Notable observation on the squares of primes of the form 10k+9

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Abstract. In this paper I conjecture that for any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime and, in case that this conjecture turns out not to be true, I considered three related “weaker” conjectures.

Conjecture:

For any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime.

Verifying the conjecture:
(for the first twenty primes p with the property mentioned)

: p = 7 and p^2 = 49; (p^2 – 5)/4 = 11, prime, so [q, r] = [5, 11];

: p = 13 and p^2 = 169; (p^2 – 5)/4 = 41, prime, so [q, r] = [5, 41];

: p = 17 and p^2 = 289; (p^2 – 7)/6 = 47, prime, so [q, r] = [7, 47];

: p = 23 and p^2 = 529; (p^2 – 5)/4 = 131, prime, so [q, r] = [5, 131]; also (p^2 – 13)/12 = 43, prime, so [q, r] = [13, 43];

: p = 37 and p^2 = 1369; (p^2 – 7)/6 = 227, prime, so [q, r] = [7, 227]; also (p^2 – 13)/12 = 113, prime, so [q, r] = [13, 113];

: p = 43 and p^2 = 1849; (p^2 – 5)/4 = 461, prime, so [q, r] = [5, 461]; also (p^2 – 7)/6 = 307, prime, so [q, r] = [7, 307]; also (p^2 – 23)/22 = 83, prime, so [q, r] = [23, 83];

: p = 47 and p^2 = 2209; (p^2 - 7)/6 = 367, prime, so [q, r] = [7, 367]; also (p^2 - 17)/16 = 137, prime, so [q, r] = [17, 137];

: p = 53 and p^2 = 2809; (p^2 - 5)/4 = 701, prime, so [q, r] = [5, 701]; also (p^2 - 7)/6 = 467, prime, so [q, r] = [7, 467]; also (p^2 - 13)/12 = 233, prime, so [q, r] = [13, 233];

: p = 67 and p^2 = 4489; (p^2 - 13)/12 = 373, prime, so [q, r] = [13, 373];

: p = 73 and p^2 = 5329; (p^2 - 7)/6 = 887, prime, so [q, r] = [7, 887]; also (p^2 - 13)/12 = 443, prime, so [q, r] = [13, 443];

: p = 83 and p^2 = 6889; (p^2 - 5)/4 = 1721, prime, so [q, r] = [5, 1721]; also (p^2 - 43)/42 = 163, prime, so [q, r] = [43, 163];

: p = 97 and p^2 = 9409; (p^2 - 5)/4 = 2351, prime, so [q, r] = [5, 2351]; also (p^2 - 7)/6 = 1567, prime, so [q, r] = [7, 1567]; also (p^2 - 17)/16 = 587, prime, so [q, r] = [17, 587]; also (p^2 - 43)/42 = 223, prime, so [q, r] = [43, 223];

: p = 103 and p^2 = 10609; (p^2 - 13)/12 = 883, prime, so [q, r] = [13, 883];

: p = 107 and p^2 = 11449; (p^2 - 5)/4 = 2861, prime, so [q, r] = [5, 2861]; also (p^2 - 7)/6 = 1907, prime, so [q, r] = [7, 1907]; also (p^2 - 13)/12 = 953, prime, so [q, r] = [13, 953]; also (p^2 - 37)/36 = 317, prime, so [q, r] = [37, 317];

: p = 113 and p^2 = 12769; (p^2 - 5)/4 = 3191, prime, so [q, r] = [5, 3191]; also (p^2 - 13)/12 = 1063, prime, so [q, r] = [13, 1063]; also (p^2 - 17)/16 = 797, prime, so [q, r] = [17, 797];

: p = 127 and p^2 = 16129; (p^2 - 7)/6 = 2687, prime, so [q, r] = [7, 2687]; also (p^2 - 43)/42 = 383, prime, so [q, r] = [43, 383]; also (p^2 - 73)/72 = 223, prime, so [q, r] = [73, 223]; also (p^2 - 97)/96 = 167, prime, so [q, r] = [97, 167];

: p = 137 and p^2 = 18769; (p^2 - 5)/4 = 4691, prime, so [q, r] = [5, 4691]

: p = 157 and p^2 = 24649; (p^2 - 13)/12 = 2053, prime, so [q, r] = [13, 2053];

: p = 163 and p^2 = 26569; (p^2 – 13)/12 = 2213, prime, so [q, r] = [13, 2213];

: p = 167 and p^2 = 27889; (p^2 – 5)/4 = 6971, prime, so [q, r] = [5, 6971].

Note:

In case that the conjecture above turns out not to be true there are three "weaker" conjectures that may be considered:

(i) For any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime or a power of prime.

Example:
: p = 73, p^2 = 5329, (p^2 – 5)/4 = 11^3.

(ii) For any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/(q – 1) is prime or semiprime m*n, n > m, with the property that n – m + 1 is prime or power of prime or n + m – 1 is prime or power of prime.

Examples:
: p = 67, p^2 = 4489, (p^2 – 5)/4 = 19*59 and 59 – 19 + 1 = 41;
: p = 53, p^2 = 2809, (p^2 – 19)/18 = 5*31 and 31 – 5 + 1 = 3^3;
: p = 127, p^2 = 16129, (p^2 – 113)/112 = 11*13 and 13 + 11 - 1 = 23.

(iii) For any square of prime of the form p^2 = 10k + 9, p greater than or equal to 7, is true that there exist at least one prime q, q lesser than p, such that r = (p^2 – q)/((q – 1)*2^n)) is prime.

Examples:
: p = 113, p^2 = 11449, (p^2 – 73)/(72*2) = 79;
: p = 137, p^2 = 18769, (p^2 – 17)/(16*2^2) = 293;
: p = 167, p^2 = 27889, (p^2 – 113)/(112*2^3) = 31.