Differential equation for 2nd-order curves.

Viktor Strohm
vfnv@mail.ru

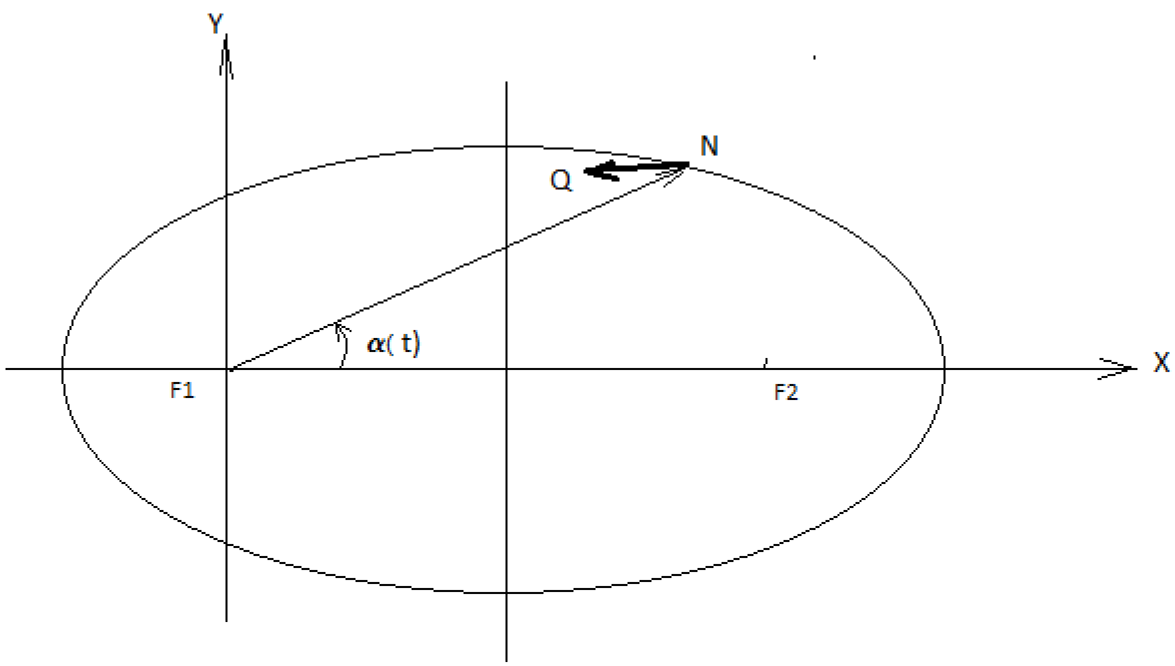
Abstract: In this article we consider a differential equation for 2nd-order curve of the motion of a parametric pendulum in zero gravity.

Introduction

"To solve the mass point motion problem we need differential equations for the motion. The way we derive these equations doesn't matter": [1,§11,п.3]. In this article we will derive a differential equation for the mass point motion in the case of zero gravity and movement on the ellipse under external force.

Let us place the point into zero gravity space.

Some external force makes this point to move along a 2nd-order curve. Let this curve be an ellipse and the point moves around a left center.



Pic.1 The mass point movement around the left point.

N - pendulum.

Q – the force acting on the pendulum.

F1- left center.

F2 - right center.

a(t) - angle between X axis and the line connecting left center and the point.

Let us place the left center into the origin of coordinates.

$$r = \frac{p}{1-e\ \cdot\cos\alpha}\, r = \frac{p}{1-e\ \cdot\cos\alpha} \qquad (1)$$

$$p = \frac{B^2}{A} p = \frac{B^2}{A} \qquad (2)$$

(*A, B* – semi-major and semi-minor axis)
r - radius.

e - eccentricity

$$m\ddot{x} = -Qcos(a(t)) m\ddot{x} = -Qcos(a(t)) \qquad (3)$$

$$m\ddot{y} = -Qsin(a(t))\, m\ddot{y} = -Qsin(a(t)) \qquad (4)$$

From (3) we can get

$$Q = \frac{-m\ddot{x}}{cos(a(t))} Q = \frac{-m\ddot{x}}{cos(a(t))} \qquad (5)$$

Let us substitute (5) into (4)

$$\ddot{y} = \frac{\ddot{x}}{cos(a(t))} sin(a(t)) \ddot{y} = \frac{\ddot{x}}{cos(a(t))} sin(a(t)) \qquad (6)$$

The point coordinates can be represented as the function of angle of deflection *α(t)* and radius *r(t)*.

$$x = r(a(t))\cdot cos(a(t)) x = r(a(t))\cdot cos(a(t)) \qquad (7)$$

$$y = r(a(t))\cdot sin(a(t)) y = r(a(t))\cdot sin(a(t)) \qquad (8)$$

Let us calculate the first and second time derivative.

$$\dot{x} = \frac{d}{dt}\left(\frac{p}{1-e*cos(a(t))}\right) cos(a(t)) = \dot{x} = \frac{d}{dt}\left(\frac{p}{1-e*cos(a(t))}\right) cos(a(t)) =$$

$$-\frac{p*cos(a(t))*e*sin(a(t))*\frac{d}{dt}a(t)}{(1-e*cos(a(t)))^2} - \frac{p*sin(a(t))*\frac{d}{dt}a(t)}{1-e*cos(a(t))}$$

$$-\frac{p*cos(a(t))*e*sin(a(t))*\frac{d}{dt}a(t)}{(1-e*cos(a(t)))^2} - \frac{p*sin(a(t))*\frac{d}{dt}a(t)}{1-e*cos(a(t))} \qquad (9)$$

$$\dot{y} = \frac{d}{dt}\left(\frac{p}{1-e*cos(a(t))}\right)*sin(a(t)) = \dot{y} = \frac{d}{dt}\left(\frac{p}{1-e*cos(a(t))}\right)*sin(a(t)) = -\frac{p*e*sin(a(t))^2*\frac{d}{dt}a(t)}{\left(1-e*cos(a(t))\right)^2}+\frac{p*cos(a(t))*\frac{d}{dt}a(t)}{1-e*cos(a(t))}-\frac{p*e*sin(a(t))^2*\frac{d}{dt}a(t)}{\left(1-e*cos(a(t))\right)^2}+\frac{p*cos(a(t))*\frac{d}{dt}a(t)}{1-e*cos(a(t))}$$

$$\ddot{x} = \frac{2*p*e^2*cos(a(t))*sin(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{\left(1-e*cos(a(t))\right)^3}+\frac{2*p*e*sin(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{\left(1-e*cos(a(t))\right)^2}-\frac{p*e*cos(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{1-e*cos(a(t))}-\frac{p*sin(a(t))*\frac{d^2}{dt^2}a(t)}{1-e*cos(a(t))}$$

$$\ddot{x} = \frac{2*p*e^2*cos(a(t))*sin(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{\left(1-e*cos(a(t))\right)^3}+\frac{2*p*e*sin(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{\left(1-e*cos(a(t))\right)^2}-\frac{p*e*cos(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{1-e*cos(a(t))}-\frac{p*sin(a(t))*\frac{d^2}{dt^2}a(t)}{1-e*cos(a(t))}$$

(11)

$$\ddot{y} = \frac{2*p*e^2*sin(a(t))^3*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*cos(a(t)))^3}-\frac{3*p*e*sin(a(t))*cos(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*cos(a(t)))^2}-\frac{p*e*sin(a(t))^2*\frac{d^2}{dt^2}a(t)}{(1-e*cos(a(t)))^2}-\frac{p*sin(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{1-e*cos(a(t))}+\frac{p*cos(a(t))*\frac{d^2}{dt^2}a(t)}{1-e*cos(a(t))}$$

$$\ddot{y} = \frac{2*p*e^2*sin(a(t))^3*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*cos(a(t)))^3}-\frac{3*p*e*sin(a(t))*cos(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*cos(a(t)))^2}-\frac{p*e*sin(a(t))^2*\frac{d^2}{dt^2}a(t)}{(1-e*cos(a(t)))^2}-\frac{p*sin(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{1-e*cos(a(t))}+\frac{p*cos(a(t))*\frac{d^2}{dt^2}a(t)}{1-e*cos(a(t))}$$

(12)

Let us substitute (11) and (12) into (6)

$$\frac{2*p*e^2*\sin(a(t))^3*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*\cos(a(t)))^3} - \frac{3*p*e*\sin(a(t))*\cos(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*\cos(a(t)))^2} - \frac{p*e*\sin(a(t))^2*\frac{d^2}{dt^2}a(t)}{(1-e*\cos(a(t)))^2} - \frac{p*\sin(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{1-e*\cos(a(t))} + \frac{p*\cos(a(t))*\frac{d^2}{dt^2}a(t)}{1-e*\cos(a(t))} - \frac{\sin(a(t))}{\cos(a(t))}\left(\frac{2*p*e^2*\cos(a(t))*\sin(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*\cos(a(t)))^3} + \frac{2*p*e*\sin(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*\cos(a(t)))^2} - \frac{p*e*\cos(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{1-e*\cos(a(t))} - \frac{p*\sin(a(t))*\frac{d^2}{dt^2}a(t)}{1-e*\cos(a(t))}\right) = 0$$

$$\frac{2*p*e^2*\sin(a(t))^3*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*\cos(a(t)))^3} - \frac{3*p*e*\sin(a(t))*\cos(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*\cos(a(t)))^2} - \frac{p*e*\sin(a(t))^2*\frac{d^2}{dt^2}a(t)}{(1-e*\cos(a(t)))^2} - \frac{p*\sin(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{1-e*\cos(a(t))} + \frac{p*\cos(a(t))*\frac{d^2}{dt^2}a(t)}{1-e*\cos(a(t))} - \frac{\sin(a(t))}{\cos(a(t))}\left(\frac{2*p*e^2*\cos(a(t))*\sin(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*\cos(a(t)))^3} + \frac{2*p*e*\sin(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{(1-e*\cos(a(t)))^2} - \frac{p*e*\cos(a(t))^2*\left(\frac{d}{dt}a(t)\right)^2}{1-e*\cos(a(t))} - \frac{p*\sin(a(t))*\frac{d^2}{dt^2}a(t)}{1-e*\cos(a(t))}\right) = 0$$

$$\frac{(e*\cos(a(t))-1)\frac{d^2}{dt^2}a(t)+2*e*\sin(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{\cos(a(t))*(1-e*\cos(a(t)))^2}\mathrm{p} = 0$$

$$\frac{(e*\cos(a(t))-1)\frac{d^2}{dt^2}a(t)+2*e*\sin(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{\cos(a(t))*(1-e*\cos(a(t)))^2}\mathrm{p} = 0$$

(13)

$$\frac{d^2}{dt^2}a(t) = \frac{2*e*\sin(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{1-e*\cos(a(t))}$$ $$\frac{d^2}{dt^2}a(t) = \frac{2*e*\sin(a(t))*\left(\frac{d}{dt}a(t)\right)^2}{1-e*\cos(a(t))}$$ (14)

(14) is the differential equation of the 2nd-order curve.

Different values of the eccentricity will lead into a different shape of the curve.

Conclusion

The constant sectoral velocity is one of equation properties. This property allows us to model orbits using Kepler's laws.

This equation makes it possible to calculate a track taking a precession into consideration .

Some video samples can be found here

http://www.fayloobmennik.net/4823487

Some executables samples can be found http://www.fayloobmennik.net/4909818

In paper „ The Algorithm Simulation of the Orbits of Objects Differential Equation Second-Order Curves " consider the algorithm for calculation of motion of bodies, using the coherent graph. The graph is built through solving a differential equation of second order curves , using theorem of the center of mass and transition from a consecutive number of relative coordinates to absolute system of coordinates.
Article available at http://vixra.org/abs/1512.0011

List of reference

1. Sivukhin D. V. General course of physics. Studies. grant: For higher education institutions. In 5 t. T. I. Mechanics. — 4 prod., — M.: FIZMATLIT; Publishing house of MIPT, 2005. - ISBN 5-9221-0225-7; 5-89155-078-4.