Examination of combination rules for the purpose of Information Fusion in C2 systems

Application of Dezert-Smarandache Theory

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Abstract—**This paper presents an analysis of known rules of combination as well as a new method of combining uncertain evidence.**

The author concentrates on examination of the rules with accordance to target threat models. The examination have been taken with usage of the predefined measuring scenarios applied to information sources.

Keywords-attribute fusion, Theory of Evidence, DSmT, rules of combination, threat models, Command & Control systems

I. INTRODUCTION

Contemporary Command & Control systems should be prepared for integration of information gathered from diverse sources. It is obvious that together with technological progress new generation sensors need to be plugged-in in order to keep the defence and security up-to-date on the required level. On the other hand, the existing verified and certified sources do not become useless, and still provide valuable information. This variety of information sources, diversified ontologically, causes that specific processing (including lexical translations) needs to be performed in order to keep particular subsystems compatible. This in turn often generates errors, and in the consequence raises uncertainty of the elaborated final decisions.

During last three decades many solutions for dealing with the above mentioned uncertainty have been proposed. Omnipresence of the uncertainty, even while determining technical parameters of the sources, had made many researchers found Theory of Evidence (Dempster-Shafer Theory) very attractive. Dezert-Smarandache Theory (*DSmT*) performs an extension of the original Theory of Evidence by Shafer, and proposes several modification of attributes model construction and hypotheses conflict distribution. As the result of these modification there are many fusion formulas in Theory of Evidence, called combination rules.

Diversity of the existing combination rules bears testimony to the fact that there is no universal combination rule, adequate in every fusion case, and in every condition.

Combination rules perform tools for integration of so called basic belief assignments (bbas) i.e. substitutes of probability distributions in Evidence Theory, which are obtained based on qualitative parameters of the sources (constant and variable), observation distances and many other factors that influence on the process of observation.

In the preliminary stage of the research, not presented in this paper, the author had selected certain rules of combination based on their mathematical properties and relevance for C2 systems applications. This had been performed in order to distinguish rules which have potential to be applied in C2 systems.

However, the actual choice of the particular rule should not be made without regarding target attribute models and structures of bbas. The closer to reality the model is the more precise fusion result may be expected. On the other hand: the more extensive bba is the more precise fusion result.

The problem of target threat assessment in C2 systems seems to be especially suitable to be solved within the *DSmT* framework for the matter the attribute of target threat according to standards like [7] and [10] consists of values that are in large degree mutually dependent (e.g. FRIEND and ASSUMED FRIEND). Additionally, hierarchy of these values is quite easy to be revealed, distinguishing primary hypotheses: {FRIEND, HOSTILE, and UNKNOWN} and secondary hypotheses {ASSUMED FRIEND, SUSPECT, JOKER, and FAKER}.

Nevertheless, not all (but selected) target threat values known in military literature are taken into account in order to avoid the blackout the idea of the paper which is the examination of combination rules with accordance to particular threat model.

II. THREAT MODELS

In order to compare combination rules it is necessary to define the model of the considered attribute. In the next stages of the research works the following models of the target threat attribute are going to be taken into account:

DSmT free model, where the subsequent secondary hypotheses (ASSUMED FRIEND, SUSPECT, FAKER, and

JOKER) perform subsets of the main classes (primary hypotheses: FRIEND, HOSTILE, UKNOWN).

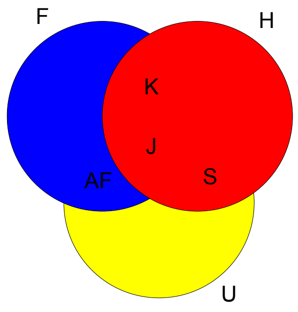

Figure 1. Venn's diagram for the *threat* attribute – the free model

DSmT hybrid model, where the classes FRIEND and HOSTILE are assumed to be disjoint. The rest of the hypotheses is defined in the same manner as in the free model.

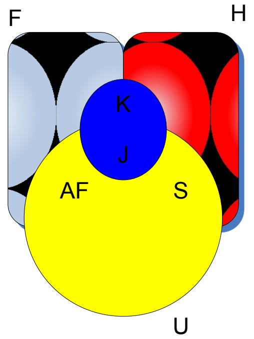

Figure 2. Venn's diagram for the *threat* attribute – the hybrid model

III. Numerical Experiments

The classic rule of combination works with the free model Figure 1. If not all of the hypotheses conjunctions exist in the reality the authors of *DSmT* suggest to use the hybrid rule of combination or any of proportional conflict redistribution *PCR* rules.

During the research works another evolving mechanism for resolving evidence conflicts has also been verified. The mechanism, called decomposition of the conflicting hypothesis, is based on separation of the total mass referring to conflict for two components: strictly conflicting and supporting primary hypotheses. The fundamental difference between this mechanism and *PCR* resides in fact that *PCR* rules operate on *bba* level, where the conflicting mass is transferred with respect to normalization. On the contrary, the decomposition mechanism operates on the belief function level. That means the particular masses are not subject to normalization and they support the respective primary hypotheses according to the belief function calculation procedure.

Due to the fact that the mechanisms of proportional conflict redistribution and decomposition of conflicting hypothesis do not operate on the same level of information processing the research works have been based on comparison of the respective belief function changeability for these two methods.

Particularly, *PCR5* and *DSmC* with two-element decomposition were subject to the comparison. The result of the 'pure' *DSmC* has also been presented as the reference.

In the examination, two sensor fusion scenario was under consideration. It was assumed the first sensor provides a constant *bba*, defined as follows:

$m_1(F) = 0.275$ $m_1(H) = 0.275$ $m_1(U) = 0.05$
$m_1(J) = 0.1$ $m_1(K) = 0.1$ $m_1(AF) = 0.1$
$m_1(S) = 0.1$

The second sensor was assumed to provide the following constant *bba*:

$m_2(U) = 0.03$ $m_2(J) = 0.1$ $m_2(K) = 0.1$
$m_2(AF) = 0.1$ $m_2(S) = 0.1$

Additionally, the mass corresponding to FRIEND hypothesis was gradually increased within <0.01, 0.41> with a step of 0.02 and simultaneous reduction of HOSTILE hypothesis, which may be defined as follows:

$m_2(F) = 0.01 : 0.41$ $m_2(H) = 0.56 : 0.16$

Application of the free *DSmT* model (see Figure 1.) with the classic rule of combination *DSmC* leads to the following belief function changeability

Figure 3. presents changeability of belief functions for two the most dominant hypotheses: FRIEND and HOSTILE. Due to the fact that the *bba* obtained from the first sensor does not show any predominance of one of the mentioned hypotheses over the other, the result of the combination strongly depends on the preset masses of FRIEND and HOSTILE for the second sensor. Based on Figure 3. it is clearly seen that for the mass of $m_2(F)$ residing within <0.01, 0.275> HOSTILE hypothesis should be accepted. Exceeding the value of 0.275 causes a decision change from HOSITLE to FRIEND, which according to the definition of *bba* for the first sensor is intuitive.

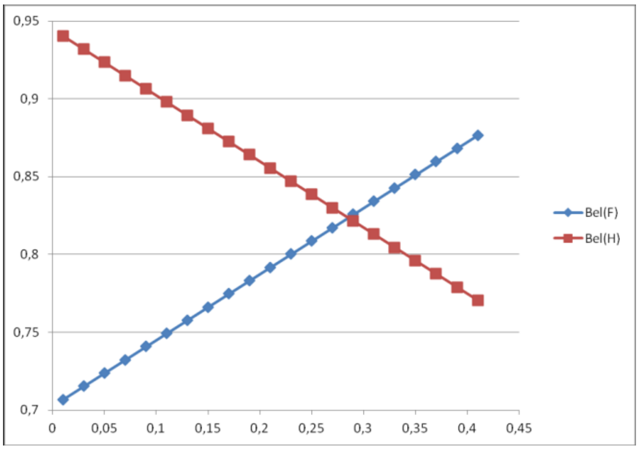

Figure 3. Changeability of belief functions for *DSmC*

Figure 4. presents changeability of belief functions for the hybrid model with *PCR5* applied. Similarly, as in case of

application of the classic rule the decision change is observable for $m_2(F) \cong 0.275$. It is important to notice that maximal values of the respective belief functions paradoxically have lower values than for the classic rule. Given the fact that in *PCR* rules transfer the conflicting mass to the corresponding primary hypotheses it is intuitive to expect relatively higher values of the belief functions. However, the opposite happens as the belief function calculation performs the decisive factor. In case of *PCR5* the primary hypotheses are supplied by relatively lower masses of the secondary hypotheses even though in normalized bbas the primary hypotheses of FRIEND and HOSTILE take higher values than for the classic rule.

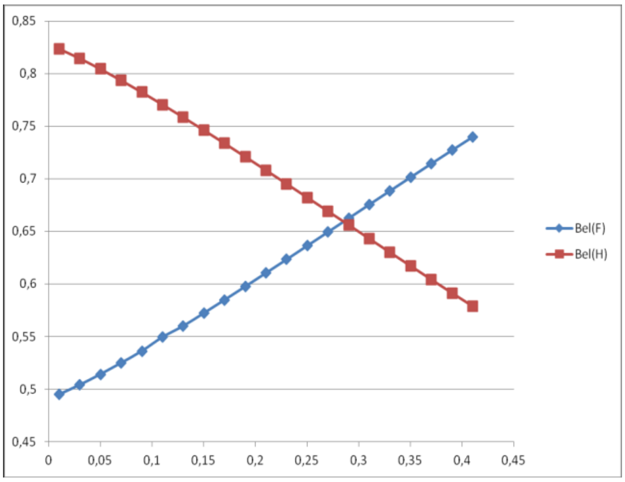

Figure 4. Changeability of belief functions for *PCR5*

Figure 5. presents the changeability of belief functions for the hybrid model (see Figure 2.) with application of the classic *DSmC* rule and two-element conflicting hypothesis decomposition mechanism. In the considered case the conflicting hypothesis is $F \cap H$, which causes the necessity of two secondary hypotheses: FAKER and JOKER.

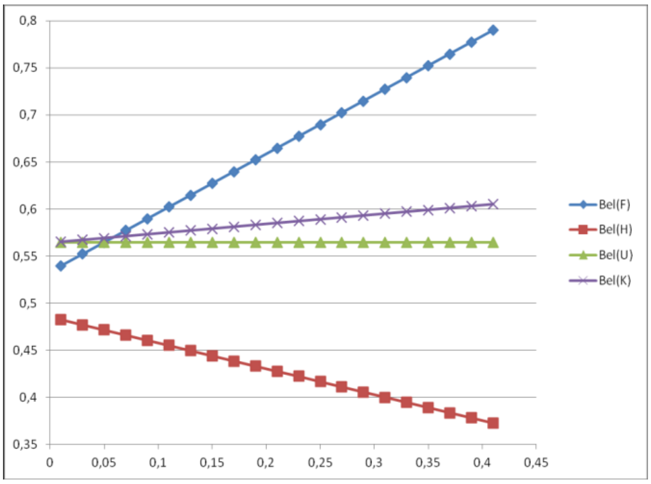

Figure 5. Changeability of belief functions for *DSmC* with two-element FAKER decomposition

The two-element decomposition of FAKER hypothesis is defined as follows:

$$K = K_{CONF} + K_{SPEC} \qquad (1)$$

where:

K_{CONF} - 'conflicting' FAKER i.e. $F \cap H$,
K_{SPEC} - 'specific' FAKER i.e. $\{K \cap K, F \cap K, K \cap H\}$

Analogically, the two-element JOKER decomposition may be performed in the same manner:

$$J = J_{CONF} + J_{SPEC} \qquad (2)$$

where:

J_{CONF} - 'conflicting' JOKER i.e. $\{F \cap S, AF \cap H\}$

J_{SPEC} – 'specific' JOKER i.e. $\{J \cap J, F \cap J, J \cap H, J \cap U, K \cap J, J \cap S, J \cap AF\}$

Thus the corresponding belief functions may be calculated as follows:

$$Bel(F) = m_{12}(F) + m_{12}(AF) + m_{12}(K) + m_{12}(J) \qquad (3)$$
$$Bel(H) = m_{12}(H) + m_{12}(S) + m_{12}(K_{CONF}) + m_{12}(J_{CONF}) \qquad (4)$$

where:

$m_{12}(.)$ – the resulting mass as a combination of evidence from the first sensor and second sensor.

From Figure 5. it can be seen that the decision change from FAKER to FRIEND takes place at $m_2(F) \cong 0.06$. A relatively fast increase of FRIEND hypothesis is observable comparing to slow decrease of HOSTILE hypothesis, which in the considered case is never accepted. This disproportion is due to the fact that FRIEND hypothesis is supplied by conflicting masses and specific masses, corresponding to decomposed training classes while HOSTILE hypothesis is supplied only by the conflicting masses.

Figure 6. presents the changeability of belief functions for the hybrid model (see Figure 2.) with application of the classic *DSmC* rule and three-element conflicting hypothesis decomposition mechanism. Similarly as in the previous experiment two secondary hypotheses: FAKER and JOKER are subject to decomposition.

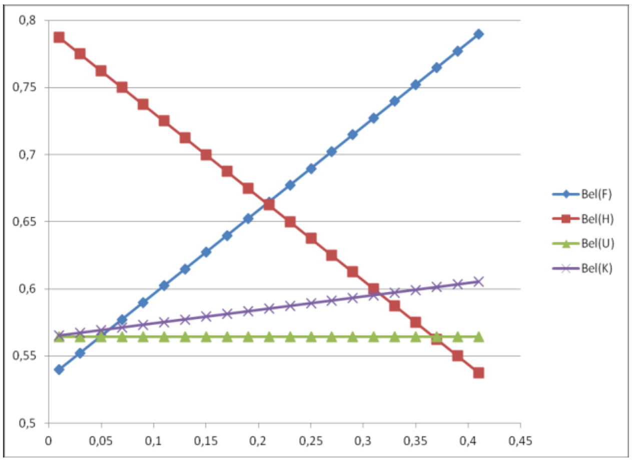

Figure 6. Changeability of belief functions for *DSmC* with three-element FAKER decomposition

The tree-element decomposition of FAKER hypothesis is defined as follows:

$$K = K_{CONF} + K_{KK} + K_{KF} + K_{KH} \quad (5)$$

where:

K_{CONF} - ‘conflicting’ FAKER i.e. $F \cap H$,
K_{KK} - ‘pure’ FAKER i.e. $K \cap K$
K_{KF} - ‘friendly’ FAKER i.e. $F \cap K$
K_{KH} - ‘hostile’ FAKER i.e. $K \cap H$

Analogically, the three-element JOKER decomposition may be performed in the same manner:

$$J = J_{CONF} + J_{JJ} + J_{JF} + J_{JH} \quad (6)$$

where:

J_{CONF} – ‘conflicting’ JOKER i.e. $\{F \cap S, AF \cap H\}$

J_{JJ} - ‘pure’ JOKER i.e. $J \cap J$

J_{JF} - ‘friendly’ JOKER $\{F \cap J, AF \cap J, K \cap J, U \cap J\}$

J_{JH} - ‘hostile’ JOKER $\{J \cap H, J \cap S, K \cap S\}$

Thus the corresponding belief function may be calculated as follows:

$$Bel(F) = m_{12}(F) + m_{12}(AF) + m_{12}(K_{CONF}) + m_{12}(K_{KK}) + m_{12}(K_{KF}) + m_{12}(J_{CONF}) + m_{12}(J_{JJ}) + m_{12}(J_{JF}) \quad (7)$$

$$Bel(H) = m_{12}(H) + m_{12}(S) + m_{12}(K_{CONF}) + m_{12}(K_{KH}) + m_{12}(J_{CONF}) + m_{12}(J_{KH}) \quad (8)$$

where:

$m_{12}(.)$ – the resulting mass as a combination of evidence from the first sensor and second sensor.

From Figure 6. it can be seen that the decision change from HOSTILE to FRIEND takes place at $m_2(F) \cong 0.21$ which is insignificantly lower than in case of applying *PCR5* and the classic rule of combination without the decomposition mechanism. The observed increase of mass corresponding to FRIEND hypothesis is equal to decrease of HOSTILE hypothesis.

IV. Summary of the Research Works

The results presented herein indicate significant differences in changeability of the belief functions corresponding to particular rules of combination. Taking the changeability of belief functions for *DSmC* as the baseline it is important to notice that for the next of the examined rules: *PCR5* and *DSmC* + decomposition lower maximum values of the belief functions were observed. In particular, for *DSmC* + decomposition (equally for two-element and three-element decomposition) maximal belief function values were below 0.8.

The mechanism of two-element decomposition of the conflicting hypothesis does not seem to very useful in practical applications due to significant values of so called decision deviation i.e. a measure of the symmetry of the decision for all possible fusion scenarios (see [9]). It was presented mainly as the reference for three-element decomposition mechanism.

Application of *DSmC* with three-element conflicting hypothesis decomposition mechanism provides similar results as *PCR5*. However, the intersection of straight lines of maximal belief functions, and thus the decision change, occurs with slightly lower value than for the examined conflict redistribution rule. It is worth of consideration which of these results better fits reality. With given *bba* for the first sensor the masses of the contradictory hypotheses of FRIEND and HOSTILE are equal to 0.275. The decision change at 0.275 seems to be intuitive. It is important that the rest of the hypotheses included in *bba* i.e. SUSPECT, ASSUMED FRIEND, JOKER, and FAKER supplies the primary hypotheses in diverse degree. Even though they are equally distributed FRIEND hypothesis is supported by larger number of secondary hypotheses i.e. ASSUMED FRIEND, JOKER, and FAKER than HOSTILE hypothesis (supplied only by SUSPECT). Thus application of *DSmC* with three-element conflicting hypothesis decomposition mechanism may be more adequate in the considered fusion case.

V. Conclusions

In this paper an analysis of known rules of combination as well as a new method of combining uncertain evidence has been presented. The examination have been taken with usage of the predefined measuring scenarios applied to information sources.

After preliminary comparative analysis and numerical experiments there have been selected rules which may be useful in C2 systems. However the results are not satisfactory for unambiguous appointment of the optimal rule for the considered fusion case. In the author’s opinion the final decision should be taken after scrutiny with usage of simulators, which enable to establish the necessary statistics, and also to compare the elaborated fusion results with the ground truth.

Acknowledgment

This work was supported by the National Centre for Research and Development for the years 2009-2011 under Commissioned Research Project MNiSW-OR00007509.

References

[1] Shafer G.: *A mathematical theory of evidence*, Princeton U.P., Princeton, NJ, 1976

[2] Smarandache F., Dezert J., *Advances and Applications of DSmT for Information Fusion*, Vol 1, American Research Press Rehoboth, 2004.

[3] Smarandache F., Dezert J., *Advances and Applications of DSmT for Information Fusion*, Vol 2, American Research Press Rehoboth, 2006.

[4] Smarandache F., Dezert J., *Advances and Applications of DSmT for Information Fusion*, Vol 3, American Research Press Rehoboth, 2009

[5] Inagaki T., Independence between safety-control policy and multiple-sensor schemes via Dempster-Shafer theory, IEEE Trans. On reliability, Vol. 40, no. 2 pp. 182-188, 1991

[6] Sentz K., Ferson S. Combination of Evidence in Dempster-Shafer Theory, SAND 2002-0835

[7] NATO Standardization Agency, *Tactical Data Exchange – Link 16*, STANAG No. 5516, Ed. 3

[8] Krenc K., A. Kawalec: An evaluation of the attribute information for the purpose of DSmT fusion in C&C systems, Fusion2008, Cologne, ISBN 978-3-00-024883-2, 2008.

[9] Krenc K., A. Kawalec, T. Pietkiewicz: *Does Basic Belief Assignments definition affect Information Fusion quality?*, Military Communications and Information Technology: A Comprehensive Approach Enabler, Warszawa 2011, ISBN 978-83-62954-20-9

[10] The Joint C3 Information Exchange Data Model, Edition 3.1b, 2007