Trapezoidal neutrosophic set and its application to multiple attribute decision making

Jun Ye*

Department of Electrical and Information Engineering, Shaoxing University, 508 Huancheng West Road, Shaoxing, Zhejiang Province 312000, P.R. China

Abstract

Based on the combination of trapezoidal fuzzy numbers and a single valued neutrosophic set, this paper proposes a trapezoidal neutrosophic set, some operational rules, score and accuracy functions for trapezoidal neutrosophic numbers. Then, a trapezoidal neutrosophic number weighted arithmetic averaging (TNNWAA) operator and a trapezoidal neutrosophic number weighted geometric averaging (TNNWGA) operator are proposed to aggregate the trapezoidal neutrosophic information and their properties are investigated. Furthermore, a multiple attribute decision-making method based on the TNNWAA and TNNWGA operators and the score and accuracy functions of a trapezoidal neutrosophic number is established to deal with the multiple attribute decision making problems in which the evaluation values of alternatives on the attributes are represented by the form of trapezoidal neutrosophic numbers. Finally, an illustrative example about software selection is given to demonstrate the application and effectiveness of the developed method.

* Corresponding author: Jun Ye
Tel.: +86-575-88327323
E-mail address: yehjun@aliyun.com (Jun Ye)

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1. Introduction

Atanassov [1] introduced an intuitionistic fuzzy set as a generalization of the Zadeh's fuzzy set [2]. Later, Liu and Yuan [3] developed triangular intuitionistic fuzzy sets based on the combination of triangular fuzzy numbers and intuitionistic fuzzy sets. The fundamental characteristic of the triangular intuitionistic fuzzy set is that the values of its membership function and nonmembership function are triangular fuzzy numbers rather than exact numbers. Then, Wang [4, 5] put forward some aggregation operators, including the triangular intuitionistic fuzzy weighted geometric (TIFWG) operator, triangular intuitionistic fuzzy ordered weighted geometric (TIFOWG) operator and triangular intuitionistic fuzzy hybrid geometric (TIFHG) operator, and established an approach based on the TIFWG and the TIFHG operators to deal with multiple attribute group decision-making problems with triangular intuitionistic fuzzy information, and then proposed the fuzzy number intuitionistic fuzzy weighted averaging (FIFWA) operator, fuzzy number intuitionistic fuzzy ordered weighted averaging (FIFOWA) operator and fuzzy number intuitionistic fuzzy hybrid aggregation (FIFHA) operator and applied the FIFHA operator to multiple attribute decision-making problems with triangular intuitionistic fuzzy information. Wei et al. [6] further introduced an induced triangular intuitionistic fuzzy ordered weighted geometric (I-TIFOWG) operator and applied the I-TIFOWG operator to group decision making problems with triangular intuitionistic fuzzy information. Furthermore, Ye [7] extended the triangular intuitionistic fuzzy set to the trapezoidal intuitionistic fuzzy set, where its fundamental characteristic is that the values of its membership function and nonmembership function are trapezoidal fuzzy numbers rather than triangular fuzzy numbers, and proposed the trapezoidal intuitionistic fuzzy prioritized weighted averaging (TIFPWA) operator and trapezoidal intuitionistic fuzzy prioritized weighted geometric (TIFPWG) operator and their multicriteria decision-making method, in which the criteria are in different priority level.

Recently, Wang et al. [8] introduced a single valued neutrosophic set, which is a subclass of a neutrosophic set presented by Smarandache [9], as a generalization of the classic set, fuzzy set, and intuitionistic fuzzy set. The single valued neutrosophic set can independently express truth-membership degree, indeterminacy-membership degree, and false-membership degree and deal with incomplete, indeterminate, and inconsistent information. All the factors described by the single valued neutrosophic set are very suitable for human thinking due to the imperfection of knowledge that human receives or observes from the external world. For example, for a given proposition "Movie X would be hit", in this situation human brain certainly cannot generate precise answers in terms of yes or no, as indeterminacy is the sector of unawareness of a proposition's value between truth and falsehood. Obviously, the neutrosophic components are best fit in the representation of indeterminacy and inconsistent information, while the intuitionistic fuzzy set cannot represent and handle indeterminacy and inconsistent information. Hence, the single valued neutrosophic set has been a rapid development and a wide range of applications [10, 11].

However, we can see that the trapezoidal fuzzy number and the single valued neutrosophic set are very useful tools to deal with incomplete, indeterminacy, and inconsistent information. Therefore, based on the combination of the trapezoidal fuzzy number and the single valued neutrosophic set, the purposes of this paper are: (1) to propose a trapezoidal neutrosophic set as the extension of the trapezoidal intuitionistic fuzzy set and the score and accuracy functions of a trapezoidal neutrosophic set, (2) to develop a trapezoidal neutrosophic number weighted arithmetic averaging (TNNWAA) operator and a trapezoidal neutrosophic number weighted geometric averaging (TNNWGA) operator, and (3) to establish a trapezoidal neutrosophic multiple attribute decision-making method. To do so, the remainder of this paper is organized as follows. Section 2 introduces some basic concepts related to trapezoidal intuitionistic fuzzy sets and single valued neutrosophic sets. Section 3 proposes a trapezoidal neutrosophic set as a generalization of a trapezoidal intuitionistic fuzzy set, some operational rules of trapezoidal neutrosophic numbers, and the score and accuracy functions of a trapezoidal neutrosophic number. In Section 4, the TNNWAA and TNNWGA operators are proposed to aggregate trapezoidal neutrosophic information and their properties are investigated. Section 5 develops a multiple attribute decision-making method with trapezoidal neutrosophic information based on the TNNWAA and TNNWGA operators and the score and accuracy functions of a trapezoidal neutrosophic number. In Section 6, an illustrative example is provided to demonstrate the application and effectiveness of the developed method. Conclusions and future work are given in Section 7.

2. Preliminaries

2.1 Some concepts of trapezoidal intuitionistic fuzzy sets

In this section, we shortly describe some concepts of trapezoidal intuitionistic fuzzy sets, which are preferred in practice, and the score function and accuracy function of a trapezoidal intuitionistic fuzzy number.

As a generalization of a triangular intuitionistic fuzzy set, Ye [7] introduced a trapezoidal intuitionistic fuzzy set and gave its definition.

Definition 1 [7]. Let X be a universe of discourse, a trapezoidal intuitionistic fuzzy set $\tilde{A}$ in X is defined as

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \mid x \in X\},$$

where $\mu_{\tilde{A}}(x) \subset [0,1]$ and $\nu_{\tilde{A}}(x) \subset [0,1]$ are two trapezoidal fuzzy numbers $\mu_{\tilde{A}}(x) = \left(\mu_{\tilde{A}}^1(x), \mu_{\tilde{A}}^2(x), \mu_{\tilde{A}}^3(x), \mu_{\tilde{A}}^4(x)\right) : X \to [0,1]$ and $\nu_{\tilde{A}}(x) = \left(\nu_{\tilde{A}}^1(x), \nu_{\tilde{A}}^2(x), \nu_{\tilde{A}}^3(x), \nu_{\tilde{A}}^4(x)\right) : X \to [0,1]$ with the condition $0 \le \mu_{\tilde{A}}^4(x) + \nu_{\tilde{A}}^4(x) \le 1, \quad x \in X$.

For convenience, let $\mu_{\tilde{A}}(x) = (a,b,c,d)$ and $\nu_{\tilde{A}}(x) = (l,m,n,p)$ be two trapezoidal fuzzy numbers, thus a trapezoidal intuitionistic fuzzy number can be denoted by $\tilde{a} = \left\langle (a,b,c,d),(l,m,n,p)\right\rangle$, which is basic element in a trapezoidal intuitionistic fuzzy set.

If $b = c$ and $m = n$ hold in a trapezoidal intuitionistic fuzzy number $\tilde{a}$, it reduces to the triangular intuitionistic fuzzy number, which is a special case of the trapezoidal intuitionistic fuzzy number.

Definition 2 [7]. Let $\tilde{a}_1 = \left\langle (a_1,b_1,c_1,d_1),(l_1,m_1,n_1,p_1)\right\rangle$ and $\tilde{a}_2 = \left\langle (a_2,b_2,c_2,d_2),(l_2,m_2,n_2,p_2)\right\rangle$ be two trapezoidal intuitionistic fuzzy numbers. Then there are the following operational rules:

(1) $\tilde{a}_1 \oplus \tilde{a}_2 = \left\langle (a_1+a_2-a_1a_2, b_1+b_2-b_1b_2, c_1+c_2-c_1c_2, d_1+d_2-d_1d_2),(l_1l_2, m_1m_2, n_1n_2, p_1p_2)\right\rangle$;

(2)

$\tilde{a}_1 \otimes \tilde{a}_2 = \left\langle (a_1a_2, b_1b_2, c_1c_2, d_1d_2),(l_1+l_2-l_1l_2, m_1+m_2-m_1m_2,, n_1+n_2-n_1n_2, p_1+p_2-p_1p_2)\right\rangle$;

(3) $\lambda\tilde{a}_1 = \left\langle \left(1-(1-a_1)^\lambda, 1-(1-b_1)^\lambda, 1-(1-c_1)^\lambda, 1-(1-d_1)^\lambda\right),\left(l_1^\lambda, m_1^\lambda, n_1^\lambda p_1^\lambda\right),\right\rangle, \lambda > 0$;

(4) $\tilde{a}_1^\lambda = \left\langle \left(a_1^\lambda, b_1^\lambda, c_1^\lambda, d_1^\lambda\right),\left(1-(1-l_1)^\lambda, 1-(1-m_1)^\lambda, 1-(1-n_1)^\lambda, 1-(1-p_1)^\lambda\right)\right\rangle, \lambda \ge 0$.

Definition 3 [7]. Let $\tilde{a} = \left\langle (a,b,c,d),(l,m,n,p)\right\rangle$ be a trapezoidal intuitionistic fuzzy number. Then a score function of a trapezoidal intuitionistic fuzzy number can be defined by

$$s(\tilde{a}) = \frac{a+b+c+d}{4} - \frac{l+m+n+p}{4}, s(\tilde{a}) \in [-1,1], \tag{1}$$

where the larger the value of $s(\tilde{a})$, the bigger the trapezoidal intuitionistic fuzzy number $\tilde{a}$. Especially when $b = c$ and $m = n$ in a trapezoidal intuitionistic fuzzy number $\tilde{a}$, Eq. (1) reduces to the score function of the triangular intuitionistic fuzzy number, which is a special case of $s(\tilde{a})$.

Definition 4 [7]. Let $\tilde{a} = \langle (a,b,c,d),(l,m,n,p) \rangle$ be a trapezoidal intuitionistic fuzzy number. Then an accuracy function of a trapezoidal intuitionistic fuzzy number can be defined by

$$h(\tilde{a}) = \frac{a+b+c+d}{4} + \frac{l+m+n+p}{4}, h(\tilde{a}) \in [0,1], \quad (2)$$

where the larger the value of $h(\tilde{a})$, the higher the degree of accuracy of the trapezoidal intuitionistic fuzzy number $\tilde{a}$. Especially when $b = c$ and $m = n$ in a trapezoidal intuitionistic fuzzy number $\tilde{a}$, Eq. (2) reduces to the accuracy function of the triangular intuitionistic fuzzy number, which is a special case of $h(\tilde{a})$.

2.2 Some concepts of single valued neutrosophic sets

From philosophical point of view, Smarandache [9] originally presented the concept of a neutrosophic set A in a universal set X, which is characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, such that $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$. Obviously, it is difficult to apply the neutrosophic set to practical problems. To easily apply it in science and engineering fields, Wang et al. [8] introduced the concept of a single valued neutrosophic set as a subclass of the neutrosophic set and gave the following definition.

Definition 5 [8]. A single valued neutrosophic set A in a universal set X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a single valued neutrosophic set A can be denoted by

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle \mid x \in X \},$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Let $A=\left\{\left\langle x, T_A(x), I_A(x), F_A(x)\right\rangle \mid x\in X\right\}$ and $B=\left\{\left\langle x, T_B(x), I_B(x), F_B(x)\right\rangle \mid x\in X\right\}$ be two single valued neutrosophic sets, and then there are the following relations [8, 11]:

(1) Complement: $A^c=\left\{\left\langle x, F_A(x), 1-I_A(x), T_A(x)\right\rangle \mid x\in X\right\}$;

(2) Inclusion: $A\subseteq B$ if and only if $T_A(x)\le T_B(x)$, $I_A(x)\ge I_B(x)$, $F_A(x)\ge F_B(x)$ for any x in X;

(3) Equality: $A = B$ if and only if $A\subseteq B$ and $B\subseteq A$;

(4) Union: $A\bigcup B=\left\{\left\langle x, T_A(x)\vee T_B(x), I_A(x)\wedge I_B(x), F_A(x)\wedge F_B(x)\right\rangle \mid x\in X\right\}$;

(5) Intersection: $A\bigcap B=\left\{\left\langle x, T_A(x)\wedge T_B(x), I_A(x)\vee I_B(x), F_A(x)\vee F_B(x)\right\rangle \mid x\in X\right\}$;

(6) Addition: $A\oplus B=\left\{\left\langle x, T_A(x)+T_B(x)-T_A(x)T_B(x), I_A(x)I_B(x), F_A(x)F_B(x)\right\rangle \mid x\in X\right\}$;

(7) Multiplication:

$$A\otimes B=\left\{\left\langle x, T_A(x)T_B(x), I_A(x)+I_B(x)-I_A(x)I_B(x), F_A(x)+F_B(x)-F_A(x)F_B(x)\right\rangle \mid x\in X\right\}.$$

3. Trapezoidal neutrosophic sets

This section extend a trapezoidal intuitionistic fuzzy set, which is preferred in practice, to a single valued neutrosophic set to present a trapezoidal neutrosophic set based on the combination of trapezoidal fuzzy numbers and a single valued neutrosophic set and its score and accuracy functions.

As a generalization of a trapezoidal intuitionistic fuzzy set, we propose the following definition of a trapezoidal neutrosophic set.

Definition 6. Let X be a universe of discourse, a trapezoidal neutrosophic set $\tilde{A}$ in X is defined as the following form:

$$\tilde{N}=\{\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x)\rangle \mid x\in X\},$$

where $T_{\tilde{N}}(x)\subset[0,1]$, $I_{\tilde{N}}(x)\subset[0,1]$ and $F_{\tilde{N}}(x)\subset[0,1]$ are three trapezoidal fuzzy numbers $T_{\tilde{N}}(x)=\left(t_{\tilde{N}}^1(x), t_{\tilde{N}}^2(x), t_{\tilde{N}}^3(x), t_{\tilde{N}}^4(x)\right)$: $X\to[0, 1]$, $I_{\tilde{N}}(x)=\left(i_{\tilde{N}}^1(x), i_{\tilde{N}}^2(x), i_{\tilde{N}}^3(x), i_{\tilde{N}}^4(x)\right)$: $X\to[0, 1]$, and $F_{\tilde{N}}(x)=\left(f_{\tilde{N}}^{1}(x), f_{\tilde{N}}^{2}(x), f_{\tilde{N}}^{3}(x), f_{\tilde{N}}^{4}(x)\right)$: $X \rightarrow [0, 1]$ with the condition $0 \le t_{\tilde{N}}^{4}(x)+i_{\tilde{N}}^{4}(x)+f_{\tilde{N}}^{4}(x) \le 3$, $x \in X$.

For convenience, the three trapezoidal fuzzy numbers are denoted by $T_{\tilde{N}}(x)=(a,b,c,d)$, $I_{\tilde{N}}(x)=(e,f,g,h)$ and $F_{\tilde{N}}(x)=(l,m,n,p)$. Thus, a trapezoidal neutrosophic number is denoted by $\tilde{n}=\langle(a,b,c,d),(e,f,g,h),(l,m,n,p)\rangle$, which is a basic element in the trapezoidal neutrosophic set.

If $b = c$, $f = g$, and $m = n$ hold in a trapezoidal neutrosophic number $\tilde{a}$, it reduces to the triangular neutrosophic number, which is considered as a special case of the trapezoidal neutrosophic number.

Definition 7. Let $\tilde{n}_1=\langle(a_1,b_1,c_1,d_1),(e_1,f_1,g_1,h_1),(l_1,m_1,n_1,p_1)\rangle$ and $\tilde{n}_2=\langle(a_2,b_2,c_2,d_2),(e_2,f_2,g_2,h_2),(l_2,m_2,n_2,p_2)\rangle$ be two trapezoidal neutrosophic numbers. Then there are the following operational rules:

(1) $$\tilde{n}_1 \oplus \tilde{n}_2=\left\langle \begin{array}{l}(a_1+a_2-a_1a_2, b_1+b_2-b_1b_2, c_1+c_2-c_1c_2, d_1+d_2-d_1d_2),\\ (e_1e_2, f_1f_2, g_1g_2, h_1h_2),(l_1l_2, m_1m_2, n_1n_2, p_1p_2)\end{array}\right\rangle;$$

(2)

$$\tilde{n}_1 \otimes \tilde{n}_2=\left\langle \begin{array}{l}(a_1a_2, b_1b_2, c_1c_2, d_1d_2),(e_1+e_2-e_1e_2, f_1+f_2-f_1f_2, g_1+g_2-g_1g_2, h_1+h_2-h_1h_2),\\ (l_1+l_2-l_1l_2, m_1+m_2-m_1m_2, n_1+n_2-n_1n_2, p_1+p_2-p_1p_2)\end{array}\right\rangle;$$

(3)

$$\lambda\tilde{n}_1=\left\langle\left(1-(1-a_1)^{\lambda}, 1-(1-b_1)^{\lambda}, 1-(1-c_1)^{\lambda}, 1-(1-d_1)^{\lambda}\right),\left(e_1^{\lambda}, f_1^{\lambda}, g_1^{\lambda} h_1^{\lambda}\right),\left(l_1^{\lambda}, m_1^{\lambda}, n_1^{\lambda} p_1^{\lambda}\right)\right\rangle, \lambda>0$$

;

(4) $$\tilde{n}_1^{\lambda}=\left\langle \begin{array}{l}\left(a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}\right),\left(1-(1-e_1)^{\lambda}, 1-(1-f_1)^{\lambda}, 1-(1-g_1)^{\lambda}, 1-(1-h_1)^{\lambda}\right),\\ \left(1-(1-l_1)^{\lambda}, 1-(1-m_1)^{\lambda}, 1-(1-n_1)^{\lambda}, 1-(1-p_1)^{\lambda}\right)\end{array}\right\rangle, \lambda \ge 0.$$

Based on expected value of a trapezoidal fuzzy number [12] and the score and accuracy functions of a neutrosophic number [13], we propose the following definitions of the score and accuracy functions for a trapezoidal neutrosophic number.

Definition 8. Let $\tilde{n} = \langle (a,b,c,d),(e,f,g,h),(l,m,n,p) \rangle$ be a trapezoidal neutrosophic number, then a score function of a trapezoidal neutrosophic number can be defined as

$$S(\tilde{n}) = \frac{1}{3}\left(2 + \frac{a+b+c+d}{4} - \frac{e+f+g+h}{4} - \frac{l+m+n+p}{4}\right), S(\tilde{n}) \in [0,1], \quad (3)$$

where the larger the value of $S(\tilde{n})$, the bigger the trapezoidal neutrosophic number $\tilde{n}$. Especially when $b = c$, $f = g$, and $m = n$ hold in a trapezoidal neutrosophic number $\tilde{n}$, Eq. (3) reduces to the following score function of the triangular neutrosophic number:

$$S(\tilde{n}) = \frac{1}{3}\left(2 + \frac{a+2b+d}{4} - \frac{e+2f+h}{4} - \frac{l+2m+p}{4}\right), S(\tilde{n}) \in [0,1], \quad (4)$$

which is a special case of Eq. (3).

Definition 9. Let $\tilde{n} = \langle (a,b,c,d),(e,f,g,h),(l,m,n,p) \rangle$ be a trapezoidal neutrosophic number, an accuracy function of a trapezoidal neutrosophic number can be defined by

$$H(\tilde{n}) = \frac{a+b+c+d}{4} - \frac{l+m+n+p}{4}, H(\tilde{n}) \in [-1,1], \quad (5)$$

where the larger the value of $H(\tilde{n})$, the higher the degree of accuracy of the trapezoidal neutrosophic number $\tilde{n}$. Especially when $b = c$, $f = g$, and $m = n$ hold in a trapezoidal neutrosophic number $\tilde{n}$, Eq. (5) reduces to the following accuracy function of the triangular neutrosophic number:

$$H(\tilde{n}) = \frac{a+2b+d}{4} - \frac{l+2m+p}{4}, H(\tilde{n}) \in [-1,1], \quad (6)$$

which is considered as a special case of Eq. (5).

Based on the score function S and the accuracy function H, we give an order relation between two trapezoidal neutrosophic numbers.

Definition 10. Let $\tilde{n}_1 = \langle (a_1,b_1,c_1,d_1),(e_1,f_1,g_1,h_1),(l_1,m_1,n_1,p_1) \rangle$ and $\tilde{n}_2 = \langle (a_2,b_2,c_2,d_2),(e_2,f_2,g_2,h_2),(l_2,m_2,n_2,p_2) \rangle$ be two trapezoidal neutrosophic numbers. Thus, $S(\tilde{n}_1)$ and $S(\tilde{n}_2)$ are the scores of $\tilde{n}_1$ and $\tilde{n}_2$, respectively, and $H(\tilde{n}_1)$ and $H(\tilde{n}_2)$ are the accuracy degrees of $\tilde{n}_1$ and $\tilde{n}_2$, respectively. Then the order relation between two trapezoidal neutrosophic numbers is defined as follows:

(1) If $S(\tilde{n}_1) > S(\tilde{n}_2)$, then $\tilde{n}_1 > \tilde{n}_2$;

(2) If $S(\tilde{n}_1) = S(\tilde{n}_2)$, and

(a) if $H(\tilde{n}_1) = H(\tilde{n}_2)$, then $\tilde{n}_1 = \tilde{n}_2$;

(b) if $H(\tilde{n}_1) > H(\tilde{n}_2)$, then $\tilde{n}_1 > \tilde{n}_2$.

4. Aggregation operators of trapezoidal neutrosophic numbers

The weighted arithmetic averaging operator and the weighted geometric averaging operator are usually used for information aggregation in decision making. Based on Definition 7, we propose the following two aggregation operators of trapezoidal neutrosophic numbers.

4.1. Trapezoidal neutrosophic number weighted arithmetic averaging operator

Definition 11. Let $\tilde{n}_j = \langle (a_j, b_j, c_j, d_j), (e_j, f_j, g_j, h_j), (l_j, m_j, n_j, p_j) \rangle$ (j = 1, 2, …, n) be a collection of trapezoidal neutrosophic numbers. Then a trapezoidal neutrosophic number weighted arithmetic averaging (TNNWAA) operator is defined as follows:

$$TNNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = w_1\tilde{n}_1 \oplus w_2\tilde{n}_2 \oplus \cdots \oplus w_n\tilde{n}_n = \bigoplus_{j=1}^{n} \left(w_j \tilde{n}_j \right), \quad (7)$$

where w_j (j =1, 2, …, n) is the weight of the jth trapezoidal neutrosophic number $\tilde{n}_j$ (j =1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Based on the operational rules of trapezoidal neutrosophic numbers in Definition 7, we can derive the following theorem.

Theorem 1. Let $\tilde{n}_j = \langle (a_j, b_j, c_j, d_j), (e_j, mf_j, g_j, h_j), (l_j, m_j, n_j, p_j) \rangle$ (j = 1, 2, …, n) be a collection of trapezoidal neutrosophic numbers. Thus, their aggregated value using the TNNWAA operator is also a trapezoidal neutrosophic number, and then

$$TNNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = w_1\tilde{n}_1 \oplus w_2\tilde{n}_2 \oplus \cdots \oplus w_n\tilde{n}_n = \bigoplus_{j=1}^{n}\left(w_j\tilde{n}_j\right)$$

$$= \left\langle \left(1-\prod_{j=1}^{n}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-b_j\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-c_j\right)^{w_j}, 1-\prod_{j=1}^{n}\left(1-d_j\right)^{w_j}\right), \right. \tag{8}$$
$$\left. \left(\prod_{j=1}^{n}e_j^{w_j}, \prod_{j=1}^{n}f_j^{w_j}, \prod_{j=1}^{n}g_j^{w_j}, \prod_{j=1}^{n}h_j^{w_j}\right), \left(\prod_{j=1}^{n}l_j^{w_j}, \prod_{j=1}^{n}m_j^{w_j}, \prod_{j=1}^{n}n_j^{w_j}, \prod_{j=1}^{n}p_j^{w_j}\right) \right\rangle$$

where w_j (j =1, 2, …, n) is the weight of the jth trapezoidal neutrosophic number $\tilde{n}_j$ (j =1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Proof: The proof of Eq. (8) can be done by means of mathematical induction.

(1) When n = 2, then

$$w_1\tilde{n}_1 = \left\langle (1-(1-a_1)^{w_1}, 1-(1-b_1)^{w_1}, 1-(1-c_1)^{w_1}, 1-(1-d_1)^{w_1}) \right.$$
$$\left. (e_1^{w_1}, f_1^{w_1}, g_1^{w_1}, h_1^{w_1}), (l_1^{w_1}, m_1^{w_1}, n_1^{w_1}, p_1^{w_1}) \right\rangle,$$

$$w_2\tilde{n}_2 = \left\langle (1-(1-a_2)^{w_2}, 1-(1-b_2)^{w_2}, 1-(1-c_2)^{w_2}, 1-(1-d_2)^{w_2}) \right.$$
$$\left. (e_2^{w_2}, f_2^{w_2}, g_2^{w_2}, h_2^{w_2}), (l_2^{w_2}, m_2^{w_2}, n_2^{w_2}, p_2^{w_2}) \right\rangle.$$

Thus,

$$TNNWAA\left(\tilde{n}_1, \tilde{n}_2\right) = w_1\tilde{n}_1 \oplus w_2\tilde{n}_2$$
$$= \left\langle (1-(1-a_1)^{w_1} + 1-(1-a_2)^{w_2} - (1-(1-a_1)^{w_1})(1-(1-a_2)^{w_2}), \right.$$
$$1-(1-b_1)^{w_1} + 1-(1-b_2)^{w_2} - (1-(1-b_1)^{w_1})(1-(1-b_2)^{w_2}),$$
$$1-(1-c_1)^{w_1} + 1-(1-c_2)^{w_2} - (1-(1-c_1)^{w_1})(1-(1-c_2)^{w_2}),$$
$$1-(1-d_1)^{w_1} + 1-(1-d_2)^{w_2} - (1-(1-d_1)^{w_1})(1-(1-d_2)^{w_2})), \tag{9}$$
$$\left. (e_1^{w_1}e_2^{w_2}, f_1^{w_1}f_2^{w_2}, g_1^{w_1}g_2^{w_2}, h_1^{w_1}h_2^{w_2}), (l_1^{w_1}l_2^{w_2}, m_1^{w_1}m_2^{w_2}, n_1^{w_1}n_2^{w_2}, p_1^{w_1}p_2^{w_2}) \right\rangle$$
$$= \left\langle (1-(1-a_1)^{w_1}(1-a_2)^{w_2}, 1-(1-b_1)^{w_1}(1-b_2)^{w_2}, \right.$$
$$1-(1-c_1)^{w_1}(1-c_2)^{w_2}, 1-(1-d_1)^{w_1}(1-d_2)^{w_2}),$$
$$\left. \left(\prod_{j=1}^{2}e_j^{w_j}, \prod_{j=1}^{2}f_j^{w_j}, \prod_{j=1}^{2}g_j^{w_j}, \prod_{j=1}^{2}h_j^{w_j}\right), \left(\prod_{j=1}^{2}l_j^{w_j}, \prod_{j=1}^{2}m_j^{w_j}, \prod_{j=1}^{2}n_j^{w_j}, \prod_{j=1}^{2}p_j^{w_j}\right) \right\rangle$$

(2) When $n = k$, by using Eq. (8), we obtain

$$TNNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_k) = w_1\tilde{n}_1 \oplus w_2\tilde{n}_2 \oplus \cdots \oplus w_k\tilde{n}_k = \bigoplus_{j=1}^{k}\left(w_j\tilde{n}_j\right)$$

$$= \left\langle \left(1-\prod_{j=1}^{k}\left(1-a_j\right)^{w_j}, 1-\prod_{j=1}^{k}\left(1-b_j\right)^{w_j}, 1-\prod_{j=1}^{k}\left(1-c_j\right)^{w_j}, 1-\prod_{j=1}^{k}\left(1-d_j\right)^{w_j}\right), \right. \tag{10}$$

$$\left. \left(\prod_{j=1}^{k} e_j^{w_j}, \prod_{j=1}^{k} f_j^{w_j}, \prod_{j=1}^{k} g_j^{w_j}, \prod_{j=1}^{k} h_j^{w_j}\right), \left(\prod_{j=1}^{k} l_j^{w_j}, \prod_{j=1}^{k} m_j^{w_j}, \prod_{j=1}^{k} n_j^{w_j}, \prod_{j=1}^{k} p_j^{w_j}\right) \right\rangle$$

(3) When $n = k + 1$, by applying Eqs. (9) and (10), we can get

$$TNNWAA(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_{k+1})$$

$$= \left\langle \left(1-\prod_{j=1}^{k}(1-a_j)^{w_j} + 1-(1-a_{k+1}))^{w_{k+1}} - (1-\prod_{j=1}^{k}(1-a_j)^{w_j})(1-(1-a_{k+1})^{w_{k+1}}), \right. \right.$$

$$1-\prod_{j=1}^{k}(1-b_j)^{w_j} + 1-(1-b_{k+1})^{w_{k+1}} - (1-\prod_{j=1}^{k}(1-b_j)^{w_j})(1-(1-b_{k+1})^{w_{k+1}}),$$

$$1-\prod_{j=1}^{k}(1-c_j)^{w_j} + 1-(1-c_{k+1})^{w_{k+1}} - (1-\prod_{j=1}^{k}(1-c_j)^{w_j})(1-(1-c_{k+1})^{w_{k+1}}),$$

$$\left. 1-\prod_{j=1}^{k}(1-d_j)^{w_j} + 1-(1-d_{k+1})^{w_{k+1}} - (1-\prod_{j=1}^{k}(1-d_j)^{w_j})(1-(1-d_{k+1})^{w_{k+1}}) \right),$$

$$\left. \left(\prod_{j=1}^{k+1} a_j^{w_j}, \prod_{j=1}^{k+1} b_j^{w_j}, \prod_{j=1}^{k+1} c_j^{w_j}, \prod_{j=1}^{k+1} d_j^{w_j}\right), \left(\prod_{j=1}^{k+1} l_j^{w_j}, \prod_{j=1}^{k+1} m_j^{w_j}, \prod_{j=1}^{k+1} n_j^{w_j}, \prod_{j=1}^{k+1} p_j^{w_j}\right) \right\rangle$$

$$= \left\langle \left(1-\prod_{j=1}^{k+1}(1-a_j)^{w_j}, 1-\prod_{j=1}^{k+1}(1-b_j)^{w_j}, 1-\prod_{j=1}^{k+1}(1-c_j)^{w_j}, 1-\prod_{j=1}^{k+1}(1-d_j)^{w_j}\right), \right.$$

$$\left. \left(\prod_{j=1}^{k+1} e_j^{w_j}, \prod_{j=1}^{k+1} f_j^{w_j}, \prod_{j=1}^{k+1} g_j^{w_j}, \prod_{j=1}^{k+1} h_j^{w_j}\right), \left(\prod_{j=1}^{k+1} l_j^{w_j}, \prod_{j=1}^{k+1} m_j^{w_j}, \prod_{j=1}^{k+1} n_j^{w_j}, \prod_{j=1}^{k+1} p_j^{w_j}\right) \right\rangle .$$

Therefore, according to the above results, we obtain Eq. (8) for any n. This completes the proof. □

Especially when $\boldsymbol{W} = (1/n, 1/n, \ldots, 1/n)^{\mathrm{T}}$, then TNNWAA operator reduces to a trapezoidal neutrosophic number arithmetic averaging operator.

It is obvious that the TNNWAA operator has the following properties (P1)-(P3):

(P1) Idempotency: Let $\tilde{n}_j = \left\langle \left(a_j, b_j, c_j, d_j\right), \left(e_j, f_j, g_j, h_j\right), \left(l_j, m_j, n_j, p_j\right) \right\rangle$ (j = 1, 2, …, n) be a collection of trapezoidal neutrosophic numbers. If each $\tilde{n}_j$ (j = 1, 2, …, n) is equal to $\tilde{n}$, i.e. $\tilde{n}_j = \tilde{n}$ for j = 1, 2, …, n, then

$$TNNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \tilde{n}\,. \quad (11)$$

(P2) Boundedness: Let $\tilde{n}_j = \langle (a_j, b_j, c_j, d_j), (e_j, f_j, g_j, h_j), (l_j, m_j, n_j, p_j) \rangle$ (j = 1, 2, …, n) be a collection of trapezoidal neutrosophic numbers. Let

$$\tilde{n}^- = \left\langle \left(\min_j a_j, \min_j b_j, \min_j c_j, \min_j d_j \right), \left(\max_j e_j, \max_j f_j, \max_j g_j, \max_j h_j \right), \left(\max_j l_j, \max_j m_j, \max_j n_j, \max_j p_j \right) \right\rangle$$

,

$$\tilde{n}^+ = \left\langle \left(\max_j a_j, \max_j b_j, \max_j c_j, \max_j d_j \right), \left(\min_j e_j, \min_j f_j, \min_j g_j, \min_j h_j \right), \left(\min_j l_j, \min_j m_j, \min_j n_j, \min_j p_j \right) \right\rangle$$

.

Then

$$\tilde{n}^- \le TNNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) \le \tilde{n}^+\,. \quad (12)$$

(P3) Monotonity: Let $\tilde{n}_j$ (j = 1, 2, …, n) and $\tilde{n}_j^*$ (j = 1, 2, …, n) be two collections of trapezoidal neutrosophic numbers. If $\tilde{n}_j \le \tilde{n}_j^*$ for j = 1, 2, …, n, then

$$TNNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) \le TNNWAA(\tilde{n}_1^*, \tilde{n}_2^*, \cdots, \tilde{n}_n^*)\,. \quad (13)$$

Proof:

(P1) Since $\tilde{n}_j = \tilde{n}$ for j = 1, 2, . . . , n, we have

$$TNNWAA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = w_1\tilde{n}_1 \oplus w_2\tilde{n}_2 \oplus \cdots \oplus w_n\tilde{n}_n = \bigoplus_{j=1}^{n} \left(w_j \tilde{n}_j \right)$$
$$= \left\langle \left(1 - \prod_{j=1}^{n} (1-a_j)^{w_j}, 1 - \prod_{j=1}^{n} (1-b_j)^{w_j}, 1 - \prod_{j=1}^{n} (1-c_j)^{w_j}, 1 - \prod_{j=1}^{n} (1-d_j)^{w_j} \right), \right.$$
$$\left. \left(\prod_{j=1}^{n} e_j^{w_j}, \prod_{j=1}^{n} f_j^{w_j}, \prod_{j=1}^{n} g_j^{w_j}, \prod_{j=1}^{n} h_j^{w_j} \right), \left(\prod_{j=1}^{n} l_j^{w_j}, \prod_{j=1}^{n} m_j^{w_j}, \prod_{j=1}^{n} n_j^{w_j}, \prod_{j=1}^{n} p_j^{w_j} \right) \right\rangle \quad .c$$
$$= \left\langle \left(1 - (1-a)^{\sum_{j=1}^{n} w_j}, 1 - (1-b)^{\sum_{j=1}^{n} w_j}, 1 - (1-c)^{\sum_{j=1}^{n} w_j}, 1 - (1-d)^{\sum_{j=1}^{n} w_j} \right), \right.$$
$$\left. \left(e^{\sum_{j=1}^{n} w_j}, f^{\sum_{j=1}^{n} w_j}, g^{\sum_{j=1}^{n} w_j}, h^{\sum_{j=1}^{n} w_j} \right), \left(l^{\sum_{j=1}^{n} w_j}, m^{\sum_{j=1}^{n} w_j}, n^{\sum_{j=1}^{n} w_j}, p^{\sum_{j=1}^{n} w_j} \right) \right\rangle$$
$$= \langle (a, b, c, d), (e, f, g, h), (l, m, n, p) \rangle = \tilde{n}$$

(P2) Since there is $\tilde{n}^- \le \tilde{n}_j \le \tilde{n}^+$ for j = 1, 2, . . . , n. Thus, there exists $\sum_{j=1}^{n} w_j\tilde{n}^- \le \sum_{j=1}^{n} w_j\tilde{n}_j \le \sum_{j=1}^{n} w_j\tilde{n}^+$. This is $\tilde{n}^- \le \sum_{j=1}^{n} w_j\tilde{n}_j \le \tilde{n}^+$ according to (P1), i.e., $\tilde{n}^- \le TNNWAA(\tilde{n}_1, \tilde{n}_2, ..., \tilde{n}_n) \le \tilde{n}^+$.

(P3) Since $\tilde{n}_j \le \tilde{n}_j^*$ for j = 1, 2, . . . , n, there is $\sum_{j=1}^{n} w_j\tilde{n}_j \le \sum_{j=1}^{n} w_j\tilde{n}_j^*$, i.e., $TNNWAA(\tilde{n}_1, \tilde{n}_2, ..., \tilde{n}_n) \le TNNWAA(\tilde{n}_1^*, \tilde{n}_2^*, ..., \tilde{n}_n^*)$.

Thus, we complete the proofs of these properties. □

4.2. Trapezoidal neutrosophic number weighted geometric averaging operator

Definition 12. Let $\tilde{n}_j = \langle (a_j, b_j, c_j, d_j), (e_j, f_j, g_j, h_j), (l_j, m_j, n_j, p_j) \rangle$ (j = 1, 2, …, n) be a collection of trapezoidal neutrosophic numbers. Then a trapezoidal neutrosophic number weighted geometric averaging (TNNWGA) operator is defined by

$$TNNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \tilde{n}_1^{w_1} \otimes \tilde{n}_2^{w_2} \otimes \cdots \otimes \tilde{n}_n^{w_n} = \bigotimes_{j=1}^{n} \tilde{n}_j^{w_j}, \tag{14}$$

where w_j (j =1, 2, …, n) is the weight of the jth trapezoidal neutrosophic number $\tilde{n}_j$ (j =1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Based on the operational rules of trapezoidal neutrosophic numbers described in Definition 7, we can derive the following theorem.

Theorem 2. Let $\tilde{n}_j = \langle (a_j, b_j, c_j, d_j), (e_j, f_j, g_j, h_j), (l_j, m_j, n_j, p_j) \rangle$ (j = 1, 2, …, n) be a collection of trapezoidal neutrosophic numbers. Thus, their aggregated value using the TNNWGA operator is also a trapezoidal neutrosophic number, and then

$$TNNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \tilde{n}_1^{w_1} \otimes \tilde{n}_2^{w_2} \otimes \cdots \otimes \tilde{n}_n^{w_j} = \bigotimes_{j=1}^{n} \tilde{n}_j^{w_j}$$

$$= \left\langle \left(\prod_{j=1}^{n} a_j^{w_j}, \prod_{j=1}^{n} b_j^{w_j}, \prod_{j=1}^{n} c_j^{w_j}, \prod_{j=1}^{n} d_j^{w_j} \right), \left(1-\prod_{j=1}^{n}(1-e_j)^{w_j}, 1-\prod_{j=1}^{n}(1-f_j)^{w_j}, 1-\prod_{j=1}^{n}(1-g_j)^{w_j}, 1-\prod_{j=1}^{n}(1-h_j)^{w_j} \right), \right.$$
$$\left. \left(1-\prod_{j=1}^{n}(1-l_j)^{w_j}, 1-\prod_{j=1}^{n}(1-m_j)^{w_j}, 1-\prod_{j=1}^{n}(1-n_j)^{w_j}, 1-\prod_{j=1}^{n}(1-p_j)^{w_j} \right) \right\rangle$$

, (15)

where w_j (j =1, 2, …, n) is the weight of the jth trapezoidal neutrosophic number $\tilde{n}_j$ (j =1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

By a similar proof manner of Theorem 1, we can proof Theorem 2, which is not repeated here.

Especially when $\boldsymbol{W} = (1/n, 1/n, \ldots, 1/n)^{\mathrm{T}}$, the TNNWGA operator reduces to a trapezoidal neutrosophic number geometric averaging operator.

It is obvious that the TNNWGA operator has the following properties (P1)-(P3):

(P1) Idempotency: Let $\tilde{n}_j = \left\langle (a_j, b_j, c_j, d_j), (e_j, f_j, g_j, h_j), (l_j, m_j, n_j, p_j) \right\rangle$ (j = 1, 2, …, n) be a collection of trapezoidal neutrosophic numbers. If each $\tilde{n}_j$ (j = 1, 2, …, n) is equal to $\tilde{n}$, i.e. $\tilde{n}_j = \tilde{n}$ for j = 1, 2, …, n, then

$$TNNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) = \tilde{n}. \quad (16)$$

(P2) Boundedness: Let $\tilde{n}_j = \left\langle (a_j, b_j, c_j, d_j), (e_j, f_j, g_j, h_j), (l_j, m_j, n_j, p_j) \right\rangle$ (j = 1, 2, …, n) be a collection of trapezoidal neutrosophic numbers. Let

$$\tilde{n}^- = \left\langle \left(\min_j a_j, \min_j b_j, \min_j c_j, \min_j d_j\right), \left(\max_j e_j, \max_j f_j, \max_j g_j, \max_j h_j\right), \left(\max_j l_j, \max_j m_j, \max_j n_j, \max_j p_j\right) \right\rangle$$

,

$$\tilde{n}^+ = \left\langle \left(\max_j a_j, \max_j b_j, \max_j c_j, \max_j d_j\right), \left(\min_j e_j, \min_j f_j, \min_j g_j, \min_j h_j\right), \left(\min_j l_j, \min_j m_j, \min_j n_j, \min_j p_j\right) \right\rangle$$

.

Then

$$\tilde{n}^- \le TNNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) \le \tilde{n}^+. \quad (17)$$

(P3) Monotonity: Let $\tilde{n}_j$ (j = 1, 2, …, n) and $\tilde{n}_j^*$ (j = 1, 2, …, n) be two collections of trapezoidal neutrosophic numbers. If $\tilde{n}_j \le \tilde{n}_j^*$ for j = 1, 2, …, n, then

$$TNNWGA(\tilde{n}_1, \tilde{n}_2, \cdots, \tilde{n}_n) \le TNNWGA(\tilde{n}_1^*, \tilde{n}_2^*, \cdots, \tilde{n}_n^*). \quad (18)$$

By a similar proof manner of the properties in Subsection 4.1, we can proof these properties, which are not repeated here.

5. Decision-making method based on the TNNWAA and TNNWGA operators

In this section, we develop an approach based on the TNNWAA and TNNWGA operators and the score and accuracy functions to deal with multiple attribute decision-making problems with trapezoidal neutrosophic information.

In a multiple attribute decision-making problem with trapezoidal neutrosophic information, there is a set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$, which satisfies a set of attributes $C = \{C_1, C_2, \ldots, C_n\}$. An alternative on attributes is evaluated by the decision maker and the evaluation values are represented by the form of trapezoidal neutrosophic numbers. Then, we can establish a trapezoidal neutrosophic decision matrix $D = \left(\tilde{d}_{ij}\right)_{m\times n} = \left(\left\langle\left(a_{ij}, b_{ij}, c_{ij}, d_{ij}\right), \left(e_{ij}, f_{ij}, g_{ij}, h_{ij}\right), \left(l_{ij}, m_{ij}, n_{ij}, p_{ij}\right)\right\rangle\right)_{m\times n}$, where $\left(a_{ij}, b_{ij}, c_{ij}, d_{ij}\right) \subset [0,1]$ indicates the degree that the alternative A_i satisfies the attribute C_j, $\left(e_{ij}, f_{ij}, g_{ij}, h_{ij}\right) \subset [0,1]$ indicates the degree that the alternative A_i is uncertain about the attribute C_j, and $\left(l_{ij}, m_{ij}, n_{ij}, p_{ij}\right) \subset [0,1]$ indicates the degree that the alternative A_i doesn't satisfy the attribute C_j with $0 \le d_{ij} + h_{ij} + p_{ij} \le 3$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

In the following, we apply the TNNWAA and TNNWGA operators and the score and accuracy functions to a multiple attribute decision-making problem with trapezoidal neutrosophic information, which can be described as the following procedures:

Step 1. Utilize the TNNWAA operator $\tilde{d}_i = \left\langle\left(a_i, b_i, c_i, d_i\right), \left(e_i, f_i, g_i, h_i\right), \left(l_i, m_i, n_i, p_i\right)\right\rangle = TNNWAA\left(\tilde{d}_{i1}, \tilde{d}_{i2}, \cdots, \tilde{d}_{in}\right)$ or the TNNWGA operator $\tilde{d}_i = \left\langle\left(a_i, b_i, c_i, d_i\right), \left(e_i, f_i, g_i, h_i\right), \left(l_i, m_i, n_i, p_i\right)\right\rangle = TNNWGA\left(\tilde{d}_{i1}, \tilde{d}_{i2}, \cdots, \tilde{d}_{in}\right)$ (i = 1, 2, …, m) to obtain the collective overall trapezoidal neutrosophic numbers of $\tilde{d}_i$ (i = 1, 2, …, m) for each alternative A_i (i = 1, 2, …, m).

Step 2. Calculate the score $S(\tilde{d}_i)$ (i = 1, 2, …, m) of the collective overall trapezoidal neutrosophic numbers of $\tilde{d}_i$ (i = 1, 2, …, m) to rank the alternatives of A_i (i = 1, 2, …, m) (if there is no difference between two scores $S(\tilde{d}_i)$ and $S(\tilde{d}_j)$, then we need to calculate the accuracy degrees $H(\tilde{d}_i)$ and $H(\tilde{d}_j)$ of the collective overall trapezoidal neutrosophic numbers, respectively, to rank the alternatives A_i and A_j according to the accuracy degrees $H(\tilde{d}_i)$ and $H(\tilde{d}_j)$.).

Step 3. Rank all the alternatives of A_i (i = 1, 2, …, m) according to $S(\tilde{d}_i)$ ($H(\tilde{d}_i)$) (i = 1, 2, …, m) and select the best one(s).

Step 4. End.

6. Illustrative example

In this section, an illustrative example of a software selection problem adapted from Ye [7] for a multiple attribute decision-making problem is provided to demonstrate the application and effectiveness of the developed multiple attribute decision-making method under a trapezoidal neutrosophic environment.

Let us consider a software selection problem for a multiple attribute decision-making problem, where five candidate software systems are given as the set of five alternatives A = (A_1, A_2, A_3, A_4, A_5) and the investment company must take a decision according to four attributes: (1) C_1 (the contribution to organization performance); (2) C_2 (the effort to transform from current system); (3) C_3 (the costs of hardware/software investment); (4) C_4 (the outsourcing software developer reliability). Assume that the weighted vector of the four attributes is $\boldsymbol{W}$ = (0.25, 0.25, 0.3, 0.2)$^{\mathrm{T}}$. Then, the five alternatives with respect to the four attributes are evaluated by the decision maker or expert under the trapezoidal neutrosophic environment, thus we can establish the following trapezoidal neutrosophic decision matrix:

$$D=\left[\begin{array}{l}
\langle(0.4,0.5,0.6,0.7),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.1)\rangle \\
\langle(0.3,0.4,0.5,0.5),(0.1,0.2,0.3,0.4),(0.0,0.1,0.1,0.1)\rangle \\
\langle(0.1,0.1,0.1,0.1),(0.1,0.1,0.1,0.1),(0.6,0.7,0.8,0.9)\rangle \\
\langle(0.7,0.7,0.7,0.7),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.1)\rangle \\
\langle(0.0,0.1,0.2,0.2),(0.1,0.1,0.1,0.1),(0.5,0.6,0.7,0.8)\rangle
\end{array}\right.$$

$$\begin{array}{l}
\langle(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3),(0.2,0.3,0.4,0.5)\rangle \\
\langle(0.2,0.3,0.4,0.5),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle \\
\langle(0.0,0.1,0.1,0.2),(0.0,0.1,0.2,0.3),(0.3,0.4,0.5,0.6)\rangle \\
\langle(0.4,0.5,0.6,0.7),(0.1,0.1,0.1,0.1),(0.0,0.1,0.2,0.2)\rangle \\
\langle(0.4,0.4,0.4,0.4),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle
\end{array}$$

$$\begin{array}{l}
\langle(0.3,0.4,0.5,0.6),(0.0,0.1,0.2,0.3),(0.1,0.1,0.1,0.1)\rangle \\
\langle(0.0,0.1,0.1,0.2),(0.1,0.1,0.1,0.1),(0.5,0.6,0.7,0.8)\rangle \\
\langle(0.2,0.3,0.4,0.5),(0.0,0.1,0.2,0.3),(0.1,0.2,0.2,0.3)\rangle \\
\langle(0.2,0.3,0.4,0.5),(0.0,0.1,0.2,0.3),(0.1,0.2,0.3,0.3)\rangle \\
\langle(0.6,0.7,0.7,0.8),(0.1,0.1,0.1,0.1),(0.0,0.1,0.1,0.2)\rangle
\end{array}.$$

$$\left.\begin{array}{l}
\langle(0.3,0.4,0.5,0.6),(0.1,0.1,0.1,0.1),(0.1,0.2,0.3,0.4)\rangle \\
\langle(0.3,0.4,0.5,0.5),(0.0,0.1,0.2,0.3),(0.0,0.1,0.1,0.2)\rangle \\
\langle(0.1,0.2,0.3,0.4),(0.1,0.1,0.1,0.1),(0.3,0.4,0.5,0.6)\rangle \\
\langle(0.1,0.2,0.3,0.4),(0.1,0.1,0.1,0.1),(0.4,0.5,0.6,0.6)\rangle \\
\langle(0.1,0.2,0.3,0.3),(0.1,0.2,0.3,0.4),(0.2,0.3,0.4,0.5)\rangle
\end{array}\right]$$

Hence, we utilize the developed method to obtain the most desirable software system (s), which can be described as follows:

Step 1. Utilize the TNNWAA operator to obtain the collective overall trapezoidal neutrosophic numbers of $\tilde{d}_i$ (i =1, 2, 3, 4, 5) for a software system A_i (i =1, 2, 3, 4, 5) as follows:

$$\tilde{d}_1=\langle(0.2636,\ 0.3656,\ 0.4682,\ 0.5719),\ (0,\ 0.1,\ 0.1741,\ 0.2408),\ (0.1189,\ 0.1512,\ 0.1762,\ 0.1973)\rangle$$

,

$$\tilde{d}_2 = \langle (0.1945, 0.2958, 0.3758, 0.4243), (0, 0.1189, 0.1798, 0.2319), (0, 0.1712, 0.2132, 0.2821) \rangle$$

,

$$\tilde{d}_3 = \langle (0.1081, 0.1848, 0.2421, 0.3245), (0, 0.1, 0.1464, 0.183), (0.2566, 0.3737, 0.4272, 0.5393) \rangle$$

,

$$\tilde{d}_4 = \langle (0.4035, 0.4652, 0.5298, 0.5983), (0, 0.1, 0.1464, 0.183), (0, 0.1699, 0.2366, 0.2366) \rangle,$$

$$\tilde{d}_5 = \langle (0.3454, 0.4287, 0.4599, 0.5218), (0, 0.1149, 0.1481, 0.1737), (0, 0.195, 0.2552, 0.376) \rangle$$

.

Or utilize the TNNWGA operator to obtain the collective overall trapezoidal neutrosophic numbers of $\tilde{d}_i$ (i = 1, 2, 3, 4, 5) for a software system A_i (i =1, 2, 3, 4, 5) as follows:

$$\tilde{d}_1 = \langle (0, 0.2991, 0.4162, 0.5244), (0.0209, 0.1, 0.1809, 0.2639), (0.1261, 0.1745, 0.2266, 0.2835) \rangle$$

,

$$\tilde{d}_2 = \langle (0, 0.2456, 0.2918, 0.3798), (0.0563, 0.1261, 0.1984, 0.2737), (0.1877, 0.2944, 0.3715, 0.4743) \rangle$$

,

$$\tilde{d}_3 = \langle (0, 0.1597, 0.1888, 0.2543), (0.0463, 0.1, 0.1565, 0.2162), (0.3437, 0.45, 0.5422, 0.6655) \rangle$$

,

$$\tilde{d}_4 = \langle (0.2832, 0.3885, 0.4807, 0.5658), (0.0463, 0.1, 0.1565, 0.2162), (0.148, 0.2276, 0.3109, 0.3109) \rangle$$

,

$$\tilde{d}_5 = \langle (0, 0.2912, 0.3756, 0.391), (0.076, 0.121, 0.169, 0.2206), (0.1958, 0.3012, 0.3877, 0.502) \rangle$$

.

Step 2. Calculate the score values of $S(\tilde{d}_i)$ (i = 1, 2, 3, 4, 5) for the collective overall trapezoidal neutrosophic numbers of $\tilde{d}_i$ (i = 1, 2, 3, 4, 5), which are shown in Table 1.

Table 1. The score values for the alternatives utilizing the TNNWAA and TNNWGA operators

Alternative A_i	Score value (TNNWAA)	Score value (TNNWGA)
A_1	0.7092	0.6553
A_2	0.6744	0.5779
A_3	0.5694	0.5069
A_4	0.7437	0.6835
A_5	0.7077	0.5904

Step 3. Rank all the software systems of A_i (i = 1, 2, 3, 4, 5) according to the score values in Table 1, which are shown in Table 2. Note that "$\succ$" means "preferred to". We can see that two kinds of ranking orders of the alternatives are identical and the most desirable software system is the alternative A_4.

Table 2. Ranking orders of the alternatives

Aggregation operator	Ranking order
TNNWAA	$A_4 \succ A_1 \succ A_5 \succ A_2 \succ A_3$
TNNWGA	$A_4 \succ A_1 \succ A_5 \succ A_2 \succ A_3$

Compared with the relevant paper [7] which proposed the trapezoidal intuitionistic fuzzy decision-making approach, the decision information used in [7] is trapezoidal intuitionistic fuzzy sets, whereas the decision information in this paper is trapezoidal neutrosophic sets. As mentioned above, the trapezoid neutrosophic set is a further generalization of a trapezoid intuitionistic fuzzy set. So the decision-making method proposed in this paper is more typical in applications. Furthermore, the decision-making approach proposed in this paper can be used to solve not only decision-making problems with triangular and trapezoid intuitionistic fuzzy information but also decision-making problems with triangular and trapezoidal neutrosophic information, whereas the decision-making method in [7] is only suitable for decision-making problems with triangular and trapezoidal intuitionistic fuzzy information and a special case of the decision-making method proposed in this paper. Therefore, the decision-making method proposed in the paper is a generalization of existing decision-making methods with triangular and trapezoidal intuitionistic fuzzy information.

7. Conclusion

This paper presented a trapezoidal neutrosophic set and its score and accuracy functions. Then, the TNNWAA and TNNWGA operators were proposed to aggregate the trapezoidal neutrosophic information. Furthermore, based on the TNNWAA and TNNWGA operators and the score and accuracy functions we have developed a trapezoidal neutrosophic multiple attribute decision-making approach, in which the evaluation values of alternatives on the attributes take the form of trapezoidal neutrosophic numbers. The TNNWAA and TNNWGA operators are utilized to aggregate the trapezoidal neutrosophic information corresponding to each alternative to obtain the collective overall values of the alternatives, and then the alternatives are ranked according to the values of the score and accuracy functions to select the most desirable one(s). Finally, an illustrative example of software selection was given to demonstrate the application and effectiveness of the developed method.

The advantage of the proposed method is more suitable for solving multiple attribute decision-making problems with trapezoidal neutrosophic information because trapezoidal neutrosophic sets can handle indeterminate and inconsistent information and are the extension of trapezoidal intuitionistic fuzzy sets. The future work is to develop other aggregated algorithms for some other practical decision-making problems, such as supply chain management and water resource schedule.

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