\omega_2 g_x y}{V_y} - \frac{\lambda_3 \rho g_x y}{B_y} + \frac{\omega_3 g_x z}{V_z} - \frac{\lambda_9 \rho g_x z}{B_z} + \underbrace{\frac{\psi_z(V_z)}{V_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(B_z)}{B_z}}_{\text{arbitrary functions}} + C_1;$$

(integral from sub–equation #5)

$$P(x) = b\rho g_x x + C_2$$

(sum of integrals from sub–equations #2,#23)

$$V_y(y) = \beta_2 \rho g_x y + \frac{\lambda_5 \rho g_x y}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_3$$

(sum of integrals from sub–equations #3, #25)

$$V_z(z) = \beta_3 \rho g_x z + \frac{\lambda_7 \rho g_x z}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_4$$

(sum of integrals from sub-equations #6, #7, #8, #9, #10, #16,#19, #22, #24)

$B_x(x,y,z,t) =$

$$B_x = -\frac{\rho g_x}{2\eta}(dx^2 + fy^2 + mz^2) + q\rho g_x x + C_2 x + C_4 y + C_6 z + +c g_x t - \frac{\lambda_1 \mu \rho g_x y}{B_y} + \frac{\lambda_4 \rho g_x y}{V_y} - \frac{\omega_4 \mu \rho g_x z}{B_z} +$$

$$\frac{\lambda_6 \rho g_x z}{V_z} + \underbrace{\frac{\psi_z(B_z)}{B_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}} + C_7$$

(sum of integrals from sub–equations #11,#18,#20)

$$B_y = r\rho g_x y \pm \sqrt{2\omega_6 \mu \rho g_x x} - \frac{\lambda_2 \rho g_x y}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_8$$

(sum of integrals from sub–equations #12,#17,#26)

$$B_z = s\rho g_x z \pm \sqrt{2\omega_5 \mu \rho g_x x} - \frac{\lambda_8 \rho g_x z}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_{21}$$

Step 7: Find the test derivatives for the linear part

1.	2.	3.	4.	5.	6.
$\frac{\partial V_x}{\partial x} = (\beta_1 \rho g_x)$	$\frac{\partial V_y}{\partial y} = (\beta_2 \rho g_x)$	$\frac{\partial V_z}{\partial z} = (\beta_3 \rho g_x)$	$\frac{\partial V_x}{\partial t} = (a g_x)$	$\frac{\partial p}{\partial x} = (b \rho g_x)$	$\frac{dB_x}{dt} = (c g_x)$

7.	8.	9.	10.	11.	12.
$\frac{\partial^2 B_x}{\partial x^2} = -\frac{d\rho g_x}{\eta}$	$\frac{\partial^2 B_x}{\partial y^2} = -\frac{f\rho g_x}{\eta}$	$\frac{\partial^2 B_x}{\partial z^2} = -\frac{m\rho g_x}{\eta}$	$\frac{\partial B_x}{\partial x} = q\rho g_x$	$\frac{\partial B_y}{\partial y} = r\rho g_x$	$\frac{\partial B_z}{\partial z} = s\rho g_x$

Test derivatives for the nonlinear part

13.	14.	15.	16	17.
$\frac{\partial V_x}{\partial x} = \frac{\omega_1 g_x}{V_x}$	$\frac{\partial V_x}{\partial y} = \frac{\omega_2 g_x}{V_y}$	$\frac{\partial V_x}{\partial z} = \frac{\omega_3 g_x}{V_z}$	$\frac{\partial B_x}{\partial z} = -\frac{\omega_4 \mu\rho g_x}{B_z}$	$\frac{\partial B_z}{\partial x} = \frac{\omega_5 \mu\rho g_x}{B_z}$

18.	19.	20.	21.	22.
$\frac{\partial B_y}{\partial x} = \frac{\omega_6 \mu\rho g_x}{B_y}$	$\frac{\partial B_x}{\partial y} = -\frac{\lambda_1 \mu\rho g_x}{B_y}$	$\frac{\partial B_y}{\partial y} = -\frac{\lambda_2 \rho g_x}{V_x}$	$\frac{\partial V_x}{\partial y} = -\frac{\lambda_3 \rho g_x}{B_y}$	$\frac{\partial B_x}{\partial y} = \frac{\lambda_4 \rho g_x}{V_y}$

23.	24.	25.	26.	27.
$\frac{\partial V_y}{\partial y} = \frac{\lambda_5 \rho g_x}{B_x}$	$\frac{\partial B_x}{\partial z} = \frac{\lambda_6 \rho g_x}{V_z}$	$\frac{\partial V_z}{\partial z} = \frac{\lambda_7 \rho g_x}{B_x}$	$\frac{\partial B_z}{\partial z} = -\frac{\lambda_8 \rho g_x}{V_x}$	$\frac{\partial V_x}{\partial z} = -\frac{\lambda_9 \rho g_x}{B_z}$

Step 8: Substitute the above test derivatives respectively in the following 28-term equation

$$\begin{cases} \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} + \rho\frac{\partial V_x}{\partial t} + \frac{\partial p}{\partial x} + \frac{\rho\partial B_x}{\partial t} - \frac{\eta\partial^2 B_x}{\partial x^2} - \frac{\eta\partial^2 B_x}{\eta\partial y^2} - \frac{\eta\partial^2 B_x}{\eta\partial z^2} + \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \\ +\rho V_x\frac{\partial V_x}{\partial x} + \rho V_y\frac{\partial V_x}{\partial y} + \rho V_z\frac{\partial V_x}{\partial z} - \frac{1}{\mu}B_z\frac{\partial B_x}{\partial z} + \frac{1}{\mu}B_z\frac{\partial B_z}{\partial x} + \frac{1}{\mu}B_y\frac{\partial B_y}{\partial x} - \frac{1}{\mu}B_y\frac{\partial B_x}{\partial y} - V_x\frac{\partial B_y}{\partial y} - B_y\frac{\partial V_x}{\partial y} \\ +V_y\frac{\partial B_x}{\partial y} + B_x\frac{\partial V_y}{\partial y} + V_z\frac{\partial B_x}{\partial z} + B_x\frac{\partial V_z}{\partial z} - V_x\frac{\partial B_z}{\partial z} - B_z\frac{\partial V_x}{\partial z} = \rho g_x \end{cases}$$

(Three lines per equation)

$$\begin{cases} (\beta_1\rho g_x) + (\beta_2\rho g_x) + (\beta_3\rho g_x) + \rho(a g_x) + (b\rho g_x) + \rho(c g_x) - \eta(-\frac{d\rho g_x}{\eta}) - \eta(-\frac{f\rho g_x}{\eta}) - \eta(-\frac{m\rho g_x}{\eta}) + \\ (q\rho g_x) + (r\rho g_x) + (s\rho g_x) + \rho V_x(\frac{\omega_1 g_x}{V_x}) + \rho V_y(\frac{\omega_2 g_x}{V_y}) + \rho V_z(\frac{\omega_3 g_x}{V_z}) - \frac{1}{\mu}B_z(-\frac{\omega_4\mu\rho g_x}{B_z}) + \\ \frac{1}{\mu}B_z(\frac{\omega_5\mu\rho g_x}{B_z}) + \frac{1}{\mu}B_y(\frac{\omega_6\mu\rho g_x}{B_y}) - \frac{1}{\mu}B_y(-\frac{\lambda_1\mu\rho g_x}{B_y}) - V_x(-\frac{\lambda_2\rho g_x}{V_x}) - B_y(-\frac{\lambda_3\rho g_x}{B_y}) + V_y(\frac{\lambda_4\rho g_x}{V_y}) + \\ B_x(\frac{\lambda_5\rho g_x}{B_x}) + V_z(\frac{\lambda_6\rho g_x}{V_z}) + B_x(\frac{\lambda_7\rho g_x}{B_x}) - V_x(-\frac{\lambda_8\rho g_x}{V_x}) - B_z(-\frac{\lambda_9\rho g_x}{B_z}) \stackrel{?}{=} \rho g_x \end{cases}$$

(Four lines per equation)

$$\begin{cases} \beta_1\rho g_x + \beta_2\rho g_x + \beta_3\rho g_x + a\rho g_x + b\rho g_x + c\rho g_x + d\rho g_x + f\rho g_x + m\rho g_x q\rho g_x + r\rho g_x + s\rho g_x + \omega_1\rho g_x \\ +\omega_3\rho g_x + \omega_5\rho g_x + \omega_6\rho g_x + \lambda_1\mu\rho g_x + \lambda_2\rho g_x + \lambda_3\rho g_x + \lambda_4\rho g_x + \lambda_5\rho g_x + \omega_2\rho g_x + \omega_3\rho g_x \\ +\lambda_6\rho g_x + \lambda_7\rho g_x + \lambda_8\rho g_x + \lambda_9\rho g_x \stackrel{?}{=} \rho g_x \end{cases}$$

(Three lines per equation)

$$\begin{cases}\beta_1 g_x + \beta_2 g_x + \beta_3 g_x + a g_x + b g_x + c g_x + d g_x + f g_x + m g_x q g_x + r g_x + s g_x + \omega_1 g_x + \omega_3 g_x + \omega_5 g_x \\ +\omega_6 g_x + \lambda_1 g_x + \lambda_2 g_x + \lambda_3 g_x + \lambda_4 g_x + \lambda_5 g_x + \omega_2 g_x + \omega_3 g_x + \lambda_6 g_x + \lambda_7 g_x + \lambda_8 g_x + \lambda_9 g_x \stackrel{?}{=} g_x \quad \text{(2 lines)}\end{cases}$$

$$\begin{cases}g_x(\beta_1 + \beta_2 + \beta_3 + a + b + c + d + f + m + q + r + s + \omega_1 + \omega_3 + \omega_5 + \lambda_3 + \lambda_4 + \lambda_5 + \omega_2 + \omega_3 + \lambda_6 + \lambda_7 \\ +\omega_6 + \lambda_1 + \lambda_2 + \lambda_8 + \lambda_9) \stackrel{?}{=} g_x \qquad \text{(Two lines per equation)}\end{cases}$$

$$g_x(1) \stackrel{?}{=} g_x \qquad \text{(Sum of the ratio terms = 1)}$$

$$g_x \stackrel{?}{=} g_x \quad \text{Yes}$$

Since an identity is obtained, the solutions to the 28-term equation are as follows

$V_x(x,y,z,t) =$ (sum of integrals from sub - equations #1, #4,#13,#14,#15,#21,#27)

$$\beta_1 \rho g_x x + a g_x t \pm \sqrt{2\omega_1 g_x x} + \frac{\omega_2 g_x y}{V_y} - \frac{\lambda_3 \rho g_x y}{B_y} + \frac{\omega_3 g_x z}{V_z} - \frac{\lambda_9 \rho g_x z}{B_z} + \underbrace{\frac{\psi_z(V_z)}{V_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(B_z)}{B_z}}_{\text{arbitrary functions}} + C_1;$$

(integral from sub–equation #5)

$$P(x) = b\rho g_x x + C_2$$

(sum of integrals from sub–equations #2,#23)

$$V_y = \beta_2 \rho g_x y + \frac{\lambda_5 \rho g_x y}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_3$$

(sum of integrals from sub–equations #3, #25)

$$V_z = \beta_3 \rho g_x z + \frac{\lambda_7 \rho g_x z}{B_x} + \underbrace{\frac{\psi_x(B_x)}{B_x}}_{\text{arbitrary function}} + C_4$$

(sum of integrals from sub - equations #6, #7, #8, #9, #10, #16,#19, #22, #24)

$B_x(x,y,z,t) =$

$$B_x = -\frac{\rho g_x}{2\eta}(dx^2 + fy^2 + mz^2) + q\rho g_x x + C_2 x + C_4 y + C_6 z + + c g_x t - \frac{\lambda_1 \mu \rho g_x y}{B_y} + \frac{\lambda_4 \rho g_x y}{V_y} - \frac{\omega_4 \mu \rho g_x z}{B_z} +$$

$$\frac{\lambda_6 \rho g_x z}{V_z} + \underbrace{\frac{\psi_z(B_z)}{B_z} + \frac{\psi_y(B_y)}{B_y} + \frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}} + C_7$$

(sum of integrals from sub–equations #11,#18,#20)

$$B_y = r\rho g_x y \pm \sqrt{2\omega_6 \mu \rho g_x x} - \frac{\lambda_2 \rho g_x y}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_8$$

(sum of integrals from sub–equations #12,#17,#26)

$$B_z = s\rho g_x z \pm \sqrt{2\omega_5 \mu \rho g_x x} - \frac{\lambda_8 \rho g_x z}{V_x} + \underbrace{\frac{\psi_x(V_x)}{V_x}}_{\text{arbitrary function}} + C_{21}$$

Step 9: The linear part of the relation satisfies the linear part of the equation (in Step 8; and the non-linear part of the relation satisfies the non-linear part of the equati. The solutions are above.

Analogy for the Identity Checking Method: If one goes shopping with American dollars and Japanese yens (without any currency conversion) and after shopping, if one wants to check the cost of the items purchased, one would check the cost of the items purchased with dollars against the receipts for the dollars; and one would also check the cost of the items purchased with yens against the receipts for the yens purchase. However, if one converts one currency to the other, one would only have to check the receipts for only a single currency, dollars or yens. This conversion case is similar to the linearized N-S equations, where there was no partitioning in identity checking.

Important insight

One observes above that the most important insight of the above solutions is the indispensability of the gravity term in MHD.. Observe that if gravity, g_x, were zero, all the non-constant terms in each solution would be zero. These results can be stated emphatically that without gravity forces on earth, there would be no magnetohydrodynamics on earth as is known. It would not therefore be meaningful to write a system of MHD equations without the gravity term, since there would be no magnetohydrodynmics.

Supporter Equation Contributions (see also viXra:1405.0251)

Note above that there are 28 terms in the driver equation, and 27 supporter equations, Each supporter equation provides useful information about the driver equation. The more of these supporter equations that are integrated, the more the information one obtains about the driver equation. However, without solving a supporter equation, one can sometimes write down some characteristics of the integration relation of the supporter equation by referring to the subjects of the supporter equations of the Navier-Stokes equations. For example, if one uses $(\eta\partial^2 B_x/\partial x^2)$ as the subject of a supporter equation here, the curve for the integration relation obtained would be parabolic, periodic, and decreasingly exponential. Using $\rho(\partial V/\partial t)$ as the subject of the supporter equation, the curve would be periodic and decreasingly exponential. Using $(\partial p/\partial x)$, the curve would be parabolic.

Comparison of Solutions of Navier-Stokes Equations
and
Solutions of Magnetohydrodynamic Equations

Navier-Stokes x–direction **solution**

$$V_{x(x,y,z,t)} = -\frac{\rho g_x}{2\mu}(ax^2+by^2+cz^2) + C_1x + C_3y + C_5z + fg \pm \sqrt{2hgx} + \frac{ngy}{V_y} + \frac{qgz}{V_z} + \underbrace{\frac{\psi_y(V_y)}{V_y} + \frac{\psi_z(V_z)}{V_z}}_{\text{arbitrary functions}}$$

$$P(x) = d\rho g_x x \qquad (V_y \neq 0,\ V_z \neq 0)$$

For magnetohydrodynamic solutions, see previous page

1. V_x for MHD system resembles the V_x for the Euler solution part of N-S solution.
2. $P(x)$) for N-S and MHD equations are the same.
3. V_y and V_z for MHD are different from those of N-S solution.
4. B_x is parabolic and resembles V_x for N-S, except for the absence of the square root function.
5. B_y and B_z resemble the Euler solution part of the N-S solution.

Conclusion

The author proposed and applied a new law to solve the system of magnetohydrodynamic equations. This law states that in magnetohydrodynamics, all the other terms in the system of equations divide the gravity term in a definite ratio, and each term utilizes gravity to function. The experience gained in solving the Navier-Stokes equations guided the author to solve the MHD equations.
It was shown that without gravity forces on earth, there would be no magnetohydrodynamics on earth as is known. The equations in the system of equations were added to produce a single equation which was then integrated. Ratios were used to split-up the single equation, and the resulting sub-equations were readily integrated; and even, the nonlinear sub-equations were readily integrated. Twenty-seven sub-equations were integrated. The linear part of the relation obtained from the integration of the linear part of the equation satisfied the linear part of the equation; and the relation from the integration of the non-linear part satisfied the non-linear part of the equation. Comparison of the solutions of MHD equations with the solutions of the N-S equations revealed the following: (a) V_x for MHD system resembles the V_x for the non-linear part of the N-S solution; (b) $P(x)$) for N-S and MHD equations are the same; (c) V_y and V_z for MHD are different from those of N-S solutions; (d) B_x is parabolic and resembles V_x for N-S solution, except for the absence of the square root function; and (e) B_y and B_z resemble the non-linear part of N-S solution.
By solving algebraically and simultaneously for V_x, V_y, V_z, B_x, B_y, B,, the solutions could be expressed in term of x,y,z and t.
In applications, the ratio terms may perhaps be determined using information such as initial and boundary conditions or may have to be determined experimentally. Finally, for any magnetohydrodynamic design, one should always maximize the role of gravity for cost-effectiveness, durability, and dependability. Perhaps, a law for magnetohydrodynamics should read "Sum of everything else equals ρg" ; and this would imply the proposed new law that the other terms in the system of equations divide the gravity term in a definite ratio, and each term utilizes gravity to function.

Note: The liquid pressure, P at the bottom of a liquid of depth h units is given by $\boxed{P = \rho gh}$.
From the MHD solutions in this paper, $\boxed{P(x) = b\rho gx}$ from integrating $\frac{dp}{dx} = b\rho g$ where b is ratio term. Each of the other terms in the MHD equation must also be set equal to the product of a ratio term and ρg. This result implies that the approach used in solving the MHD equations is sound.

P.S.
The author spent more time on " vector juggling" than on the integration of the equations, since no complete system without vector notation was available either in textbooks or on-line. The integration took less time because of the experience with the N-S equations. Any error in the vector juggling part, if any, can be integrated within minutes.

References:
For paper edition of the above paper, see Appendix 8 of the book entitled "Power of Ratios" by A. A. Frempong, published by Yellowtextbooks.com. Since ratios were the key to splitting the combined 28-term MHD equation into sub-equations and solving them, the solutions have also been published in the " Power of Ratios" book which covers definition of ratio and applications of ratio in mathematics, science, engineering, economics and business fields.