Using formula for searching a prime number in the interval[p_m, p_{m+1}^2]

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Abstract.There is now no a method for searching a prime number in the interval [p_m, p_{m+1}^2] by using formula, and there is no known useful formula that sets apart all prime numbers from composites. In this paper, we try to use a formula for searching a prime number in the interval [p_m, p_{m+1}^2], if a complete list of prime numbers up to p_m is known,and we also give an open problem on this formula.

1.Introduce

We have known the famous Euclid's proof by formula $N = 2.3.5...p_{m-1}.p_m + 1$, the primality test by using formula $N = n! \pm 1$. Assume $p_m \leqslant n < p_{m+1}$, then N is not divisible by all prime numbers $\leqslant p_m$, but they require more test, since $\sqrt{N} > p_{m+1}$ if $p_m \geq 7$ in these formulas, and they do not give a value which is belong to the interval [p_m, p_{m+1}^2] .

2.Using formula for searching a prime number in the interval [p_m, p_{m+1}^2]

Let be given prime numbers from 2 to p_m, we divide these prime numbers into two groups, the first group contains prime numbers $p_1, p_2, ..., p_k$, we make the first product: $p_1^{\alpha_1}.p_2^{\alpha_2}...p_k^{\alpha_k}$, the second group contains remaining prime numbers $q_1, q_2, ..., q_h$, and we make the second product: $q_1^{\beta_1}.q_2^{\beta_2}...q_h^{\beta_h}$.

Here: α_i, β_j are powers, max $\{p_k, q_h\} = p_m$

Then make absolute value of difference of two products:

$$N = |p_1^{\alpha_1}.p_2^{\alpha_2}...p_k^{\alpha_k} - q_1^{\beta_1}.q_2^{\beta_2}...q_h^{\beta_h}|$$

N is not divisible by any prime numbers from 2 to p_m, and N can be following values:
a. $N = 1$.
b. $p_m < N < p_{m+1}^2$, then N is a prime number, since N is not divisible by any prime number $\leq \sqrt{N}$.
c. $N \geq p_{m+1}^2$, then N is a prime number or a composite of two or more prime factors, each of them is equal or lager than p_{m+1}.

If p_{m+1} is unknown, since $p_{m+1} \geq p_m + 2$, so if $N < (p_m + 2)^2$, then N is certain a prime number.

Example 1: Given list of prime numbers up to $p_k = 7$

Apply above formula, we obtain some prime numbers : $7 < N < 11^2$ as follows:

$$
\begin{aligned}
N_1 &= |3.5.7 - 2^7| = 23 \\
N_2 &= |3.5.7 - 2^6| = 41 \\
N_3 &= |3.5.7 - 2^5| = 73 \\
N_4 &= |3.5.7 - 2^4| = 89 \\
N_5 &= |3.5.7 - 2^3| = 97 \\
N_6 &= |3.5.7 - 2^2| = 101 \\
N_7 &= |3.5.7 - 2| = 103 \\
N_8 &= |3.7 - 2.5| = 11 \\
N_9 &= |3^2.7 - 2.5^2| = 13 \\
N_{10} &= |3^2.7 - 2.5| = 53 \\
N_{11} &= |3^2.7 - 2^2.5^2| = 37
\end{aligned}
$$

Example 2: $p_k = 11, p_{k+1} = 13$:
We obtain some prime numbers: $11 < N < 13^2$

$$
\begin{aligned}
N_1 &= |2.5.7 - 3.11| = 37 \\
N_2 &= |2^2.5.7 - 3.11| = 107 \\
N_3 &= |2.7.11 - 3.5| = 139
\end{aligned}
$$

As Euclid's proof, this formula is the same way to prove that set of prime numbers is infinite.

3.Open problem on formula $N = |p_1^{\alpha_1}.p_2^{\alpha_2}...p_k^{\alpha_k} - q_1^{\beta_1}.q_2^{\beta_2}...q_h^{\beta_h}|$

Consider all prime numbers in the interval $[7, 11^2]$, and express them by above formula:

$$11 = 3.7 - 2.5$$
$$13 = 3^2.7 - 2.5^2$$
$$17 = 5.7 - 2.3^2$$
$$19 = 5.2^3 - 3.7$$
$$23 = 2^7 - 3.5.7$$
$$29 = 5.7 - 2.3$$
$$31 = 5^2.7 - 2^4.3^2$$
$$37 = 2.3.7 - 5$$
$$41 = 2^3.7 - 3.5$$
$$43 = 3^2.7 - 2^2.5$$
$$47 = 3.7^2 - 2^2.5^2$$
$$53 = 2^2.3.5 - 7$$
$$59 = 2^2.3.7 - 5^2$$
$$61 = 2.5.7 - 3^2$$
$$67 = 2^4.7 - 3^2.5$$
$$71 = 2^3.3.5 - 7^2$$
$$73 = 3.5.7 - 2^5$$
$$79 = 2^2.3.7 - 5$$
$$83 = 2.7^2 - 3.5$$
$$89 = 3.5.7 - 2^4$$
$$97 = 2^4.7 - 3.5$$
$$101 = 2.3^2.7 - 5^2$$
$$103 = 3.5.7 - 2$$
$$107 = 3^3.5 - 2^2.7$$
$$109 = 3^3.7 - 2^4.5$$
$$113 = 2^3.3.5 - 7$$

Open problem: Does above formula give all prime numbers in the interval[p_m, p_{m+1}^2]. In other words, can any prime numbers be expressed by this formula.

Reference

- Prime number- Wikipedia

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