

# Performance Evaluation of Fusion Rules For Multitarget Tracking in Clutter Based on Generalized Data Association\*

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**Abstract** – *In this paper, we present and compare different fusion rules which can be used for Generalized Data Association (GDA) for multitarget tracking (MTT) in clutter. Most of tracking methods including Target Identification (ID) or attribute information are based on classical tracking algorithms as PDAF, JPDAF, MHT, IMM, etc and either on the Bayesian estimation and prediction of target ID, or on fusion of target class belief assignments through the Dempster-Shafer Theory (DST) and Dempster's rule of combination. In this paper we pursue our previous works on the development of a new GDA-MTT based on Dezert-Smarandache Theory (DSmT) but compare also it with standard fusion rules (Dempster's, Dubois & Prade's, Yager's) and with a new fusion Proportional Conflict Redistribution (PCR) rule in order to assess the efficiency of all these different fusion rules for this GDA-MTT in highly conflicting situation. This evaluation is based on a Monte Carlo simulation for a difficult maneuvering MTT problem in clutter similar to the example recently proposed by Bar-Shalom, Kirubarajan and Gokberk.*

**Keywords:** Multitarget Tracking, Generalized Data Association, Dezert-Smarandache Theory (DSmT), Attribute and Kinematics fusion, Data fusion, Combination rules, Conflict management.

## 1 Introduction

The idea of incorporating Target Identification (ID) information or target attribute measurements to improve MTT systems is not new and many approaches have been proposed in the literature over the last fifteen years. For example, in [14, 15, 20] an improved PDAF (Probabilistic Data Association Filter) had been developed for autonomous navigation systems based on Target Class ID and ID Confusion matrix, and also on another version based on imprecise attribute measurements combined within Dempster's rule. At the same time Lerro in [19] developed the AI-PDAF (Amplitude Information PDAF). Since the nineties many improved versions of classical tracking algorithms like IMM, JPDAF, IMM-PDAF, MHT, etc including attribute information have been proposed (see

[12] and [6] for a recent overview). Recent contributions have been done by Blasch and al. in [7, 8, 9, 10, 28] for Group Target Tracking and classification. In last two years efforts have been done also by Hwang and al. in [16, 17, 18]. We recently discovered that the Hwang's MTIM (Multiple-target Tracking and Identity Management) algorithm is very close to our GDA-MTT. The difference between MTIM and GDA-MTT lies fundamentally in the Attribute Data Association procedure. MTIM is based on MAJPDA (Modified Approximated JPDA) coupled with RMIMM (Residual-mean Interacting Multiple Model) algorithm while the GDA-MTT is based on GNN (Global Nearest Neighbour) approach for data association incorporating both kinematics and attribute measurements (with more sophisticated fusion rules dealing with fuzzy, imprecise and potentially highly conflicting target attribute measurements), coupled with standard IMM-EKF [1]. The last recent attempt for solving the GDA-MTT problem was proposed by Bar-Shalom and al. in [6] and expressed as a multiframe assignment problem where the multiframe association likelihood was developed to include the target classification results based on the confusion matrix that specifies the prior accuracy of the target classifier. Such multiframe s-D assignment algorithm should theoretically provide performances close to the optimality for MTT systems but remains computationally greedy. The purpose of this paper is to compare the performances of several fusion rules usable into our new GDA-MTT algorithm based on a MTT scenario similar to the one given in [6] but actually more difficult since seven closed spaced maneuvering targets are considered belonging only to two classes within clutter and with only 2D kinematic measurements and attribute measurement.

This paper is organized as follows. In section 2 we present our approach for GDA-MTT algorithm emphasizing only on the new developments in comparison with our previous GDA-MTT algorithm, developed in [27, 24]. In our previous works, we proved the efficiency of GDA-MTT (in term of Track Purity Performance) based on DSm

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Hybrid rule of combination over the GDA-MTT based on Dempster's rule but also over the KDA-MTT (Kinematics-only-based Data Association) trackers on simple two targets scenarios (with and without clutter). In section 3 we remind the main fusion rules we investigate for our new GDA-MTT algorithm. Most of these rules are well-known in the literature [24, 22], but the PCR5 rule presented here is really a new one recently proposed in [25, 26]. Due to space limitations, we assume the reader familiar with basics on Target Tracking [2, 3, 4, 5, 11, 12], on DST [23] and on DSMT [24] for fusion of uncertain, imprecise and possibly highly conflicting information. Section 4 presents and compares several Monte Carlo results for different versions of our GDA-MTT algorithm based on the fusion rules proposed in section 3 for a MTT scenario similar to the one in [6]. Conclusion is given in section 5.

## 2 General principle of GDA-MTT

Classical target tracking algorithms consist mainly in two basic steps: *data association* to associate proper measurements (usually kinematics measurement  $\mathbf{z}(k)$  representing either position, distance, angle, velocity, acceleration, etc) with correct targets and *track filtering* to estimate and predict state of targets once data association has been performed. The first step is very important for the quality of tracking performances since its goal is to associate correctly (or at least as best as possible) observations to existing tracks (or eventually new born targets). The data association problem is very difficult to solve in dense multitarget and cluttered environment. To eliminate unlikely (kinematics-based) observation-to-track pairing, the classical validation test is carried on the Mahalanobis distance  $d^2(i, j) \triangleq (\mathbf{z}_j(k) - \hat{\mathbf{z}}_i(k|k-1))' \mathbf{S}^{-1}(k) (\mathbf{z}_j(k) - \hat{\mathbf{z}}_i(k|k-1)) \leq \gamma$  computed from the measurement  $\mathbf{z}_j(k)$  and its prediction  $\hat{\mathbf{z}}_i(k|k-1)$  computed by the tracker of target  $i$  (see [2] for details). Once all the validated measurements have been defined for the surveillance region, a clustering procedure defines the clusters of the tracks with shared observations. Further the decision about observation-to-track associations within the given cluster is considered. The Extended Kalman Filter coupled with a classical IMM (Interacting Multiple Model) for maneuvering target tracking is used to update the targets state vectors.

This new GDA-MTT improves data association process by adding attribute measurements (like amplitude information or RCS (radar cross section), or eventually as in [6] Target ID decision coupled with confusion matrix, to classical kinematic measurements to increase the performances of the MTT system. When attribute data are available, the generalized (kinematics and attribute) likelihood ratios are used to improve the assignment. The GNN approach is used in order to make a decision for data association. Our new GDA approach consists in choosing a set of assignments  $\{\chi_{ij}\}$ , for  $i = 1, \dots, n$  and  $j = 1, \dots, m$ , that assures maximum of the total generalized likelihood ratio sum by solving the classical assignment problem  $\min \sum_{i=1}^n \sum_{j=1}^m a_{ij} \chi_{ij}$  using extended Munkres

algorithm [13] and where  $a_{ij} = -\log(LR_{gen}(i, j))$  with  $LR_{gen}(i, j) = LR_k(i, j)LR_a(i, j)$ , where  $LR_k(i, j)$  and  $LR_a(i, j)$  are kinematics and attribute likelihood ratios respectively, and

$$\chi_{ij} = \begin{cases} 1 & \text{if measurement } j \text{ is assigned to track } i \\ 0 & \text{otherwise} \end{cases}$$

where the elements  $a_{ij}$  of the assignment matrix  $\mathbf{A} = [a_{ij}]$  take the following values [21]:

$$a_{ij} = \begin{cases} \infty & \text{if } d_{ij}^2 > \gamma \\ -\log(LR_k(i, j)LR_a(i, j)) & \text{if } d_{ij}^2 \leq \gamma \end{cases}$$

The solution of the assignment matrix is the one that minimizes the sum of the chosen elements. We solve the assignment problem by realizing the extension of Munkres algorithm, given in [13]. As a result one obtains the optimal measurements to tracks association. Once the optimal assignment is found, i.e. the (what we feel) correct association is available, then standard tracking filter is used depending on the dynamics of the target under tracking. We will not recall classical tracking filtering methods here which can be found in many standard textbooks [5, 12].

### 2.1 Kinematics Likelihood Ratios for GDA

The kinematics likelihood ratios  $LR_k(i, j)$  involved into  $a_{ij}$  are quite easy to obtain because they are based on classical statistical models for spatial distribution of false alarms and for correct measurements [5].  $LR_k(i, j)$  is evaluated as  $LR_k(i, j) = LF_{true}(i, j)/LF_{false}$  where  $LF_{true}$  is the likelihood function that the measurement  $j$  originated from target (track)  $i$  and  $LF_{false}$  the likelihood function that the measurement  $j$  originated from false alarm. At any given time  $k$ ,  $LF_{true}$  is defined<sup>1</sup> as  $LF_{true} = \sum_{l=1}^r \mu_l(k) LF_l(k)$  where  $r$  is the number of the models (in our case of two nested models  $r = 2$  is used for EKF-IMM),  $\mu_l(k)$  is the probability (weight) of the model  $l$  for the scan  $k$ ,  $LF_l(k)$  is the likelihood function that the measurement  $j$  is originated from target (track)  $i$  according to the model  $l$ , i.e.  $LF_l(k) = \frac{1}{2\pi\sqrt{|\mathbf{S}_i^l(k)|}} e^{-d_i^2(i, j)/2}$ .  $LF_{false}$  is defined as  $LF_{false} = P_{fa}/V_c$ , where  $P_{fa}$  is the false alarm probability and  $V_c$  is the resolution cell volume chosen as in [6] as  $V_c = \prod_{i=1}^{n_z} \sqrt{12R_{ii}}$ . In our case,  $n_z = 2$  is the measurement vector size and  $R_{ii}$  are sensor error standard deviations for azimuth  $\beta$  and distance  $D$  measurements.

### 2.2 Attribute Likelihood Ratios for GDA

The major difficulty to implement GDA-MTT depends on the correct derivation of coefficients  $a_{ij}$ , and more specifically the attribute likelihood ratios  $LR_a(i, j)$  for correct association between measurement  $j$  and target  $i$  based only on attribute information. When attribute data are available and their quality is sufficient, the attribute likelihood ratio helps a lot to improve MTT performances. In our case, the

<sup>1</sup>where indexes  $i$  and  $j$  have been omitted here for  $LF$  notation convenience.

target type information is utilized from RCS attribute measurement through fuzzification interface proposed in [27]. A particular confusion matrix is constructed to model the sensor's classification capability. This work presents different possible issues to evaluate  $LR_a(i, j)$  depending on the nature of the attribute information and the fusion rules used to predict and to update each of them. The specific attribute likelihood ratios are derived within both DSMT and DST frameworks.

### 2.2.1 Modeling the Classifier

The way of constructing the confusion matrix is based on some underlying decision-making process based on specific attribute features measurements. In this particular case, it is based on the fuzzification interface, described in our previous work [27, 24]. Through Monte Carlo simulations, the confusion matrix for two different average values of RCS is obtained, in terms of the first frame of hypotheses  $\Theta_1 = \{(S)mall, (B)ig\}$ . Based on the fuzzy rules, described in [27], defining the correspondence between RCS values and the respective targets' types, the final confusion matrix  $\mathbf{T} = [t_{ij}]$  in terms of the second frame of hypotheses  $\Theta_2 = \{(F)ighter, (C)argo\}$  is constructed. Their elements  $t_{ij}$  represent the probability to declare that the target type is  $i$  when its real type is  $j$ . Thus the targets type probability mass vector for classifier output is the  $j$ -th column of the confusion matrix  $\mathbf{T}$ . When false alarms arise, their mass vector consists in an equal distribution of masses among the two classes of target.

### 2.2.2 Attribute Likelihood Ratio within DSMT

The approach for deriving  $LR_a(i, j)$  within DSMT is based on relative variations of pignistic probabilities [24] for the target type hypotheses,  $H_j$  ( $j = 1$  for Fighter,  $j = 2$  for Cargo) included in the frame  $\Theta_2$  conditioned by the correct assignment. These pignistic probabilities are derived after the fusion between the generalized basic belief assignments of the tracks old attribute state history and the new attribute/ID observation, obtained within the particular fusion rule. It is proven [24] that this approach outperforms most of the well-known ones for attribute data association. It is defined as:

$$\delta_i(P^*) = \frac{|\Delta_i(P^*|Z) - \Delta_i(P^*|\hat{Z} = T_i)|}{\Delta_i(P^*|\hat{Z} = T_i)} \quad (1)$$

where

$$\begin{cases} \Delta_i(P^*|Z) = \sum_{j=1}^2 \frac{|P_{T_i}^*(H_j) - P_{T_i}^*(H_j)|}{P_{T_i}^*(H_j)} \\ \Delta_i(P^*|\hat{Z} = T_i) = \sum_{j=1}^2 \frac{|P_{T_i}^*(H_j) - P_{T_i}^*(H_j)|}{P_{T_i}^*(H_j)} \end{cases}$$

i.e.  $\Delta_i(P^*|\hat{Z} = T_i)$  is obtained by forcing the attribute observation mass vector to be the same as the attribute mass vector of the considered real target, i.e.  $m_Z(\cdot) = m_{T_i}(\cdot)$ . The decision for the right association relies on the minimum of expression (1). Because the generalized likelihood ratio  $LR_{gen}$  is looking for the maximum value, we define the

final form of the attribute likelihood ratio to be inversely proportional to the  $\delta_i(P^*)$  with  $i$  defining the number of the track, i.e.  $LR_a(i, j) = 1/\delta_i(P^*)$ .

### 2.2.3 Attribute Likelihood Ratio within DST

$LR_a(i, j)$  within DST is defined from the derived attribute likelihood function proposed in [3, 12]. If one considers the observation-to-track fusion process using Dempster's rule, the degree of conflict  $k_{ij}$  is computed as the assignment of mass committed to the conflict, i.e.  $m(\emptyset)$ . The larger this assignment is, the less likely is the correctness of observation  $j$  to track  $i$  assignment. Then, the reasonable choice for the attribute likelihood function is  $LHF_{i,j} = 1 - k_{ij}$ . The attribute likelihood function for the possibility that a given observation  $j$  originated from the false alarm is computed as  $LHF_{fa,j} = 1 - k_{fa,j}$ . Finally the attribute likelihood ratio to be used in GDA is obtained as  $LR_a(i, j) = LHF_{i,j}/LHF_{fa,j}$ .

## 3 Fusion rules proposed for GDA-MTT

Imprecise, uncertain and even contradicting information or data are characteristics of the real world situations and must be incorporated into modern MTT systems to provide a complete and accurate model of the monitored problem. On the other hand, the conflict and paradoxes management in collected knowledge is a major problem especially during the fusion of many information sources. Indeed the conflict increases with the number of sources or with the number of processed scans in MTT. Hence a reliable issue for processing and/or reassigning the conflicting probability masses is required. Such a situation involves also some decision-making procedures based on specific data bases to achieve proper knowledge extraction for a better understanding of the overall monitored problem. It is important and valuable to achieve hierarchical extraction of relevant information and to improve the decision accuracy such that highly accurate decisions can be made progressively. There are many valuable fusion rules in the literature to deal with imperfect information based on different mathematical models and on different methods for transferring the conflicting mass onto admissible hypotheses of the frame of the problem. DST [23, 22] was the first theory for combining uncertain information expressed as basic belief assignments with Dempster's rule. Recently, DSMT [24] was developed to overcome the limitations of DST (mainly due to the well-known inconsistency of Dempster's rule for highly conflicting fusion problem and the limitations of the Shafer's model itself) and for combining uncertain, imprecise and possibly highly conflicting sources of information for static or dynamic fusion applications. DSMT is actually a natural extension of DST. The major differences between these two theories is on the nature of the hypotheses of the frame  $\Theta$  on which are defined the basic belief assignments (bba)  $m(\cdot)$ , i.e. either on the power set  $2^\Theta$  for DST or on the hyper-power set (Dedekind's lattice, i.e. the lattice closed by  $\cap$  and  $\cup$  set operators)  $D^\Theta$  for DSMT. Let's consider a frame  $\Theta = \{\theta_1, \dots, \theta_n\}$  of finite number of hypotheses assumed for simplicity to be exhaustive. Let's denote  $G$

the classical power set of  $\Theta$  (if we assume Shafer's model with all exclusivity constraints between elements of  $\Theta$ ) or denote  $G$  the hyper-power set  $D^\Theta$  (if we adopt DSMT and we know that some elements can't be refined because of their intrinsic fuzzy and continuous nature). A basic belief assignment  $m(\cdot)$  is then defined as  $m : G \rightarrow [0, 1]$  with  $m(\emptyset) = 0$  and  $\sum_{X \in G} m(X) = 1$ . The differences between DST and DSMT lie in the model of the frame  $\Theta$  one wants to deal with but also in the rules of combination to apply. Here are the main fusion rules we investigate in this work:

- **Dempster's rule:** (Shafer's model)

The Dempster's rule of combination of  $m_1(\cdot)$  and  $m_2(\cdot)$  is obtained as follows:  $m_{DS}(\emptyset) = 0$  and  $\forall (X \neq \emptyset) \in 2^\Theta$  by

$$m_{DS}(X) = \frac{1}{1 - k_{12}} \cdot \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1)m_2(X_2)$$

$m_{DS}(\cdot)$  is a proper basic belief assignment if and only if the denominator  $1 - k_{12}$  is non-zero, i.e. the *degree of conflict*  $k_{12} \triangleq \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1)m_2(X_2) < 1$ .

- **Yager's rule:** (Shafer's model)

The Yager's rule of combination [24] admits that in case of conflict the result is not reliable, so that  $k_{12}$  plays the role of an absolute discounting term added to the weight of ignorance. This commutative but not associative rule, denoted here by index  $Y$  is given<sup>2</sup> by  $m_Y(\emptyset) = 0$  and  $\forall X \in 2^\Theta, X \neq \emptyset, X \neq \Theta$  by

$$m_Y(X) = \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1)m_2(X_2)$$

and when  $X = \Theta$  by

$$m_Y(\Theta) = m_1(\Theta)m_2(\Theta) + \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1)m_2(X_2)$$

- **Dubois & Prade's rule:** (Shafer's model)

Dubois & Prade's rule of combination [24] admits that the two sources are reliable when they are not in conflict, but one of them is right when a conflict occurs. Then if one observes a value in set  $X_1$  while the other observes this value in a set  $X_2$ , the truth lies in  $X_1 \cap X_2$  as long  $X_1 \cap X_2 \neq \emptyset$ . If  $X_1 \cap X_2 = \emptyset$ , then the truth lies in  $X_1 \cup X_2$ . According to this principle, the commutative (but not associative) Dubois & Prade hybrid rule of combination, denoted here by index  $DP$ , which is a reasonable trade-off between precision and reliability, is defined by  $m_{DP}(\emptyset) = 0$  and

$\forall X \in 2^\Theta, X \neq \emptyset$  by

$$m_{DP}(X) = \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = X \\ X_1 \cap X_2 \neq \emptyset}} m_1(X_1)m_2(X_2) + \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cup X_2 = X \\ X_1 \cap X_2 = \emptyset}} m_1(X_1)m_2(X_2)$$

- **Hybrid DSMT fusion rule:** (any model)

The hybrid DSMT rule of combination is the first general rule of combination developed in DSMT framework [24] which can work on any models (including Shafer's model) and for any level of conflicting information. It can deal with the potential dynamicity of the frame and its model as well. DSMT deals properly with the granularity of information and intrinsic vague/fuzzy nature of elements of the frame  $\Theta$  to manipulate. The basic idea of DSMT is to define belief assignments on the hyper-power set  $D^\Theta$  (i.e. free Dedekind's lattice) and to integrate all integrity constraints (exclusivity and/or non-existential constraints) of the model, say  $\mathcal{M}(\Theta)$ , fitting with the problem into the rule of combination. This rule for  $s \geq 2$  independent sources is defined as (see chap. 4 in [24]) for all  $X \in D^\Theta$

$$m_{\mathcal{M}(\Theta)}(X) \triangleq \phi(X) \left[ S_1(X) + S_2(X) + S_3(X) \right]$$

where  $\phi(X)$  is the *characteristic non-emptiness function* of a set  $X$ , i.e.  $\phi(X) = 1$  if  $X \notin \emptyset$  and  $\phi(X) = 0$  otherwise, where  $\emptyset \triangleq \{\emptyset_{\mathcal{M}}, \emptyset\}$ .  $\emptyset_{\mathcal{M}}$  is the set of all elements of  $D^\Theta$  which have been forced to be empty through the constraints of the model  $\mathcal{M}$  and  $\emptyset$  is the classical/universal empty set.  $S_1(X)$ ,  $S_2(X)$  and  $S_3(X)$  are defined by

$$S_1(X) \triangleq \sum_{\substack{X_1, X_2, \dots, X_s \in D^\Theta \\ (X_1 \cap X_2 \cap \dots \cap X_s) = X}} \prod_{i=1}^s m_i(X_i)$$

$$S_2(X) \triangleq \sum_{\substack{X_1, X_2, \dots, X_s \in \emptyset \\ [\mathcal{U} = X] \vee [(\mathcal{U} \in \emptyset) \wedge (X = I_t)]}} \prod_{i=1}^s m_i(X_i)$$

$$S_3(A) \triangleq \sum_{\substack{X_1, X_2, \dots, X_k \in D^\Theta \\ u(c(X_1 \cap X_2 \cap \dots \cap X_k)) = A \\ (X_1 \cap X_2 \cap \dots \cap X_k) \in \emptyset}} \prod_{i=1}^k m_i(X_i)$$

with  $\mathcal{U} \triangleq u(X_1) \cup u(X_2) \cup \dots \cup u(X_k)$  where  $u(X)$  is the union of all  $\theta_i$  that compose  $X$ ,  $I_t \triangleq \theta_1 \cup \theta_2 \cup \dots \cup \theta_n$  is the total ignorance, and  $c(X)$  is the conjunctive normal form<sup>3</sup> of  $X$ , i.e. its simplest form (for example if  $X = (A \cap B) \cap (A \cup B \cup C)$ ,

<sup>2</sup> $\Theta$  represents here the full ignorance  $\theta_1 \cup \theta_2 \cup \dots \cup \theta_n$  on the frame of discernment according the notation used in [23].

<sup>3</sup>In Boolean algebra the conjunctive normal form is a conjunction of disjunctions, in its simplest form, which is unique; in this paper we consider each disjunction formed by a singleton or by

$c(X) = A \cap B$ ). The hybrid DSm rule (which differs from Dempster's rule) can be seen actually as an improved version of Dubois & Prade's rule which mix the conjunctive and disjunctive consensus applied in DSmT framework to take into account the possibility for any dynamical integrity constraint in the model.

- **PCR fusion rules:** (any model)

The general principle of the recent PCR rules developed in [26] consists in the following steps :

- **Step 1:** compute the conjunctive rule,
- **Step 2:** compute the conflicting masses (partial and/or total), The *total conflicting mass* drawn from two sources, denoted  $k_{12}$ , is defined as follows:

$$k_{12} = \sum_{\substack{X_1, X_2 \in G \\ X_1 \cap X_2 = \emptyset}} \underbrace{m_1(X_1)m_2(X_2)}_{m(X_1 \cap X_2)}$$

which is nothing but the sum of *partial conflicting masses*  $m(X_1 \cap X_2)$ , where  $X_1 \cap X_2 = \emptyset$ , represents a partial conflict.

- **Step 3:** then proportionally redistribute the conflicting mass (total or partial) to non-empty sets according to all integrity constraints using a given strategy. We present here the most interesting and sophisticated PCR rule (denoted PCR5) among the five PCR rules proposed in [26].

**PCR5 fusion rule** for two sources is defined as [26]:

$$\forall X \in G \setminus \{\emptyset\}, m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y \in G \setminus \{X\} \\ c(Y \cap X) = \emptyset}} \left[ \frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right]$$

where  $c(x)$  represents the conjunctive normal form of  $x$ ,  $m_{12}(X)$  corresponds to the conjunctive consensus on  $X$ , and where all denominators are *different from zero*. If a denominator is zero, that fraction is discarded. The general (but more complex) PCR5 formula for  $s \geq 2$  sources and many examples are given in [25].

## 4 Simulation scenario and results

### 4.1 Simulation scenario

The simulation scenario consists in seven air targets with only two classes. The stationary sensor is located at the

union of singletons; for example:  $A \cap B \cap (C \cup D)$  is a conjunctive normal form; also,  $X = (A \cup B) \cap C \cap (A \cup C)$  is a conjunction of disjunctions, but it is not in its simplest form, then its conjunctive normal form is  $c(X) = (A \cup B) \cap C$  since  $C \cap (A \cup C) = C$ . The conjunctive normal form is introduced here in order to improve the original formula given in [24] for preserving the neutral impact of the vacuous belief mass  $m(\emptyset) = 1$  within complex hybrid models.

origin. The sampling period is  $T_{scan} = 5$  sec and measurement standard deviations are 0.3 deg and 120 m for azimuth and range respectively. The targets go from North to South with the following type order FCFCFCF from left to right (F=Fighter, C=Cargo) with constant velocity 150m/sec. They are moving in parallel with approximately 320 m inter-distance. During scans 8 – 10, 17 – 19, 26 – 28 and 34 – 36 the maneuvers are performed with transversal acceleration  $3.5m/s^2$ . Process noise standard deviations for the two nested models for constant velocity IMM are  $0.1m/s^2$  and  $3m/s^2$  respectively. The number of false alarms (FA) follows a Poisson distribution and FA are uniformly distributed spacially in the surveillance region.

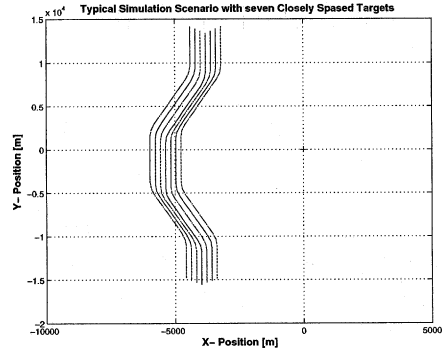


Fig. 1: Multitarget Scenario with seven targets

Monte Carlo simulations are made for the two different average values of Radar Cross Section in order to obtain the confusion matrix in terms of the first frame of hypotheses  $\Theta_1 = \{Small, Big\}$ . According to the fuzzy rules in [24, 27], defining the correspondence between Radar Cross Section values and the respective targets' types, the confusion matrix in terms of the second frame of hypotheses  $\Theta_2 = \{Fighter, Cargo\}$  is constructed. The two simulation cases correspond to the following parameters for the probability of target detection, the probability of false alarms and the confusion matrices:

- Case 1:  $P_d = 1, P_{fa} = 0, T_1 = \begin{bmatrix} 0.995 & 0.005 \\ 0.005 & 0.995 \end{bmatrix}$
- Case 2:  $P_d = 0.9, P_{fa} = 1 \cdot 10^{-5}, T_2 = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$

### 4.2 Simulation results

In this section we present and discuss simulation results for 100 Monte Carlo runs. The evaluation of fusion rules' performance is based on the criteria of tracks' purity, tracks' life, the variation of pignistic entropy in confirmed tracks attribute states, the average pignistic entropy value in a steady state and percentage of miscorrelation. Tracks purity criteria examines the ratio between the number of particular performed (observation j-track i) associations (in case of detected target) over the total number of all possible associations during the tracking scenario. Track's life is evaluated as an average number of scans before track's deletion. The tracks deletion is performed after the a priori

defined number (in our case it is assumed to be 3) of incorrect associations or missed detections. The percentage of miscorrelation examines the relative number of incorrect (observation-track) associations during the scans. The results for GDA are obtained by different fusion rules. Relying on our previous work [24, 27], where the performance of DSm Classic and DSm Hybrid (DSmH) rules were examined, in the present work the attention is directed to the well-known Dempster's rule, Yager's, Dubois & Prade's, and especially to DSmH and the new PCR5 rule. From results presented in Tables 1-4 in next sections, it is obvious that for both cases 1 and 2 the track's purity and tracks' life in the case of KDA-MTT are significantly lower with respect to all GDA-MTT, and a higher percentage of miscorrelation is obtained with KDA-MTT than with GDA-MTT. The figures below show typical tracking performances for KDA-MTT and GDA-MTT systems.

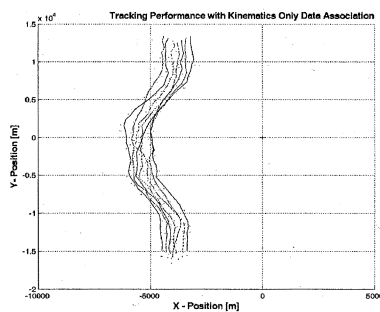


Fig. 2: Typical performance with KDA-MTT

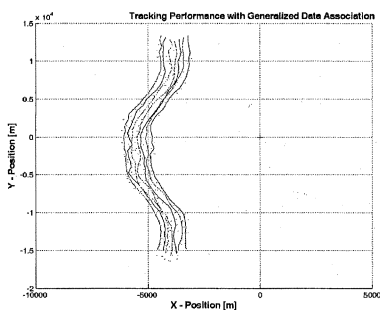


Fig. 3: Typical performance with GDA-MTT

#### 4.2.1 Simulation results for case 1

Case no. 1 is characterized by no false alarm and maximum probability of target detection, but the problem consists in the proximity of the targets (inter-distance of 320 m) with bad sensor distance resolution ( $\sigma_D = 120\text{m}$ ). It results in cross-associations. The Monte Carlo results on track purity based on KDA-MTT and on GDA-MTT (based on PCR5, Dempster (DS), Yager's rules<sup>4</sup> and DSmH rule) are given

<sup>4</sup>Yager's rule, Dubois & Prade's (DP) rule and DSmH rule coincide in our example because we are working with only a 2D specific classes frame  $\Theta_2$ . This is normal. In general, Yager's, DP and DSmH do no longer coincide when the cardinality of the frame becomes greater than two.

in Table 1. Each number of the table gives the ratio of correct measurement-target association for a given target and a given MTT algorithm and the last row of the table provides the average purity performance for all targets and for each algorithm.

One can see that the corresponding fields for results obtained via Dempster's rule of combination are empty (see Tables 1-4). There are two major reasons for this:

1. The case of increasing intrinsic conflicts between the fused bodies of evidence (generalized basic belief assignments of targets' tracks histories and new observations), yields a poor targets tracks' performance. The situation when this conflict becomes unity, is a stressful, but a real one. It is the moment, when Dempster's rule produces indefiniteness. The fusion process stops and the attribute histories of tracking tracks cannot be updated. As a result the whole tracking process corrupts. Actually in such a case there is a need of an artificial break and intervention into the real time tracking process, which could cause noncoherent results. Taking into account all these particularities, we can summarize that the fusion process within DST is not fluent and cannot be controlled without prior unjustified and artificial assumptions and some heuristic engineering tricks. As a consequence no one of the performance criteria cannot be evaluated.
2. In case when in the updated track's attribute history one of the hypotheses in the frame of the problem is supported by unity, from this point on, Dempster's rule becomes indifferent to all observations, detected during the next scans. It means, the track's attribute history remains unchanged regardless of the new observations. It is a dangerous situation, which hides the real opportunity for producing the non-adequate results.

	KDA	PCR5	DS	Y/DP/DSmH
$T_1$	0.6187	0.9650	-	0.9521
$T_2$	0.4892	0.9150	-	0.9150
$T_3$	0.4424	0.8300	-	0.8289
$T_4$	0.5500	0.9084	-	0.9350
$T_5$	0.6797	0.8861	-	0.8689
$T_6$	0.7213	0.9561	-	0.9624
$T_7$	0.8034	0.9863	-	0.9739
Average	0.6149	0.9209	-	0.9194

Table 1: Track's purity for KDA and GDA-MTT (case 1)

The results of the percentage of Track life duration and miscorrelation are given in Table 2. The third column in Table 2 gives the *Pignistic Entropy's Steady State Fluctuation* values which correspond to the ratio between the fluctuation of Pignistic entropy in steady state over its maximum range (■ indicates that no entropy fluctuations can be obtained from KDA only). The last column represents the average pignistic entropy value in a steady state.

Looking on the results achieved according to GDA, it can be underlined that :

1. The tracks' purity, obtained by PCR5 rule outperforms the tracks' purity results obtained by using all other

Trackers	Track Life [%]	MisCor [%]	PEF	APE
KDA-MTT	65.01	37.68	■	■
GDA <sub>PCR5</sub> -MTT	93.67	7.55	0.05	0.02
GDA <sub>DS</sub> -MTT	-	-	-	-
GDA <sub>Y</sub> -MTT	93.46	7.68	0.06	0.04
GDA <sub>DP</sub> -MTT	93.46	7.68	0.06	0.04
GDA <sub>DSmH</sub> -MTT	93.46	7.68	0.06	0.04

Table 2: Track's life, Miscorrelation, Pignistic Entropy Fluctuations (PEF) and Average Pignistic Entropy (APE) value in steady state (case 1)

rules. Yager's, Dubois & Prade's and DSmH rules lead to a small decrease in GDA performance and in this 2D frame case based on Shafer's model their tracks' purity results are equal which is normal.

2. According to Table 2, the average tracks' life, the percentage of miscorrelation and Pignistic Entropy's Steady State Fluctuation related to the performance of PCR5 rule are a little bit (about percent) better than all other rules' performance.
3. Using the Pignistic Entropy's Steady State Fluctuation criteria and looking at the third column of Tables 2 and 4, one can see that during the consecutive scans, the pignistic entropy obtained via PCR5, Yager's, Dubois & Prade's, DSmH rule decrease gradually during the first 3-4 scans, approaching zero and this process is a stable one. According to the criteria of average pignistic entropy value in a steady state, one can see that the entropy associated with updated tracks' attribute states by using PCR5 rule is almost 2 times less than the entropy obtained by using all other rules. It means that PCR5 rule leads to results, which are much more informative in comparison with the other ones.

#### 4.2.2 Simulation results for case 2

Case no. 2 is more difficult than case no. 1 since the presence of false alarms and missed target detections significantly degrade the process of data association even in the case of GDA. But in comparison with KDA, one sees in Table 3 that the use of the attribute type information still helps significantly to reduce the cross-associations and increase the track's purity performances.

	KDA	PCR5	DS	Y/DP/DSmH
T <sub>1</sub>	0.5576	0.6600	-	0.597
T <sub>2</sub>	0.3642	0.6011	-	0.4650
T <sub>3</sub>	0.3424	0.4726	-	0.4308
T <sub>4</sub>	0.3945	0.6750	-	0.5858
T <sub>5</sub>	0.4968	0.5937	-	0.5129
T <sub>6</sub>	0.5418	0.6668	-	0.6145
T <sub>7</sub>	0.6471	0.7689	-	0.7071
Average	0.4777	0.6340	-	0.5590

Table 3: Track's purity for KDA and GDA-MTT (case 2)

The results of the percentage of Track life duration and miscorrelation, and Pignistic Entropy's Steady State Fluctuation are given in Table 4.

## 5 Conclusions

In this paper a comparison of the performances of different fusion rules is presented and compared in order to assess their efficiency for GDA for MTT in highly conflicting

Trackers	Track Life [%]	MisCor [%]	PEF	APE
KDA-MTT	47.87	45.48	■	■
GDA <sub>PCR5</sub> -MTT	63.13	30.58	0.44	0.175
GDA <sub>DS</sub> -MTT	-	-	-	-
GDA <sub>Y</sub> -MTT	57.01	37.39	0.62	0.3
GDA <sub>DP</sub> -MTT	57.01	37.39	0.62	0.3
GDA <sub>DSmH</sub> -MTT	57.01	37.39	0.62	0.3

Table 4: Track's life, Miscorrelation, Pignistic Entropy Fluctuations (PEF) and Average Pignistic Entropy (APE) value in steady state (case 2)

situations in clutter. A model of an attribute type classifier is considered on the base of particular input fuzzification interface according to the target RCS values and on fuzzy rule base according to the target type. A generalized likelihood ratio is obtained and included in the process of GDA. The classification results rely on the confusion matrix specifying the accuracy of the classifier and on the implemented fusion rules (PCR5, Dempster's, Yager's, Dubois & Prade's, DSmH). The goal was to examine their advantages and milestones and to improve association results. This work confirms the benefits of attribute utilization and shows some hidden drawbacks, when the sources of information remain in high conflict, especially in case of using Dempster's rule of combination. In clutter-free environment with maximum of target detection probability and very good classifier quality, the results, according to the performance criteria, obtained via PCR5 rule outperform the corresponding results obtained by using all the other combination rules tested. When tracking conditions decrease (presence of clutter, missed target detections with lower classifier quality), the PCR5 fusion rule still provides the best performances with respect to other rules tested for our new GDA-MTT algorithm. This work reveals also the real difficulty to define and to choose an unique or a multiple performance criteria for the fair evaluation of different fusion rules. Actually the choice of the fusion rule is in practice highly conditioned by the performance criteria that the system designer considers as the most important for his application. More efforts on multicriteria-based methods for performance evaluation are under investigations. Further works on GDA-MTT would be to define some precise benchmark for difficult multitarget tracking and classification scenarios and to see if the recent MITM approach (i.e. RMIMM coupled with MAJPDA) can be improved by our new generalized data association method.

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