Curiosity about Henry Dudeney's Curious Numbers

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Abstract:

To find Henry Dudeney's Curious Numbers using pattern.

Introduction:

In "Amusements in Mathematics", Henry Ernest Dudeney [1] observes the following curiosity:

Curious Numbers

The number 48 has this peculiarity, that if you add 1 to it the result is a square number (49, the square of 7), and if you add 1 to its half, you also get a square number (25, the square of 5). Now, there is no limit to the numbers that have this peculiarity, and it is an interesting puzzle to find three more of them—the smallest possible numbers. What are they?

Solution:

The three smallest numbers, in addition to 48, are 1,680, 57,120, and 1,940,448. It will be found that 1,681 and 841, 57,121 and 28,561, 1,940,449 and 970,225, are respectively the squares of 41 and 29, 239 and 169, 1,393 and 985.

48, 1680, 57120, 1940448, 65918160, 2239277040, 76069501248, 2584123765440, 87784138523760, 2982076586042448, 101302819786919520, 3441313796169221280, ..

Pattern

Really these are very curious numbers and we can find numbers also by following pattern

1st Number is 48

$(\frac{48}{2}+1)=25=5^2$ and $48+1=49=7^2$

With 1st number 48, we got squares of 5 and 7.

2nd Number is 1680

$(\frac{1680}{2}+1)=841=29^2$ and $1680+1=1681=41^2$

With 2nd number 1680, we got squares of 29 and 41

5x3+7x2=29 and 5x4+7x3=41

3rd Number is 57120

$(\frac{57120}{2}+1)=169^2$ and $57120+1=239^2$

With 3rd number we got squares of 169 and 239

3x29+2x41=169 and 4x29+3x41=239

4th Number is 1940448

$(\frac{1940448}{2}+1)=985^2$ and $1940448+1=1393^2$

With 4th number we got squares of 985 and 1393

3x169+2x239=985 and 4x169+3x239=1393

Similar can be used for next numbers.

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References:

[1] Gutenberg Project. Dudeney, H.E., Amusements in Mathematics, 1917.

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