Analysis of information fusion combining rules under the DSm theory using ESM inputs

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Abstract— In the context of Electronic Support Measures, the use of the Dempster-Shafer Theory is not flexible enough to obtain a clear evaluation of the state of allegiance for a detected target. With the new theory of plausible, paradoxical, and neutrosophic reasoning, the Dezert-Smarandache Theory, we are able to get a clearer assessment. The current paper presents our research for these cases.

Keywords: Dempster-Shafer, Dezert-Smarandache, Combining rules, Electronic support measures.

I. INTRODUCTION

As introduced in [1], the Dezert-Smarandache Theory (DSmT) is able to combine information even in the presence of large conflicts and constraints. Not only does the DSmT circumvent the well known problems of the Dempster-Shafer Theory (DST) [2] reported by Zadeh in [3], but it can also help us get a more precise assessment of the possible truth as we will see in section (I-B.1). Even if the DSmT resolves Zadeh's problem with the DST, one still faces a problem with the DSmT which was exposed in [4], namely that combinations under DSmT were too complex. However, this was not true anymore after the work presented in [5]. We are now able to easily experiment with the DSmT in different cases never explored before due to its apparent complexity.

As mentioned in the abstract, the current paper will present our work on an analysis between the DST and the DSmT in a ESM input environment. We will begin by a brief review of concepts required for the paper in this section, followed by theorical adaptations and the main aspects of the implementation process in section (II). We then present a selection of results and decisions taken by the implementation for a representative example in section (III). The final section presents our conclusions and a review of our results and possible future research with the DSmT.

A. Combining rules for data fusion

1) Definitions:

- *Basic Set* (Θ) : $\Theta = \{\theta_1, \theta_2, \ldots \theta_n\}$. It's the set including every possible object θ_i. This set is exhaustive and its elements are not exclusive.
- *Power set* (2^Θ) : represents the set of all possible sets using the objects (singletons) of the basic set Θ. It includes the empty set and excludes intersections. With the basic set defined above, we get the power set:

$$2^\Theta = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \ldots \{\theta_n\}, \{\theta_1, \theta_2\},$$

$$\ldots \{\theta_1, \theta_2, \ldots, \theta_n\}, \ldots, \Theta\}.$$

- *Hyper-power set* (D^Θ) : represents the set of all possible sets using the objects of the basic set Θ and allowing intersections between singletons. It includes the empty set. With the basic set: $\Theta = \{\theta_1, \theta_2\}$, we get the hyper-power set

$$D^\Theta = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \{\theta_1 \cap \theta_2\}, \{\theta_1 \cup \theta_2\}\}.$$

- *Conjunctive Power set* $(2^\Theta_\cap)$: represents the set of all possible sets using the objects of Θ. It includes the empty set and excludes disjunctions. Defined as a mathematical object, it helps for the evaluation of the DSm cardinal. With the basic set $\{\theta_1, \theta_2, \ldots \theta_n\}$, we get the conjunctive power set:

$$2^\Theta_\cap = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \ldots \{\theta_n\}, \{\theta_1 \cap \theta_2\},$$

$$\ldots \{\theta_1 \cap \theta_2, \ldots \cap \theta_n\}\}.$$

- *Constraint* : is a set considered impossible to obtain.
- *Constraints power set* (D^Θ_c) : is the set containing all elements considered as constrained in the D^Θ.
- *Basic belief assignment* (bba) : $m : 2^\Theta \to [0, 1]$, so the mass given to a set $A \subseteq \Theta$ obeying $m(A) \in [0, 1]$.
- *Core of Θ* $(\mathcal{K})$: The set of all focal elements of Θ, where a focal element is a subset A of Θ such that $m(A) > 0$.

2) Conjunctive combining rule [7]:
When we refer to the conjunctive combining rule, we will refer to the version described as:

$$q(A) = m_1 \wedge m_2 = \sum_{\substack{B \cap C = A \\ B, C \subseteq \Theta}} m_1(B) m_2(C) \quad \forall A \subseteq \Theta. \quad (1)$$

3) Disjunctive combining rule [8]: When we refer to the disjunctive combining rule, we will refer to the version described as:

$$q(A) = m_1 \vee m_2 = \sum_{\substack{B \cup C = A \\ B,C \subseteq \Theta}} m_1(B) m_2(C) \quad \forall A \subseteq \Theta. \quad (2)$$

4) Dempster-Shafer Theory (DST): The DST rule of combination is a conjunctive normalized rule working on the power set. It combines information with intersections. This theory works with the hypothesis of mathematically independent sources of evidence. The ith bba's source of evidence is denoted m_i. The DST works within 2^Θ.

Equation (3) describes the DST rule of combination where K is the conflict and is $\forall C \subseteq \Theta$, the conflict in DST being defined by equation (4).

$$(m_1 \oplus m_2)(C) = \frac{1}{1-K} \sum_{A \cap B = C} m_1(A) m_2(B) \quad (3)$$

$$K = \sum_{A \cap B = \emptyset} m_1(A) m_2(B) \qquad A, B \subseteq \Theta \quad (4)$$

5) Dezert-Smarandache Theory (DSmT): The DSmT uses the hyper-power set being thus able to work with intersections. The DSmT possesses two rules of combination which are able to work around the mass redistribution problem of the DST in the presence of large conflicts:

- The Classical DSm rule of combination (DSmC) is based on the free model $M^f(\Theta)$

$$m(C) = \sum_{A \cap B = C} m_1(A) m_2(B) \quad \substack{A, B \in D^\Theta \\ \forall C \in D^\Theta} \quad (5)$$

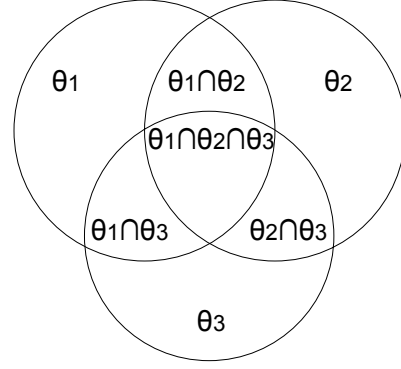- The Hybrid DSm rule of combination (DSmH) is able to work with many types of constraints

$$m_{M(\Theta)}(A) = \phi(A)[S_1(A) + S_2(A) + S_3(A)] \quad (6)$$

$$S_1(A) = \sum_{X_1 \cap X_2 = A} m_1(X_1) m_2(X_2) \quad \forall X_1, X_2 \in D^\Theta \quad (7)$$

$$S_2(A) = \sum_{\substack{[(u(X_1) \cup u(X_2)) = A] \vee \\ [((u(X_1) \cup u(X_2)) \in \emptyset) \wedge (A = I_t)] \\ \forall X_1, X_2 \in \emptyset}} m_1(X_1) m_2(X_2) \quad (8)$$

$$S_3(A) = \sum_{\substack{X_1 \cup X_2 = A \\ X_1 \cap X_2 \in \emptyset}} m_1(X_1) m_2(X_2) \quad \forall X_1, X_2 \in D^\Theta \quad (9)$$

Fig. 1. Venn diagram of degree 3

Note that $\phi(A)$ in equation (6) is a binary function resulting in 0 for empty or impossible sets, and 1 otherwise. In equation (8), $u(X)$ represents the union of all objects of set X. Equation (9) is the union of all objects of sets X_1 and X_2, when it is not empty. From equation (8), I_t represents the total ignorance, or the union of all objects part of the basic set.

The DSmH can also be viewed in an incremental way:
Step S1: if $(\theta_1 \cap \theta_2 \in D_c^\Theta)$, then continue to step S3, otherwise the mass $m_1(X_1) m_2(X_2)$ is added to the mass of $A = (\theta_1 \cap \theta_2)$. **Step S3**: if $(\theta_1 \cup \theta_2 \in D_c^\Theta)$, then continue to step S2, otherwise, the mass $m_1(X_1) m_2(X_2)$ is added to the mass $A = (\theta_1 \cup \theta_2)$. **Step S2**: if $(u(X_1) \cup u(X_2) \in D_c^\Theta)$, then add the mass to I_t, otherwise, the mass $m_1(X_1) m_2(X_2)$ is added to the mass of $A = (u(X_1) \cup u(X_2))$.

Complete examples of the DSmT are available in [6].

B. Electronic Support Measures (ESM)

An ESM is a passive sensor that captures incoming electro-magnetic energy, which after treatment, reveals informations about a detected target, such as direction (bearing), identity, and allegiance of the target that emitted the radiation, together with a belief about the identity and allegiance.

The belief information thus received is interpreted as a mass for each of the three possible ESM declarations which are friendly, neutral and hostile. In the following we will describe them as θ_1 (friendly), θ_2 (neutral) and θ_3 (hostile). Figure (1) presents us the Venn diagram for a case without any constraints.

While the ESM only gives us 3 choices for the allegiance, the decision maker requires 5 possibilities for the allegiance (according to STANAG 1241 - NATO Standardization Agreement), which, in the natural language of the DSmT makes $\theta_1 \cap \theta_2$ correspond to *assumed friendly*, with the other intersections corresponding to *suspect* (involving an intersection with θ_3).

1) DST precision problem: Figure (2) shows us the extent of the limitation of the DST in our case showing it's inability to specify when a target is 'assumed friendly' or 'suspect'. It is equivalent to take $D_c^\Theta = D^\Theta \backslash 2^\Theta$, then all intersections are constraints. Without this ability systems with DST will directly switch between allegiance states avoiding some allegiance possibilities. These inaccessible states can be accessed using the DSmT.

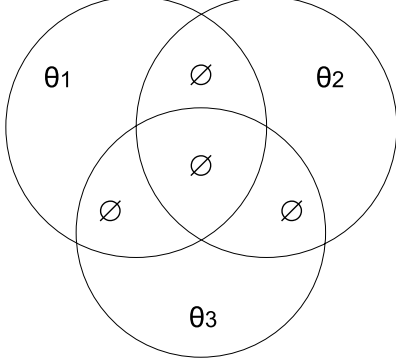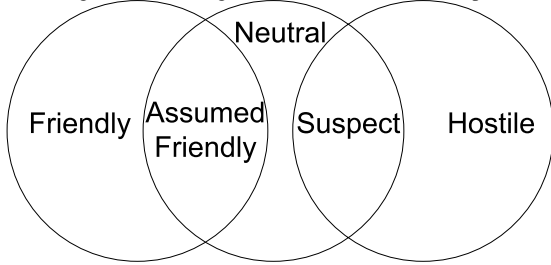Fig. 2. Venn diagram of degree 3 for DST

Fig. 3. Venn diagram for our simulated example

II. THEORY AND IMPLEMENTATION

The main challenges and difficulties encountered in this research are found on the level of the effective, or optimal, implementation of the DSmH combining rule. In the present section, we will review critical points where an original reasoning allowed the implementation of a combining rule originally considered too complex [4], and of the implementation of the Generalized Pignistic Transformation (GPT). There is also the issue of the oscillations which is addressed by two different tools, the first one being the Florea's Quasi-Associativity method [9], the second one being a filter.

A. Constraints on simulation

Our simulation, as mentioned earlier, works with the basic set described by figure (1), however we've added constraints on sets $\{[\theta_1 \cap \theta_3], [\theta_1 \cap \theta_2 \cap \theta_3]\}$. So the simulated example's Venn diagram is represented by figure (3).

B. Combining rules

1) Generated ESM information: We seek to generate as realistic ESM declarations as possible. The situation that we have in mind is one where the ESM declaration could have been caused by multiple targets trying to hide behind each other within the angular accuracy of the ESM's bearing measurement. For example, if one were to track a friendly target, ESM reports from any target within the bearing accuracy of the ESM's bearing measurement could be associated to our friendly. Thus on average we could correctly associate a friendly ESM declaration (say) 70% of the time in a dense environment, with 15% sometimes resulting from miss-association of a "neutral", and "15% from a "hostile", presumably all close in bearing, but far away in range.

Thus we have used a randomly generated ESM declaration with a probability that the object θ_1 is selected 70% of the time for the first 50 iterations, out of 100 iterations (with the remaining 30% split evenly between the other two possibilities). From the 51th iteration to the last one, the object θ_3 is selected 70% of the time (again with the remaining 30% split evenly between the other two possibilities). For each new ESM declaration, the selected allegiance is given the mass of 0.70, with the rest of the mass being given to total ignorance (I_t). In splitting up each Monte-Carlo run at the halfway point, we can see the capability of each rule in identifying a true change in allegiance.

2) Combining rule's results: To insure a certain level of computational safety, and to avoid the DST problem when the conflict approaches the value of 1, we've applied a filter at the output of all combining rules. That filter insures that the mass given to the total ignorance (I_t) is never less than 2%.

3) Data format: The information is contained in the same way described in [5]. Thus, we keep in memory only the core of the bba of the BOEs (Bodies of Evidence). Each core information is kept in a structure containing two objects, one being the matrix of objects, the other one being the vector of masses. Each one of the objects is kept as a product of sums, intersections are kept as vectors, unions are composed of multiple cells in a matrix, each cell being a vector (intersection). Obviously the structure can be recursive to support larger objects.

4) Reasoning frame generation: The generation of 2^Θ and D^Θ is not required for the evaluation of the combining rules, but are required for the evaluation of the generalized pignistic transformation, so we have developed a way of generating 2^Θ, and from it, D^Θ. However, the generation of D^Θ work only for Θ, with $|\Theta| = 3$, which is enough for our case.

To generate 2^Θ, we built a function that uses as parameter the cardinal of Θ (3, in our case) and a factor. That factor tells us which type of power set we want, 2^Θ or $2^\Theta_\cap$. We use that function to generate both, then take their intersection and union to get D^Θ. The cardinal sets the dimension of a matrix of binary numbers that we generate in numerical ascending order.

A second matrix is built, for which each line represents an object. Each odd column is a singleton, and each even column contains a code for a conjunction or a disjunction symbol. The value contained in each odd column is a numerical value coded by taking the value of a counter. That counter, counts the number of odd columns and resets when changing line. To know when to put in a value for each position, the function reads the matrix of binary numbers. In that matrix, the line n corresponds the object represented by the line n in the second matrix. The column m of the matrix of binary numbers represents the singleton θ_m.

After the second matrix is built, a treatment is made to merge columns, put the objects in the appropriate format, sort the information, and attribute temporary mass values.

5) Combining rule implementation: The implementation of rules uses the system developed and presented in [5].

C. Classical Pignistic Transformation

The Classical Pignistic Transformation (CPT) is described in [7] and modified in [1] is represented by:

$$\Pr\{A\} = \sum_{X \in 2^\Theta} \frac{|X \cap A|}{|X|} m(X), \forall A \in 2^\Theta. \quad (10)$$

In equation (10), $|X| = n$ represents the classical cardinality of a set X, and n, the number of elements in the set. The CPT can be used to transform the bba into a probability function in cases within 2^Θ.

D. Generalized Pignistic Transformation

The Generalized Pignistic Transformation (GPT) as described in [1] required modifications so its implementation would be efficient. If one takes a look at the original equation for the GPT, in section (7.3) of [1] we have:

$$\Pr\{A\} = \sum_{X \in D^\Theta} \frac{C_\mathcal{M}(X \cap A)}{C_\mathcal{M}(X)} m(X), \forall A \in D^\Theta. \quad (11)$$

For efficiency purposes, even if A is considered as a set part of D^Θ, only $A \in 2^\Theta_\cap$ are used. This choice has been made to avoid all the redundancy of evaluating pignistic probabilities for all the sets part of D^Θ, considering that we can evaluate disjunctions by simple summations and subtractions as in the probability theory. Theses evaluations can be accomplished using Theorem 2.3.4 as seen in [10] (See example case in equation (12)). It also had the effect of optimizing our simulation system.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (12)$$

For the same reason of efficiency, we've modified the range of X. Rather than $X \in D^\Theta$, which is true, we have restricted it to the core, $\mathcal{K}$. Hence, it will be for the subset of D^Θ which includes only non zero masses. In other words, only the sets given in input will be considered, since our system considers only non empty objects. Note that the GPT requires to take the decision only by choosing the object with the maximum pignistic probability.

1) DSm cardinal: The DSm cardinal could be viewed as presented in chapter 3 of [1]. It can also be viewed and implemented as in the following algorithm. For E, $C_\mathcal{M}(E)$ can be considered as the sum of the number of effective intersections with F, $\forall F \in D^\Theta_*$, where D^Θ_* is the set D^Θ without constraints ($D^\Theta_* = D^\Theta \backslash D^\Theta_c$).

2) GPT implementation: We have implemented the GPT as a function which takes into parameters the current core of Θ and current constraints and then gives us a structure containing a matrix of objects associated with a vector of computed pignistic probabilities. The function evaluates at each call DSm cardinal values for all possible sets of D^Θ_*, and then proceeds with equation (11). When required, a value of any given DSm cardinal is not evaluated but looked up in the table of DSm cardinal values previously built.

Fig. 4. Florea's Quasi-Associativity
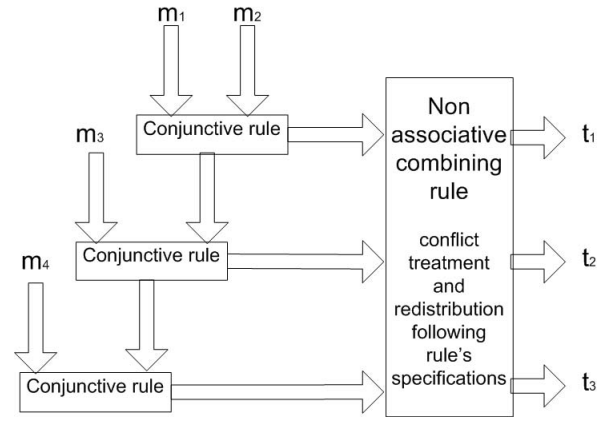
3) Power set cases resolved under the GPT: Our system has only the GPT version of pignistic transformation implemented. This is quite sufficient, since as demonstrated in both examples of section (7.4) of [1], the GPT is able to work with Shafer's model cases considering $D^\Theta_c = D^\Theta \backslash 2^\Theta$. Since our system works under D^Θ, we need to add to the list of constraints the list of all possible conjunctions so the GPT would work as the CPT.

E. Quasi-Associativity (QA)

In our implementation, we've also experimented with an algorithm to implement QA into DSmT. We've experimented with the QA version presented in [9] and a modified version. Florea's algorithm exploits the fact that the conjunctive and disjunctive rules respects the associativity property.

As we see in figure (4), Florea's QA combines the information using the conjunctive rule until time (iteration) τ where we have to make a decision. At that time, we use the new information from time τ and combine it using the non-associative rule with the combined output of the conjunctive rule of time $(\tau - 1)$. Thus turning a non-associative rule to a QA rule.

In [9], it is only shown that the result of such QA rule respect the associativity property. As shown in figure (4), the result of the QA rule at time τ is not used for subsequent combinations, it is the result of the conjunctive rule that is used in subsequently.

Two adaptations were developed from Florea's QA for our research. Since the disjunctive rule converges too quickly toward total ignorance (I_t), it cannot be used as described in [9] with both conjunction and disjunction as it would be required by DSmT. So the first adaptation we had to make, is the use of the conjunctive rule only. The second adaptation is a modification to the first one. This time, instead of the conjunctive rule, we'll be using the DST's combining rule.

1) QA implementation: Our implementation of Florea's QA is not exactly as described by figure (4). The difference is at the point where the non-associative combining rule takes it's input information. Florea's version, takes the input information from the output of the conjunction at time τ, and combines

it to get the output at time τ. In our version, since the non-associative combining rule version has it's own conjunction calculator module, we've taken the combined output of the conjunctive rule of time $(\tau - 1)$ as an input. The implemented system can thus be, and stay, modular and scalable.

F. Filters and preconditioning filter

To avoid an output with a behavior having oscillations, we've tried different types of filters. The first one is rather simple, but doesn't have a theoretical basis other than smoothing the graphical output. The filter applies an arithmetic mean on a window of the output. In a second experimented filter is a little more original, we've tried a filtering by window method on the input mass information. Theoretically, this is based on the hypothesis that the oscillations present in the output may be caused by unprecise, and/or biased input. Thus, input filtering would attenuate the effect of incorrect inputs.

1) Filters implementation: The implementation of the first type of filter, as described in the previous section, is based on the following equation:

$$m_\tau = \frac{\left(\frac{m_{\tau-3}+m_{\tau-2}+m_{\tau-1}}{3}\right) + m_{\tau 0}}{2}, \qquad (13)$$

where $m_{\tau 0}$ is the mass at time τ of the input, and m_τ is the filtered mass for time τ. The filter form was chosen arbitrarily to smoothen the graphical output. We could've tried many types of filter that kept coherent results and let the rule react in a promptly to new information. One of the possibility was to take a continuous window with an exponential weighting, instead of a discrete window.

The implementation of the second type of filter counts the number of occurrences of each of the possible singletons in the last four iterations including the current one. If the current singleton θ_a occurred more than 3 times out of the 4 possibilities, the mass of the bba of the BOE is kept intact. If θ_a occurred only 2 times, it's mass is reduced from 0.70 to 0.60, the balance always going to total ignorance (uncertainty). If θ_a occurs only 1 time, the given mass for θ_a is 0.50, thereby giving more mass to uncertainty, since the input information hasn't proven to be of high quality. Note that we shouldn't reduce the mass of θ_a to values lower than 0.5 since we don't want to give a mass too high for the ignorance.

III. RESULTS AND DECISION

This section presents us graphical outputs for a randomly generated case where we obtained a rate of occurrence of 42.57% for both θ_1 and θ_3 (close to the expected $(70 + 15)/2\%$), with θ_2 having the rest of the occurrences (14.85%). Figure (5) shows us which singleton was detected at which time index. Note that in this section, all figures (5-13) have the X-coordinate representing time index.

A. Cases using the Dempster-Shafer combining rule

1) Decision based on the DS rule: The figure (6) shows the mass of θ_1, $(m(\theta_1))$, as being the solid line. We can see from figure (6) that Dempster-Shafer combination rule has a

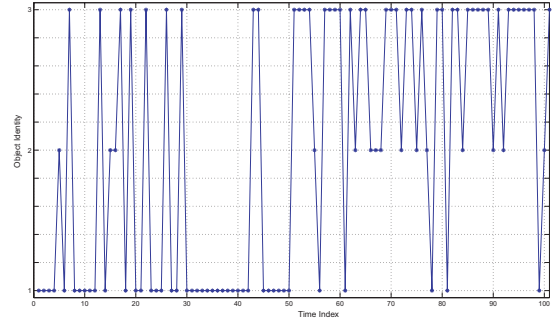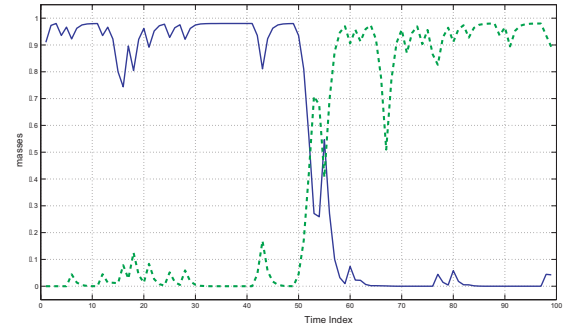Fig. 5. Occurrences of $(\theta_1, \theta_2, \theta_3)$ from the ESM

Fig. 6. $m(\theta_1)$ and $m(\theta_3)$ for DS rule

fairly quick reaction when new input information arrives. Less than ten iterations after the change of allegiance toward θ_3, the dashed-line ($m(\theta_3)$) becomes dominant. At the same time, $m(\theta_1)$ decreases to lower values. No mass ever reaches the value of 1 because of the filter which guarantees a minimum mass of 0.02 to total ignorance (I_t). However, even if the combining result never reaches 1, it gives us a direction toward a specific set with confidence.

2) Decision based on the GPT with DS rule: The dash-dot line, in figure (7), representing the pignistic probability of θ_2 ($\Pr(\theta_2)$), is relatively important through all the time index (which might possibly become a problem in other cases). The solid-line represents $\Pr(\theta_1)$ and the dashed one, $\Pr(\theta_3)$. We

Fig. 7. Pignistic probabilities for $(\theta_1, \theta_2, \theta_3)$ for GPT DS rule

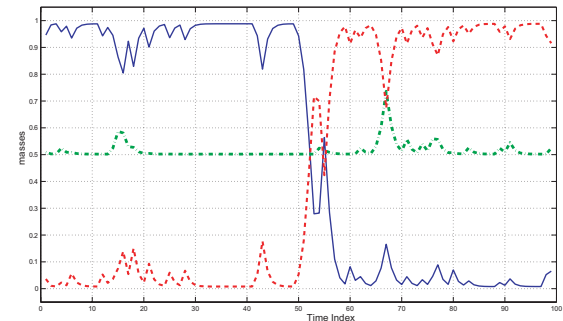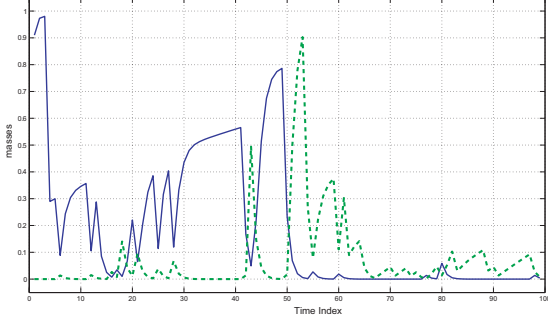Fig. 8. $m(\theta_1)$ and $m(\theta_3)$ for DSmH

Fig. 10. Filtered sum of masses including θ_1 and θ_3 for DSmH

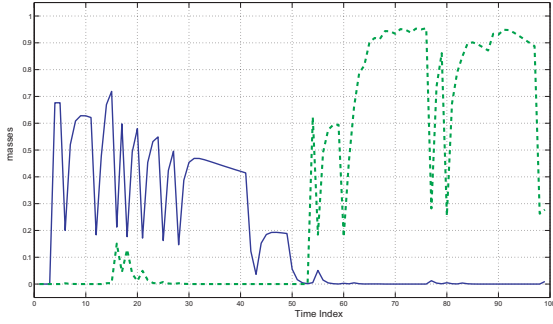Fig. 9. $m(\theta_1 \cap \theta_2)$ and $m(\theta_2 \cap \theta_3)$ for DSmH

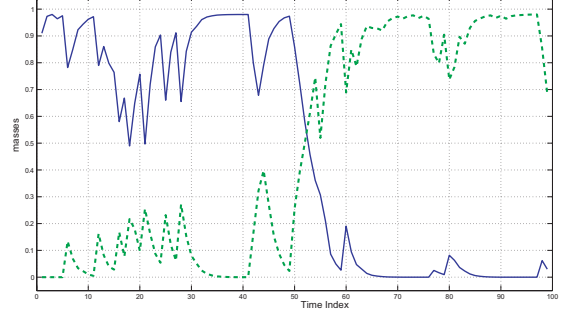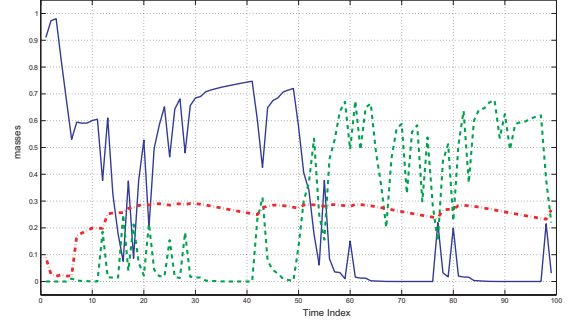Fig. 11. Filtered θ_1, θ_3 and I_t for DSmH using Florea's QA

should also observe that sometimes $\sum_i \Pr(\theta_i) \geq 1$ because we must take into account the pignistic probability given to intersections. Note that, as explained in the introduction, our case requires such basic belief assignment. However, for cases requiring empty intersection, contraints could be added to the process. As it is required by the GPT decision process, we should take the object with the most significant amount of pignistic probability to make the decision at time τ. We can see from the figure (7) that the decision would be taken correctly through all the simulation.

B. Cases using the DSmH combining rule

1) Decision case based on the DSmH: Figures (8, 9) shows masses of four objects for the DSmH combining rule case. The first figure shows $m(\theta_1)$ and $m(\theta_3)$ for which there is little activity with $m(\theta_1)$ being higher than $m(\theta_3)$ for the first half of the simulation, both having little confidence. On the other hand, the second figure, which shows us $m(\theta_1 \cap \theta_2)$, (the solid-line), and $m(\theta_2 \cap \theta_3)$, (the dashed-line), accumulates most of the mass. $m(\theta_1 \cap \theta_2)$ is dominant in the first half of the simulation. $m(\theta_1 \cap \theta_2)$ and $m(\theta_2 \cap \theta_3)$ represents "assumed friend" and "suspect" allegiance state respectively. As we can see, the DSmH has the same problem as the conjunctive rule: accumulating the mass on conjunctions.

2) Decision case based on sum of masses for DSmH: Figure (10) shows two curves, both represents sum of masses including θ_i such as the sum $m(\theta_i) + m(\theta_i \cap \theta_j) + m(\theta_i \cap \theta_k) + m(\theta_i \cup \theta_j) + m(\theta_i \cup \theta_k)$ for $i, j, k \in \{1, 2, 3\}$ where

$i \neq j \neq k$.

Figure (10) shows that we can make decisions bases on DSmH's results after a few simple arithmetics. If we use a threshold of 70% before making a decision, we can make the correct decision most of the time. It should be noted that procceding this way gives the curves of figure (10) which ressembles the DS seen on figure (6). Using curves based on summations using fewer objects lessens the decision's efficiency and increases the oscillation behavior of the output after combination.

3) Decision based on Florea's QA: Figure (11) gives the filtered values for the mass given to θ_1, θ_3 and I_t for a DSmH combination using Florea's QA. Filtering the output was necessary since the unfiltered version was too unstable with an excessive oscillation preventing us from making any decision.

In figure (11), we're able to take a decision. However, the confidence is lower than expected. Except a few times, it never exceeds 70%. It should also be observed that with this combining method, we obtained a high level of $m(I_t)$ as showed by the dash-dot line around mass of 30%. This effect hasn't been observed with the QA version using the DS rule shown in the following section. Further research would be required to investigate the source of the problem.

4) Decision based on Florea's QA using DS rule: The modified version of QA using the DS combining rule presented in figure (12) has the same behavior as the DS combining rule with an increased oscillatory behavior. The solid-line

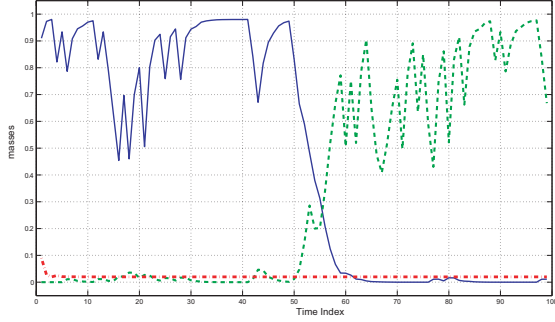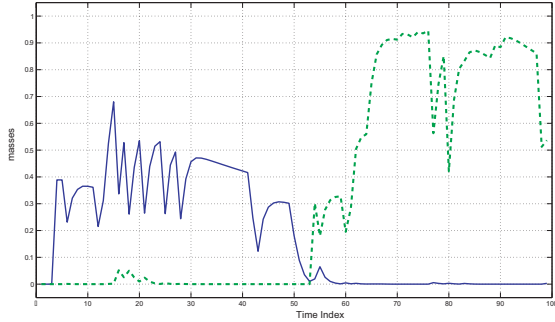Fig. 12. Filtered θ_1, θ_3 and I_t for DSmH with Florea's QA using DS rule

Fig. 13. $m(\theta_1 \cap \theta_2)$ and $m(\theta_2 \cap \theta_3)$ for DSmH with filtered ESM

represents $m(\theta_1)$, $m(\theta_3)$ being the dashed one. The dash-dot line near 0.02 represents the mass given to total ignorance $m(I_t)$.

5) Decision based on filtered ESM input: Figure (13) should not be considered alone but in comparison with figure (9) while looking at the ESM inputs shown in figure (5). We can see that the more an event is isolated, the less that event's occurrence has an impact on the combining rule. As described in section (II-F), the filter has the effect of reducing the mass of locally rare events.

IV. CONCLUSIONS

As introduced earlier, our research covered the use of the considered too complex Dezert-Smarandache Hybrid combining rule for the evaluation of the state of allegiance of detected targets by electronic support measures (ESM). We have compared the behavior of the DSmT in such context with the Dempster-Shafer rule and with different versions of combination rules. We have explored the use of two types of quasi-associativity algorithm (Florea's QA [9], Florea's QA with DS rule), two types of filtering methods (on the combining output's masses, on ESM's bba) and three types of decisions (with masses, with sum of masses, using generalized pignistic transformation).

A. Results review

1) Using the Dempster-Shafer combining rule: As we have seen the DS combining rule enables us to take directly from the output mass an efficient and quick decision. By quick, we mean that it is easy to modify the decision quickly after a change of allegiance. The rule is also resistant to random errors in detection keeping the decision right as it should. When we base our decision on the pignistic probability function the decision is taken correctly even with high level of pignistic probability given to some objects.

2) Using the Dezert-Smarandache hybrid rule: With a threshold of 70%, we are unable to make a decision based on bba. In rare cases, we can do so with intersections, however, even in those cases, the confidence isn't very high. However, when we base the decision on sum of masses as described in section (III-B.2), we are able to make a good decision. Since summed masses with this method adds masses from different objects, we are unable to choose precisely one of them.

3) DSmH using Florea's QA: As in the non-QA version of the DSmH rule, we are faced with a level of confidence not high enough to make a decision based on a threshold of 70%. If we consider a lower threshold, we are able to make the right decision. Note that even filtered, the output has quite an oscillatory behavior. A stronger filter might be recommended. However, we must consider the side effects of such filters, which would slow down the reaction time.

4) DSmH using Florea's QA with DS rule: This version of the Florea's QA uses the DS combining rule instead of the conjunctive rule. As a consequence, the modified DSmH behaves quite like the DS rule with little oscillation of the output. A filter at the output is already in use, a stronger one could be tried. This version of DSmH behaves quite well, making the right decision at the right time and changing it's decision in 10 iterations from a change of allegiance.

5) DSmH using a filter on ESM's bba: In the cases where we used a filter on ESM's basic belief assignment, we've observed a better reaction to locally rare singletons occurrences as it was expected. This filter attenuates the effect of events that occurs rarely in a fixed-size filtering window. Further research should be done to find out how well and how much such filter can help.

B. Encountered problems to investigate

1) With the DSm hybrid rule: The DSmH seems unable to concentrate a wide proportion of the mass to a specific object. Most of the time, it has divided the confidence in too many objects to get a high enough confidence on a single one. To palliate to this problem, we've tried to add the mass of different objects as described earlier, in section (III-B.2). However, this method has the problem of removing the advantage of using the DSmH rule, which was to get a clearer assessment on a target's allegiance. A possible method to investigate would be to lower the decision threshold when using the DSmH.

2) With DSmH using Florea's QA: As said earlier, the DSmH case using Florea's QA doesn't give high confidence to θ_1 nor θ_3. A careful look at figure (11), shows that a great level of mass is given to ignorance. We haven't found errors or mistakes in the implementation of the rule, so we may have to investigate further the rule's behavior to find the cause.

C. Possible future works on the DSmT

Our work did not cover comparative analysis of the complexity of different methodology, nor the possibility of using different decision process than the use of the GPT. Work should be done in this area to be able to completely assess the efficiency of a complete system.

There are two main possible avenues to take for future works on the topic. One is the possibility of comparing DS and DSmH to other combining rules in the same study context. There is also the possibility of re-evaluating different versions of Florea's QA, different types of filters or the presented ones but with different parameters.

More work could be done at the decision level to evaluate the rate of successful decisions. Obviously, we should also explore different simulation runs, thus trying different distributions of bba.

REFERENCES

[1] F. Smarandache and J. Dezert (Editors), *Advances and Applications of DSmT for Information Fusion*, (Collected Works), vol. 1, USA : American Research Press, Rehoboth, 2004.

[2] G. Shafer, *A Mathematical Theory of Evidence*, Princeton Univ. Press, Princeton, NJ, USA, 1976.

[3] L. Zadeh, "On the validity of Dempster's rule of combination," *Memo M 79/24*, Univ. of California, Berkeley, 1979.

[4] F. Smarandache, "Unification of Fusion Theories," *ArXiv Computer Science e-prints*, arXiv: cs/ 0409040, 2004.

[5] P. Djiknavorian and D. Grenier, "Reducing DSmT hybrid rule complexity through optimisation of the calculation algorithm," Chapter in [6], 2006.

[6] F. Smarandache and J. Dezert (Editors), *Advances and Applications of DSmT for Information Fusion*, (Collected Works), vol. 2, USA : American Research Press, Rehoboth, 2006.

[7] Ph. Smets and R. Kennes, "The transferable belief model," *Artificial Intelligence*, Vol. 66, pp 191-234, 1994

[8] Ph. Smets, "Belief functions: the disjunctive rule of combination and the generalized Bayesian theorem," *International Journal of Approximate reasoning*, Vol. 9, pp 1-35, 1993

[9] M. C. Florea, D. Grenier and A. L. Jousselme, "Adaptive combination rule for the evidence theory," *Rapport Annuel d'Activités - Laboratoire de Radiocommunications et de Traitement du Signal*, Université Laval, pp 143 - 150, 2006

[10] I. Guttman and S.S. Wilks, *Introductory Engineering Statistics*, USA : John Wiley & Sons, Inc., Second Printing, 1967.