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Correlation coefficients of single valued neutrosophic refined soft sets and their applications in clustering analysis

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Abstract Neutrosophic set theory was introduced by Smarandache [33] based on neutrosophy which is a branch of philosophy. The concept of single valued neutrosophic refined set was defined by Ye [46] as an extension of single valued neutrosophic sets introduced by Wang [39]. In this study, the concept of single valued neutrosophic refined soft set is defined as an extension of single valued neutrosophic refined set. Also set theoretic operations between two single valued neutrosophic refined soft sets are defined and some basic properties of these operations are investigated. Furthermore, two methods to calculate correlation coefficient between two single valued neutrosophic refined soft sets are proposed and based on method given by Xu et al. in [48], an application of one of proposed methods is given in clustering analysis

Keywords Soft set · neutrosophic soft set · single valued neutrosophic refined set · single valued neutrosophic refined soft set · correlation coefficient · clustering analysis.

1 Introduction

To cope with uncertainty and inconsistency has been very important matter for researchers that study on mathematical modeling. Researchers have proposed many approximations to make mathematical model some problems containing uncertainty and inconsistency data. Some of well-known approximations are fuzzy set theory proposed by Zadeh [40] and intuitionistic fuzzy set theory introduced by Atanassov [2]. A fuzzy set is identified by membership function and an intuitionistic fuzzy set is identified by membership

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and non-membership functions. But fuzzy sets and intuitionistic fuzzy sets don't handle the indeterminant and inconsistent information. Therefore, neutrosophic set theory was introduced by Smarandache [33] as a generalization of fuzzy sets and intuitionistic fuzzy sets based on Neutrosophy which is a branch of philosophy. In 2005, Smarandache [34] shown that neutrosophic set is a generalization of paraconsistent set and intuitionistic fuzzy set. Wang [38] defined the concept of interval neutrosophic set (INS) and gave set theoretic operations of INSs. Zhang et al. [41] presented an application of INS in multicriteria decision making problems. Broumi and Smarandache [7] gave some new operations on interval valued neutrosophic sets. Intuitionistic neutrosophic sets and their set theoretical operations such as complement, union and intersection were defined by Bhowmik and Pal [4]. They also defined to deal with the engineering problem relations of four special type of intuitionistic neutrosophic sets and gave some properties of these relations. In 2010, concept of single-valued neutrosophic set and its set operations were defined by Wang et al. [39]. Ansari et al. [1] gave an application of neutrosophic set theory to medical AI. Ye [47] proposed concept of trapezoidal neutrosophic set by combining trapezoidal fuzzy set with single valued neutrosophic set. He also presented some operational rules related to this new sets and proposed score and accuracy function for trapezoidal neutrosophic numbers.

In classical set theory, if there are repeated elements in a set, each of repeated elements is represented a representative element. Therefore, elements of a classical set are different from each other. However, in some situations, we need a structure containing repeated elements. For instance, while search in a dad name-number of children-occupation relational basis. To express these cases, we use a structure called bags defined by Yager [44]. In 1998, Baowen [3] defined fuzzy bags and their operations based on Peizhuang's theory of set-valued statistics [29] and Yager's bags theory [44]. Concept of intuitionistic fuzzy bags (multi set) and its operations were defined by Shinoj and Sunil [36], and they gave an application in medical diagnosis under intuitionistic fuzzy multi environment. Rajarajeswari and Uma [31] introduced the Normalized Hamming Similarity measure for intuitionistic fuzzy multi sets based on the geometrical elucidation of intuitionistic fuzzy sets and gave an application in medical diagnosis.

To model problems containing uncertainty, notion of soft set was first proposed by Molodtsov [23] as a new mathematical tool which is an alternative approach to fuzzy set and intuitionistic fuzzy set. Maji et al. [24,25] defined some new operations of soft sets and gave an application for decision making problem. Then, studies on soft sets have progressed increasingly. For examples; Çağman and Enginoğlu [13] redefined soft sets operations and improved a new decision making method called uni-int decision making method. Qin et al. [30] gave some algorithms which require relatively fewer calculations compared with the existing decision making algorithms, Zhi et al. [42] presented a decision making approach for incomplete soft sets. Neutrosophic set and soft set were combined by Maji [26] in 2013. He also gave an application to decision making problem under neutrosophic soft environment. Broumi [5] was defined concept of generalized neutrosophic sets by combining Molodtsov's [23] soft set definition and Salama's [32] neutrosophic set definition. Şahin and Küçük [37] proposed generalized neutrosophic soft set based on Maji's neutrosophic soft set definition. Intuitionistic neutrosophic soft set and its operations were defined by Broumi and Smarandache [6]. Interval-valued neutrosophic soft set was defined by Deli [15] and it was generalized by Broumi et al. [8]. Then, Broumi et al. [9] extended definition of interval valued intuitionistic fuzzy soft relation to interval valued neutrosophic soft sets and also they defined neutrosophic parameterized soft sets and investigated their set theoretical properties in [10]. In 2014, Karaaslan [21] redefined operations of neutrosophic soft sets and gave applications in decision making problem and group decision making problem. In 2015, Maji [27] proposed concept of weighted neutrosophic soft set as a hybridization of neutrosophic sets with soft sets corresponding to weighted parameters and gave an application in multicriteria decision making problem.

In 2013, Smarandache [35] refined the neutrosophic set to n components: $t_1, t_2, t_j; i_1, i_2 i_k;\ f_1, f_2, f_l$, with $j + k + l = n > 3$. Single valued neutrosophic multiset (refined) (SVNM) was proposed by Ye and Ye [45] as a generalization of single valued neutrosophic sets. They also proposed Dice similarity measure and weighed Dice similarity measure of SVNMs and investigated their properties. Neutrosophic soft multi set theory was introduced by Deli et al. [16] and its an application was made to decision making.

Chiang and Lin [14] considered the fuzzy correlation under fuzzy environment and Mitchell [28] proposed a procedure to compute correlation coefficient between two intuitionistic fuzzy sets. Bustince and Burillo [12] studied on correlation coefficient of interval-valued intuitionistic fuzzy sets and introduced two decomposition theorems of the correlation of interval valued intuitionistic fuzzy sets. Hung and Wu [20] extended the "centroid" method to interval-valued intuitionistic fuzzy sets and gave a formula to compute the correlation coefficient between interval-valued intuitionistic fuzzy sets. Hanafy et al. [17] suggested a procedure to compute correlation coefficient of generalized intuitionistic fuzzy sets by means of "centroid and extended the centroid method to interval-valued generalized intuitionistic fuzzy sets. Also, they discussed and derived formula for correlation coefficient between two neutrosophic sets based on centroid method [18] and derived formula for correlation coefficient between neutrosophic sets in probability space [19]. Karaaslan [22] proposed a method to compute correlation coefficient between two possibility neutrosophic soft sets. Chen et al. [43] gave a formula to compute correlation coefficient of hesitant fuzzy sets and applied the formula to clustering analysis. Ye [46] improved to compute correlation coefficients of single valued neutrosophic sets and interval valued neutrosophic sets based on existing correlation coefficient and clustering analysis methods not being defined phenomenon or not consistent result in some cases. Broumi and Deli [11] developed a method to compute correlation between two neutrosophic refined (multi) sets as an extension of correlation measure of neutrosophic set and intuitionistic fuzzy multi sets.

In this study, a new structure called single valued neutrosophic refined soft set (SVNRS-set) which is a generalization of the single valued neutrosophic refined sets is defined, and some properties of SVNRS-sets in term of set theoretical operations are obtained based on Ye's [45] definitions and operations. $SVNRS-$set is an important structure to model some multicriteria decision making problems. Also two formulas are given to compute correlation coefficient between two SVNRS-sets and a clustering method is developed based on the proposed formulas. In the last section of the paper an example is presented to show calculation of proposed correlation coefficient and operation of clustering method.

2 Preliminary

In this section, a brief overview of the concepts of soft set, single valued neutrosophic set and single valued neutrosophic refined (multi) set are presented and their set theoretical operations required in subsequent sections are recalled.

Throughout paper, X denotes initial universe, E is a set of parameters and $I_p = \{1, 2, ...p\}$ is an index set.

Definition 1 [23] Let E be parameter set and $\emptyset \neq A \subseteq E$. A pair (f, A) is called a soft set over X, where f is a mapping given by $f : A \to \mathcal{P}(X)$.

Definition 2 [39] Let X be an initial universe. A single-valued neutrosophic set $(SVNS)$ $A \subseteq X$ is characterized by a truth membership function $t_A(x)$, an indeterminacy membership function $i_A(x)$, and a falsity membership function $f_A(x)$ with $t_A(x)$, $i_A(x)$, $f_A(x) \in [0, 1]$ for all $x \in X$.

It should be noted that for $SVNS$ A, the relation

$$0 \leq t_A(x) + i_A(x) + f_A(x) \leq 3 \quad \text{for all } x \in X$$

holds good. When X is discrete a $SVNS$ A can be written as

$$A = \sum_x \langle t_A(x), i_A(x), f_A(x) \rangle / x, \quad \text{for all } x \in X.$$

$SVNS$ has the following pattern: A=$\{\langle x, t_A(x), i_A(x), f_A(x)\rangle : x \in X\}$.

Thus, finite $SVNS$ A can be presented as follows:
$A = \{\langle x_1, t_A(x_1), i_A(x_1), f_A(x_1)\rangle, \ldots, \langle x_M, t_A(x_M), i_A(x_M), f_A(x_M)\rangle\}$ for all$x \in X$. The following definitions are given in [39] for $SVNSs$ A and B as follows:

1. $A \subseteq B$ if and only if $t_A(x) \leq t_B(x),\ i_A(x) \geq i_B(x),\ f_A(x) \geq f_B(x)$ for any $x \in X$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$ for all $x \in X$.
3. A^c=$\{\langle x, f_A(x), 1 - i_A(x), t_A(x)\rangle : x \in X\}$.
4. $A \cup B$= $\{\langle x, (t_A(x) \vee t_B(x)), (i_A(x) \wedge i_B(x)), (f_A(x) \wedge f_B(x))\rangle : x \in X\}$
5. $A \cap B$= $\{\langle x, (t_A(x) \wedge t_B(x)), (i_A(x) \vee i_B(x)), (f_A(x) \vee f_B(x))\rangle : x \in X\}$.

Definition 3 [45] Let X be a nonempty set with generic elements in X denoted by x. A single valued neutrosophic refined set (SVNR-set) f is defined as follows:

$$A = \Big\{ \Big\langle x, (t_A^1(x), t_A^2(x), ..., t_A^p(x)), (i_A^1(x), i_A^2(x), ..., i_A^p(x)), (f_A^1(x), f_A^2(x), ..., f_A^p(x)) \Big\rangle : x \in X \Big\}.$$

Here, $t_A^1, t_A^2, ..., t_A^p : X \to [0,1]$, $i_A^1, i_A^2, ..., i_A^p : X \to [0,1]$ and $f_A^1, f_A^2, ..., f_A^p : X \to [0,1]$ such that $0 \leq t_A^i(x) + i_A^i(x) + f_A^i(x) \leq 3$ for all $x \in X$ and $i \in I_p$. $(t_A^1(x), t_A^2(x), ..., t_A^k(x))$, $(i_A^1(x), i_A^2(x), ..., i_A^l(x))$ and $(f_A^1(x), f_A^2(x), ..., f_A^m(x))$ are the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x. These sequences may be in decreasing or increasing order.

A $SVNR$-set A drawn from X is characterized by the tree functions: count truth-membership of CT_A, count indeterminacy-membership of CI_A, and count falsity-membership of CF_A such that $CT_A(x) : X \to R$, $CI_A(x) : X \to R$ and $CF_A(x) : X \to R$ for $x \in X$, where R is the set of all real number refined set in real unit $[0,1]$.

For convenience, a $SVNR$-set A can be denoted by the simplified for:

$$A = \Big\{ \langle x, t_A^i(x), i_A^i(x), f_A^i(x) \rangle : x \in X, i \in I_p \Big\}$$

Set of all single valued neutrosophic refined sets over X will be denoted by $SVNR_X$.

Definition 4 [45] The length of an element x in $SVNR$-set A is defined as cardinality of $CT_A(x)$ or $CI_A(x)$, or $CF_A(x)$ and denoted by $L(x : A)$. Then $L(x : A) = |CT_A(x)| = |CI_A(x)| = |CF_A(x)|$.

Definition 5 [45] Let $A = \Big\{ \langle x, t_A^i(x), i_A^i(x), f_A^i(x) \rangle : x \in X, i \in I_p \Big\}$ and $B = \Big\{ \langle x, t_B^i(x), i_B^i(x), f_B^i(x) \rangle : x \in X, i \in I_p \Big\}$ be two SVNR-sets over X. Then,

1. A is said to be SVNR-subset of B is denoted by $A \subseteq B$ if $t_A^i(x) \leq t_B^i(x)$, $i_A^i(x) \geq i_B^i(x)$, $f_A^i(x) \geq f_B^i(x)$ for all $i \in I_p$ and $x \in X$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
3. The complement of A denoted by A^c and is define as follows:

$$A = \Big\{ \langle x, f_A^i(x), 1 - i_A^i(x), t_A^i(x) \rangle : x \in X, i \in I_p \Big\}.$$

Definition 6 Let $A = \Big\{ \langle x, t_A^i(x), i_A^i(x), f_A^i(x) \rangle : x \in X, i \in I_p \Big\}$ be a SVNR-set in X. Then,

1. A is said to be a null SVNR-set, if $t_A^i(x) = 0$, $i_A^i(x) = 1$ and $f_A^i(x) = 1$ for all $i \in I_p$ and $x \in X$, and denoted by $\hat{\Phi}$.

2. A is said to be a universal SVNR-set, if $t^i_A(x) = 1$, $i^i_A(x) = 0$ and $f^i_A(x) = 0$ for all $i \in I_p$ and $x \in X$, and denoted by $\hat{X}$.

Definition 7 [45] Let $A = \Big\{\big\langle x, t^i_A(x), i^i_A(x), f^i_A(x)\big\rangle : x \in X, i \in I_p\Big\}$ and $B = \Big\{\big\langle x, t^i_B(x), i^i_B(x), f^i_B(x)\big\rangle : x \in X, i \in I_p\Big\}$ be two SVNR-sets in X. Then,

1. Union:

$$A \cup B = \Big\{\big\langle x, t^i_A(x) \vee t^i_B(x), i^i_A(x) \wedge i^i_B(x), f^i_A(x) \wedge f^i_B(x)\big\rangle : x \in X, i \in I_p\Big\}$$

2. Intersection:

$$A \cap B = \Big\{\big\langle x, t^i_A(x) \wedge t^i_B(x), i^i_A(x) \vee i^i_B(x), f^i_A(x) \vee f^i_B(x)\big\rangle : x \in X, i \in I_p\Big\}$$

3 Single valued neutrosophic refined soft sets

In this section, the concept of single valued neutrosophic refined soft set and set theoretical operations between single valued neutrosophic refined soft sets are defined. Also some properties of the defined operations are investigated.

Definition 8 Let X be an initial universe and E be a parameter set. A single valued neutrosophic refined soft set (SVNRS-set) $\tilde{f}$ is defined by a function as follows:

$$\tilde{f} : E \to SVNR_X.$$

Here SVNRS-set $\tilde{f}$ as a family of SVNR-sets on X can be written as follows:

$$\tilde{f} = \Big\{\big(e, \{\langle x, t^i_{f(e)}(x), i^i_{f(e)}(x), f^i_{f(e)}(x)\rangle : x \in X, i \in I_p\}\big) : e \in E\Big\}.$$

Note that $\tilde{f}(e) = \{\langle x, (t^1_{f(e)}(x), t^2_{f(e)}(x), ..., t^p_{f(e)}(x)), (i^1_{f(e)}(x), i^2_{f(e)}(x), ..., i^p_{f(e)}(x)), (f^1_{f(e)}(x), f^2_{f(e)}(x), ..., f^p_{f(e)}(x))\rangle : x \in X, i \in I_p\}$.

From now on set of all $SVNRS-$sets on initial universe X and parameter set E will be denoted by $SVNRS^E_X$.

Example 1 Let $X = \{x_1, x_2, x_3, x_4\}$ be the set of houses and $E = \{e_1, e_2, e_3\}$ be a set of qualities where $e_1 = cheap$, $e_2 = big$ and $e_3 = repearing$. Then, SVNRS-set $\tilde{f}$ can be considered as follows:

$$\tilde{f} = \left\{ \begin{array}{c} \Big(e_1\{\langle x_1, (0.4, 0.3, 0.2, 0.1), (0.6, 0.4, 0.1, 0.0), (0.8, 0.5, 0.5, 0.3)\rangle, \langle x_2, (0.3, 0.1, 0.1, 0.0), \\ (0.5, 0.5, 0.4, 0.1), (0.6, 0.5, 0.2, 0.1)\rangle, \langle x_3, (0.9, 0.3, 0.1, 0.1), (0.7, 0.3, 0.3, 0.1), \\ (0.7, 0.5, 0.2, 0.0)\rangle, \langle x_4, (0.6, 0.5, 0.1, 0.0), (0.9, 0.8, 0.7, 0.6), (0.2, 0.1, 0.0, 0.0)\rangle\} \Big), \\ \Big(e_2\{\langle x_1, (0.4, 0.3, 0, 3, 0.1), (0.6, 0.6, 0.4, 0.4), (0.8, 0.8, 0.6, 0.6)\rangle, \langle x_2, (0.9, 0.5, 0.5, 0.3), \\ (0.9, 0.7, 0.7, 0.6), (0.8, 0.2, 0.2, 0.1)\rangle, \langle x_3, (0.6, 0.5, 0.1, 0.0), (0.3, 0.2, 0.1, 0.0), \\ (0.9, 0.5, 0.5, 0.0)\rangle, \langle x_4, (0.7, 0.6, 0.6, 0.5), (0.9, 0.4, 0.3, 0.2), (0.7, 0.3, 0.3, 0.0)\rangle\} \Big), \\ \Big(e_3\{\langle x_1, (0.9, 0.7, 0.7, 0.1), (0.8, 0.7, 0.7, 0.2), (0.1, 0.0, 0.0, 0.0)\rangle, \langle x_2, (0.6, 0.6, 0.6, 0.6), \\ (0.8, 0.2, 0.2, 0.1), (0.5, 0.1, 0.1, 0.0)\rangle, \langle x_3, (0.6, 0.3, 0.2, 0.0), (0.8, 0.8, 0.7, 0.4), \\ (0.8, 0.8, 0.3, 0.0)\rangle, \langle x_4, (0.4, 0.3, 0.1, 0.1), (0.5, 0.4, 0.4, 0.2), (0.7, 0.5, 0.0, 0.0)\rangle\} \Big), \end{array} \right\}.$$

Definition 9 Let $\tilde{f} = \Big\{ \big(e, \{\langle x, t^i_{f(e)}(x), i^i_{f(e)}(x), f^i_{f(e)}(x)\rangle : x \in X, i \in I_p\}\big) : e \in E \Big\}$ and $\tilde{g} = \Big\{ \big(e, \{\langle x, t^i_{g(e)}(x), i^i_{g(e)}(x), f^i_{g(e)}(x)\rangle : x \in X, i \in I_p\}\big) : e \in E \Big\}$ be two $SVNRS-$sets. Then,

1. $\tilde{f}$ is said to be SVNRS-subset of $\tilde{g}$ and denoted by $\tilde{f}\hat{\subseteq}g$, if $t^i_{f(e)}(x) \leq t^i_{g(e)}(x)$, $i^i_{f(e)}(x) \geq i^i_{g(e)}(x)$, $f^i_{f(e)}(x) \geq f^i_{g(e)}(x)$ for all $i \in I_p$, $x \in X$ and $e \in E$.
2. $\tilde{f} = \tilde{g}$ if and only if $\tilde{f}\hat{\subseteq}\tilde{g}$ and $\tilde{g}\hat{\subseteq}\tilde{f}$;
3. The complement of $\tilde{f}$, denoted by $\tilde{f}^c$, is defined as follows:
$$\tilde{f}^c = \Big\{ (e, f^c(e)) : e \in E \Big\}.$$
Here $f^c(e)$ is a SVNR-set over X, for each $e \in E$.

Definition 10 Let $\tilde{f} = \Big\{ \big(e, \{\langle x, t^i_{f(e)}(x), i^i_{f(e)}(x), f^i_{f(e)}(x)\rangle : x \in X, i \in I_p\}\big) : e \in E \Big\}$ be a $SVNRS-$set. Then,

1. $\tilde{f}$ is said to be a null SVNRS-set, if $\tilde{f}(e) = \hat{\Phi}$ for all $e \in E$, and denoted by $\tilde{\Phi}$.
2. $\tilde{f}$ is said to be a universal SVNRS-set, if $\tilde{f}(e) = \hat{X}$ for all $e \in E$, and denoted by $\tilde{X}$.

Definition 11 Let $\tilde{f} = \Big\{ \big(e, \{\langle x, t^i_{f(e)}(x), i^i_{f(e)}(x), f^i_{f(e)}(x)\rangle : x \in X, i \in I_p\}\big) : e \in E \Big\}$ and $\tilde{g} = \Big\{ \big(e, \{\langle x, (t^i_{g(e)}(x)), (i^i_{g(e)}(x)), (f^i_{g(e)}(x))\rangle : x \in X, i \in I_p\}\big) : e \in E \Big\}$ be two $SVNRS-$set. Then,

1. Union:
$$\tilde{f}\hat{\cup}\tilde{g} = \Big\{ (e, \tilde{f}(e) \cup \tilde{g}(e)) : e \in E \Big\}.$$
2. Intersection:
$$\tilde{f}\hat{\cap}\tilde{g} = \Big\{ (e, \tilde{f}(e) \cap \tilde{g}(e)) : e \in E \Big\}.$$

Example 2 Consider $SVNRS-$sets $\tilde{f}$ and $\tilde{g}$ in which their tabular representations are given below:

$\tilde{f}$	e_1	e_2	e_3
x_1	$(\langle 0.5, 0.5, 0.3, 0.2\rangle, \langle 0.6, 0.4, 0.2, 0.1\rangle, \langle 0.7, 0.5, 0.5, 0.3\rangle)$	$(\langle 0.4, 0.4, 0.0, 0.0\rangle, \langle 0.6, 0.5, 0.3, 0.1\rangle, \langle 0.7, 0.4, 0.1, 0.0\rangle)$	$(\langle 0.3, 0.2, 0.1, 0.1\rangle, \langle 0.7, 0.5, 0.1, 0.1\rangle, \langle 0.9, 0.9, 0.8, 0.3\rangle)$
x_2	$(\langle 0.8, 0.5, 0.4, 0.1\rangle, \langle 0.7, 0.7, 0.5, 0.5\rangle, \langle 0.5, 0.4, 0.3, 0.2\rangle)$	$(\langle 0.6, 0.5, 0.3, 0.1\rangle, \langle 0.5, 0.5, 0.5, 0.5\rangle, \langle 0.5, 0.3, 0.3, 0.2\rangle)$	$(\langle 0.6, 0.4, 0.4, 0.1\rangle, \langle 0.7, 0.7, 0.5, 0.5\rangle, \langle 0.9, 0.1, 0.0, 0.0\rangle)$
x_3	$(\langle 0.8, 0.5, 0.4, 0.3\rangle, \langle 0.9, 0.8, 0.7, 0.5\rangle, \langle 0.8, 0.7, 0.7, 0.4\rangle)$	$(\langle 1.0, 0.9, 0.9, 0.8\rangle, \langle 0.3, 0.2, 0.1, 0.1\rangle, \langle 0.9, 0.5, 0.1, 0.0\rangle)$	$(\langle 0.8, 0.6, 0.4, 0.2\rangle, \langle 0.8, 0.5, 0.5, 0.4\rangle, \langle 0.8, 0.5, 0.4, 0.3\rangle)$
x_4	$(\langle 0.7, 0.6, 0.5, 0.3\rangle, \langle 0.9, 0.6, 0.5, 0.4\rangle, \langle 0.8, 0.8, 0.7, 0.7\rangle)$	$(\langle 0.7, 0.5, 0.3, 0.3\rangle, \langle 0.9, 0.6, 0.5, 0.3\rangle, \langle 0.8, 0.7, 0.3, 0.1\rangle)$	$(\langle 0.6, 0.5, 0.0, 0.0\rangle, \langle 0.5, 0.4, 0.4, 0.1\rangle, \langle 0.7, 0.7, 0.7, 0.5\rangle)$

$\tilde{g}$	e_1	e_2	e_3
x_1	$(\langle 0.6, 0.5, 0.1, 0.0\rangle, \langle 0.7, 0.5, 0.3, 0.2\rangle, \langle 0.9, 0.8, 0.6, 0.3\rangle)$	$(\langle 0.5, 0.4, 0.4, 0.2\rangle, \langle 0.3, 0.1, 0.1, 0.0\rangle, \langle 0.7, 0.7, 0.2, 0.1\rangle)$	$(\langle 0.6, 0.4, 0.3, 0.1\rangle, \langle 0.9, 0.9, 0.5, 0.1\rangle, \langle 0.6, 0.5, 0.1, 0.0\rangle)$
x_2	$(\langle 0.7, 0.2, 0.1, 0.1\rangle, \langle 0.9, 0.0, 0.0, 0.0\rangle, \langle 0.8, 0.7, 0.5, 0.2\rangle)$	$(\langle 1.0, 0.5, 0.5, 0.3\rangle, \langle 0.6, 0.4, 0.1, 0.1\rangle, \langle 0.9, 0.7, 0.7, 0.0\rangle)$	$(\langle 0.9, 0.8, 0.6, 0.5\rangle, \langle 0.8, 0.7, 0.3, 0.1\rangle, \langle 0.5, 0.5, 0.5, 0.2\rangle)$
x_3	$(\langle 0.9, 0.8, 0.5, 0.5\rangle, \langle 0.5, 0.4, 0.1, 0.1\rangle, \langle 0.1, 0.1, 0.0, 0.0\rangle)$	$(\langle 0.9, 0.8, 0.8, 0.3\rangle, \langle 0.9, 0.3, 0.3, 0.2\rangle, \langle 0.3, 0.2, 0.2, 0.1\rangle)$	$(\langle 1.0, 0.3, 0.3, 0.0\rangle, \langle 0.7, 0.4, 0.2, 0.2\rangle, \langle 0.7, 0.6, 0.5, 0.2\rangle)$
x_4	$(\langle 1.0, 1.0, 0.7, 0.3\rangle, \langle 0.7, 0.5, 0.1, 0.1\rangle, \langle 0.9, 0.4, 0.2, 0.1\rangle)$	$(\langle 0.6, 0.5, 0.5, 0.2\rangle, \langle 0.2, 0.2, 0.1, 0.1\rangle, \langle 0.8, 0.4, 0.3, 0.1\rangle)$	$(\langle 0.5, 0.5, 0.4, 0.4\rangle, \langle 0.9, 0.9, 0.5, 0.3\rangle, \langle 0.6, 0.4, 0.4, 0.3\rangle)$

.

Then,

$\tilde{f}\hat{\cup}\tilde{g}$	e_1	e_2	e_3
x_1	$(\langle 0.6, 0.5, 0.3, 0.2\rangle, \langle 0.6, 0.4, 0.2, 0.1\rangle, \langle 0.7, 0.5, 0.5, 0.3\rangle)$	$(\langle 0.5, 0.4, 0.4, 0.2\rangle, \langle 0.3, 0.1, 0.1, 0.0\rangle, \langle 0.7, 0.4, 0.1, 0.0\rangle)$	$(\langle 0.7, 0.7, 0.3, 0.3\rangle, \langle 0.7, 0.6, 0.5, 0.1\rangle, \langle 0.6, 0.5, 0.1, 0.0\rangle)$
x_2	$(\langle 0.8, 0.5, 0.4, 0.1\rangle, \langle 0.7, 0.0, 0.0, 0.0\rangle, \langle 0.5, 0.4, 0.3, 0.2\rangle)$	$(\langle 1.0, 0.5, 0.5, 0.3\rangle, \langle 0.5, 0.4, 0.1, 0.1\rangle, \langle 0.5, 0.5, 0.3, 0.0\rangle)$	$(\langle 0.9, 0.8, 0.6, 0.5\rangle, \langle 0.7, 0.7, 0.3, 0.1\rangle, \langle 0.5, 0.1, 0.0, 0.0\rangle)$
x_3	$(\langle 0.9, 0.8, 0.5, 0.5\rangle, \langle 0.5, 0.4, 0.1, 0.1\rangle, \langle 0.1, 0.1, 0.0, 0.0\rangle)$	$(\langle 1.0, 0.9, 0.9, 0.8\rangle, \langle 0.3, 0.2, 0.1, 0.1\rangle, \langle 0.3, 0.2, 0.1, 0.0\rangle)$	$(\langle 0.1, 0.6, 0.4, 0.2\rangle, \langle 0.7, 0.4, 0.2, 0.2\rangle, \langle 0.7, 0.5, 0.4, 0.2\rangle)$
x_4	$(\langle 1.0, 1.0, 0.7, 0.3\rangle, \langle 0.7, 0.5, 0.1, 0.1\rangle, \langle 0.8, 0.4, 0.2, 0.1\rangle)$	$(\langle 0.7, 0.5, 0.3, 0.3\rangle, \langle 0.2, 0.2, 0.1, 0.1\rangle, \langle 0.8, 0.4, 0.3, 0.1\rangle)$	$(\langle 0.6, 0.5, 0.4, 0.4\rangle, \langle 0.5, 0.4, 0.4, 0.1\rangle, \langle 0.6, 0.4, 0.4, 0.3\rangle)$

and

$\tilde{f}\hat{\cap}\tilde{g}$	e_1	e_2	e_3
x_1	$(\langle 0.5, 0.5, 0.1, 0.0\rangle,$ $\langle 0.7, 0.5, 0.3, 0.2\rangle,$ $\langle 0.9, 0.8, 0.6, 0.3\rangle)$	$(\langle 0.4, 0.4, 0.0, 0.0\rangle,$ $\langle 0.6, 0.5, 0.3, 0.1\rangle,$ $\langle 0.7, 0.7, 0.2, 0.1\rangle)$	$(\langle 0.3, 0.3, 0.3, 0.1\rangle,$ $\langle 0.9, 0.9, 0.6, 0.7\rangle,$ $\langle 0.9, 0.9, 0.8, 0.3\rangle)$
x_2	$(\langle 0.7, 0.2, 0.1, 0.1\rangle,$ $\langle 0.9, 0.7, 0.5, 0.5\rangle,$ $\langle 0.8, 0.7, 0.5, 0.2\rangle)$	$(\langle 0.6, 0.5, 0.3, 0.1\rangle,$ $\langle 0.6, 0.5, 0.5, 0.5\rangle,$ $\langle 0.9, 0.7, 0.7, 0.2\rangle)$	$(\langle 0.6, 0.4, 0.4, 0.1\rangle,$ $\langle 0.8, 0.7, 0.5, 0.5\rangle,$ $\langle 0.9, 0.5, 0.5, 0.2\rangle)$
x_3	$(\langle 0.8, 0.5, 0.4, 0.3\rangle,$ $\langle 0.9, 0.8, 0.7, 0.5\rangle,$ $\langle 0.8, 0.7, 0.7, 0.4\rangle)$	$(\langle 0.9, 0.8, 0.8, 0.3\rangle,$ $\langle 0.9, 0.3, 0.3, 0.2\rangle,$ $\langle 0.9, 0.5, 0.2, 0.1\rangle)$	$(\langle 0.8, 0.3, 0.3, 0.0\rangle,$ $\langle 0.8, 0.5, 0.5, 0.4\rangle,$ $\langle 0.8, 0.6, 0.5, 0.3\rangle)$
x_4	$(\langle 0.7, 0.6, 0.5, 0.3\rangle,$ $\langle 0.9, 0.6, 0.5, 0.4\rangle,$ $\langle 0.9, 0.8, 0.7, 0.7\rangle)$	$(\langle 0.6, 0.5, 0.3, 0.2\rangle,$ $\langle 0.9, 0.6, 0.5, 0.3\rangle,$ $\langle 0.8, 0.7, 0.3, 0.1\rangle)$	$(\langle 0.5, 0.5, 0.0, 0.0\rangle,$ $\langle 0.9, 0.9, 0.5, 0.3\rangle,$ $\langle 0.7, 0.7, 0.7, 0.5\rangle)$

Proposition 1 *Let* $\tilde{f}, \tilde{g}, \tilde{h} \in SVNRS_X^E$. *Then,*

(1) $\tilde{\Phi}\hat{\subseteq}\tilde{f}$
(2) $\tilde{f}\hat{\subseteq}\tilde{X}$
(3) $\tilde{f}\hat{\subseteq}\tilde{f}$
(4) $\tilde{f}\hat{\subseteq}\tilde{g}$ *and* $\tilde{g}\hat{\subseteq}\tilde{h} \Rightarrow \tilde{f}\hat{\subseteq}\tilde{h}$

Proof The proof is obvious from Definition 9.

Proposition 2 *Let* $\tilde{f} \in SVNRS_X^E$. *Then,*

(1) $\tilde{\Phi}^{\tilde{c}} = \tilde{X}$
(2) $\tilde{X}^c = \tilde{\Phi}$
(3) $(\tilde{f}^c)^c = \tilde{f}$.

Proof The proof is clear from Definition 10.

Proposition 3 *Let* $\tilde{f}, \tilde{g}, \tilde{h} \in SVNRS_X^E$. *Then,*

(1) $\tilde{f}\hat{\cap}\tilde{f} = f$ *and* $\tilde{f}\hat{\cup}\tilde{f} = \tilde{f}$
(2) $\tilde{f}\hat{\cap}\tilde{g} = \tilde{g}\hat{\cap}\tilde{f}$ *and* $\tilde{f}\hat{\cup}\tilde{g} = \tilde{g}\hat{\cup}\tilde{f}$
(3) $\tilde{f}\hat{\cap}\tilde{\Phi} = \tilde{\Phi}$ *and* $\tilde{f}\hat{\cap}\tilde{X} = \tilde{f}$
(4) $\tilde{f}\hat{\cup}\tilde{\Phi} = \tilde{f}$ *and* $\tilde{f}\hat{\cup}\tilde{X} = \tilde{X}$
(5) $\tilde{f}\hat{\cap}(\tilde{g}\hat{\cap}\tilde{h}) = (\tilde{f}\hat{\cap}\tilde{g})\hat{\cap}\tilde{h}$ *and* $\tilde{f}\hat{\cup}(\tilde{g}\hat{\cup}\tilde{h}) = (\tilde{f}\hat{\cup}\tilde{g})\hat{\cup}\tilde{h}$
(6) $\tilde{f}\hat{\cap}(\tilde{g}\hat{\cup}\tilde{h}) = (\tilde{f}\hat{\cap}\tilde{g})\hat{\cup}(\tilde{f}\hat{\cap}\tilde{h})$ *and* $\tilde{f}\hat{\cup}(\tilde{g}\hat{\cap}\tilde{h}) = (\tilde{f}\hat{\cup}\tilde{g})\hat{\cap}(\tilde{f}\hat{\cup}\tilde{h})$.

Proof The proof is obtained from Definition 11.

Theorem 1 *Let* $\tilde{f}, \tilde{g} \in SVNRS_X^E$. *Then, De Morgan's law is valid.*

(1) $(\tilde{f}\hat{\cup}\tilde{g})^c = \tilde{f}^c\hat{\cap}\tilde{g}^c$
(2) $(\tilde{f}\hat{\cup}\tilde{g})^c = \tilde{f}^c\hat{\cap}\tilde{g}^c$

4 Correlation coefficient of single valued neutrosophic refined soft sets

In this section, two types of correlation coefficients between two SVNRS-sets are defined and some properties of them are given.

Definition 12 Let $\tilde{f} = \Big\{\big(e, \{\langle x, t^i_{f(e)}(x), i^i_{f(e)}(x), f^i_{f(e)}(x)\rangle : x \in X, i \in I_p\}\big) : e \in E\Big\}$ and $\tilde{g} = \Big\{\big(e, \{\langle x, (t^i_{g(e)}(x)), (i^i_{g(e)}(x)), (f^i_{g(e)}(x))\rangle : x \in X, i \in I_p\}\big) : e \in E\Big\}$ be two $SVNRS-$sets. Then, for any $e_k \in E, k \in I_m$, correlation of truth sequence (indeterminacy sequence, falsity sequence) of $SVNRS$-sets $\tilde{f}$ and $\tilde{g}$, is defined as follows:

$$C_\Lambda(\tilde{f},\tilde{g})(e_k) = \frac{1}{p^2}\sum_{j=1}^{n}\Big[\Big(\sum_{r=1}^{p}\Lambda^r_{f(e_k)}(x_j) - \frac{1}{n}\sum_{s=1}^{n}\sum_{r=1}^{p}\Lambda^r_{f(e_k)}(x_s)\Big) \times \Big(\sum_{r=1}^{p}\Lambda^r_{g(e_k)}(x_j) - \frac{1}{n}\sum_{s=1}^{n}\sum_{r=1}^{p}\Lambda^r_{g(e_k)}(x_s)\Big)\Big]. \quad (1)$$

Here, $\Lambda \in \{t = truth,\ i = indeterminacy,\ f = falsity\}$, $e_k \in E$ and $|X| = n$.

Definition 13 Let $\tilde{f} = \Big\{\big(e, \{\langle x, t^i_{f(e)}(x), i^i_{f(e)}(x), f^i_{f(e)}(x)\rangle : x \in X, i \in I_p\}\big) : e \in E\Big\}$ and $\tilde{g} = \Big\{\big(e, \{\langle x, (t^i_{g(e)}(x)), (i^i_{g(e)}(x)), (f^i_{g(e)}(x))\rangle : x \in X, i \in I_p\}\big) : e \in E\Big\}$ be two $SVNRS-$sets. Then, correlation coefficient with respect to component $\Lambda \in \{t, i, f\}$ is defined as follows:

$$\rho^{(1)}_\Lambda(\tilde{f},\tilde{g})(e_k) = \frac{C_\Lambda(\tilde{f},\tilde{g})(e_k)}{[C_\Lambda(\tilde{f},\tilde{f})(e_k)]^{\frac{1}{2}}[C_\Lambda(\tilde{g},\tilde{g})(e_k)]^{\frac{1}{2}}}. \quad (2)$$

Definition 14 Correlation coefficient between two $SVNRS-$sets $\tilde{f}$ and $\tilde{g}$ is defined as follows:

$$\rho_{SVNRS_1}(\tilde{f},\tilde{g}) = \frac{1}{3}\sum_{\forall\Lambda\in\{t,i,j\}}\frac{1}{|E|}\sum_{e_k\in E}\rho_\Lambda(\tilde{f},\tilde{g})(e_k). \quad (3)$$

Note that correlation coefficient between two $SVNRS$-sets gets values in $[-1, 1]$.

Theorem 2 *Let $\tilde{f}, \tilde{g} \in SVNRS^E_X$. Then, correlation coefficient $\rho_{SVNRS}(\tilde{f},\tilde{g})$ satisfies following properties:*

1. $\rho_{SVNRS_1}(\tilde{f},\tilde{g}) = \rho_{SVNRS_1}(\tilde{g},\tilde{f})$
2. *If* $\tilde{f} = \tilde{g}$ *then* $\rho_{SVNRS_1}(\tilde{f},\tilde{g}) = 1$
3. $|\rho_{SVNRS_1}(\tilde{f},\tilde{g})| \leq 1$.

Proof 1. Since

$$
\begin{aligned}
C_\Lambda(\tilde{f},\tilde{g})(e_k) &= \frac{1}{p^2}\sum_{j=1}^{n}\Big[\Big(\sum_{r=1}^{n}\Lambda^r_{f(e_k)}(x_j) - \frac{1}{n}\sum_{s=1}^{n}\sum_{r=1}^{p}\Lambda^r_{f(e_k)}(x_s)\Big)\\
&\quad\times\Big(\sum_{r=1}^{n}\Lambda^r_{g(e_k)}(x_j) - \frac{1}{n}\sum_{s=1}^{n}\sum_{r=1}^{p}\Lambda^r_{g(e_k)}(x_s)\Big)\Big]\\
&= \frac{1}{p^2}\sum_{j=1}^{n}\Big[\Big(\sum_{r=1}^{n}\Lambda^r_{g(e_k)}(x_j) - \frac{1}{n}\sum_{s=1}^{n}\sum_{r=1}^{p}\Lambda^r_{g(e_k)}(x_s)\Big)\\
&\quad\times\Big(\sum_{r=1}^{n}\Lambda^r_{f(e_k)}(x_j) - \frac{1}{n}\sum_{s=1}^{n}\sum_{r=1}^{p}\Lambda^r_{f(e_k)}(x_s)\Big)\Big]\\
&= C_\Lambda(\tilde{g},\tilde{f})(e_k)
\end{aligned}
$$

for all $e_k \in E$ and $\Lambda \in \{t,i,f\}$, then

$$
\rho^{(1)}_\Lambda(\tilde{f},\tilde{g})(e_k) = \frac{C_\Lambda(\tilde{g},\tilde{f})(e_k)}{[C_\Lambda(\tilde{g},\tilde{g})(e_k)]^{\frac{1}{2}}[C_\Lambda(\tilde{f},\tilde{f})(e_k)]^{\frac{1}{2}}}
$$

and so

$$
\rho_{SVNRS_1}(\tilde{f},\tilde{g}) = \rho_{SVNRS_1}(\tilde{g},\tilde{f})
$$

2. It is clear that $\rho^{(1)}_\Lambda(\tilde{f},\tilde{f})(e_k) = \frac{C_\Lambda(\tilde{f},\tilde{f})(e_k)}{[C_\Lambda(\tilde{f},\tilde{f})(e_k)]^{\frac{1}{2}}[C_\Lambda(\tilde{f},\tilde{f})(e_k)]^{\frac{1}{2}}} = 1$, for all $e_k \in E$. Therefore,

$$
\begin{aligned}
\rho_{SVNRS_1}(\tilde{f},\tilde{f}) &= \frac{1}{3}\sum_{\forall\Lambda\in\{t,i,f\}}\frac{1}{|E|}\sum_{e_k\in E}\rho^{(1)}_\Lambda(\tilde{f},\tilde{f})(e_k)\\
&= \frac{1}{3}\sum_{\forall\Lambda\in\{t,i,f\}}\frac{1}{|E|}\Big(\rho^{(1)}_\Lambda(\tilde{f},\tilde{f})(e_1) + \rho^{(1)}_\Lambda(\tilde{f},\tilde{f})(e_2) + \dots + \rho^{(1)}_\Lambda(\tilde{f},\tilde{f})(e_{|E|})\Big)\\
&= \Big(\rho^{(1)}_t(\tilde{f},\tilde{f})(e_1) + \rho^{(1)}_t(\tilde{f},\tilde{f})(e_2) + \dots + \rho^{(1)}_t(\tilde{f},\tilde{f})(e_{|E|})\Big)\\
&+ \Big(\rho^{(1)}_i(\tilde{f},\tilde{f})(e_1) + \rho^{(1)}_i(\tilde{f},\tilde{f})(e_2) + \dots + \rho^{(1)}_i(\tilde{f},\tilde{f})(e_{|E|})\Big)\\
&+ \Big(\rho^{(1)}_f(\tilde{f},\tilde{f})(e_1) + \rho^{(1)}_f(\tilde{f},\tilde{f})(e_2) + \dots + \rho^{(1)}_f(\tilde{f},\tilde{f})(e_{|E|})\Big)\\
&= \frac{1}{3|E|}3|E| = 1
\end{aligned}
$$

3. Let us adopt the following notations;

$$
\begin{aligned}
&\sum_{r=1}^{p}\Lambda^r_{f(e_k)}(x_j) = \tilde{x}_j,\\
&\frac{1}{n}\sum_{s=1}^{n}\sum_{r=1}^{p}\Lambda^r_{f(e_k)}(x_s) = \overline{f}
\end{aligned}
$$

$$\sum_{r=1}^{p} \Lambda^r_{g(e_k)}(x_j) = \tilde{y}_j,$$

$$\frac{1}{n}\sum_{s=1}^{n}\sum_{r=1}^{p} \Lambda^r_{g(e_k)}(x_s) = \overline{g},$$

$$\begin{aligned}(C_\Lambda(\tilde{f},\tilde{g})(e_k))^2 &= (\frac{1}{p^2})^2\Big(\Big[(\tilde{x}_1-\overline{f})(\tilde{y}_1-\overline{g})\Big]+\Big[(\tilde{x}_2-\overline{f})(\tilde{y}_2-\overline{g})\Big]+...+\Big[(\tilde{x}_n-\overline{f})(\tilde{y}_n-\overline{g})\Big]\Big)^2\\ &\le (\frac{1}{p^4})\Big((\tilde{x}_1-\overline{f})^2+(\tilde{x}_2-\overline{f})^2+...+(\tilde{x}_n-\overline{f})^2\Big)\Big((\tilde{y}_1-\overline{g})^2+(\tilde{y}_2-\overline{g})^2+...+(\tilde{y}_n-\overline{g})^2\Big)\\ |(C_\Lambda(\tilde{f},\tilde{g})(e_k))| &\le [C_\Lambda(\tilde{f},\tilde{f})(e_k)]^{\frac{1}{2}}[C_\Lambda(\tilde{g},\tilde{g})(e_k)]^{\frac{1}{2}}.\end{aligned}$$

Then,

$$-[C_\Lambda(\tilde{f},\tilde{f})(e_k)]^{\frac{1}{2}}[C_\Lambda(\tilde{g},\tilde{g})(e_k)]^{\frac{1}{2}} \le (C_\Lambda(\tilde{f},\tilde{g})(e_k)) \le [C_\Lambda(\tilde{f},\tilde{f})(e_k)]^{\frac{1}{2}}[C_\Lambda(\tilde{g},\tilde{g})(e_k)]^{\frac{1}{2}}$$

and

$$-1 \le \frac{(C_\Lambda(\tilde{f},\tilde{g})(e_k))}{[C_\Lambda(\tilde{f},\tilde{f})(e_k)]^{\frac{1}{2}}[C_\Lambda(\tilde{g},\tilde{g})(e_k)]^{\frac{1}{2}}} \le 1$$

Thus,

$$-1 \le \rho^{(1)}_\Lambda(\tilde{f},\tilde{g})(e_k) \le 1$$

for all $e_k \in E$ and $\Lambda \in \{t,i,f\}$ and

$$|\rho_{SVNRS_1}(\tilde{f},\tilde{g})| \le 1.$$

Corollary 1 *Let* $\tilde{f} = \Big\{\big(e, \{\langle x, t^i_{f(e)}(x), i^i_{f(e)}(x), f^i_{f(e)}(x)\rangle : x \in X, i \in I_p\}\big) : e \in E\Big\}$ *and* $\tilde{g} = \Big\{\big(e, \{\langle x, (t^i_{g(e)}(x)), (i^i_{g(e)}(x)), (f^i_{g(e)}(x))\rangle : x \in X, i \in I_p\}\big) : e \in E\Big\}$

1. *If, for any* $\Lambda_1 \in \{t,i,f\}$, Λ_1 *sequences of* $\tilde{f}$ *and* $\tilde{g}$ *is equal and* $\Lambda_{2f(e_k)}(x) = 1 - \Lambda_{2g(e_k)}(x)$, *for all* $\Lambda_2 \in \{t,i,f\} - \{\Lambda_1\}$, $e_k \in E$ *and* $x \in X$, *then*

$$\rho_{SVNRS_1}(\tilde{f},\tilde{g}) = -1.$$

2. *If, for any* $\Lambda_1, \Lambda_2 \in \{t,i,f\}$, Λ_1 *and* Λ_2 *sequences of* $\tilde{f}$ *and* $\tilde{g}$ *is equal and* $\Lambda_{3f(e_k)}(x) = 1 - \Lambda_{3g(e_k)}(x)$, *for all* $\Lambda_3 \in \{t,i,f\} - \{\Lambda_1, \Lambda_2\}$, $e_k \in E$ *and* $x \in X$, *then*

$$\rho_{SVNRS_1}(\tilde{f},\tilde{g}) = 1.$$

Example 3 Consider SVNRS-sets $\tilde{f}$ and $\tilde{g}$ as in Example 2. Correlation coefficient between SVNSR-sets $\tilde{f}$ and $\tilde{g}$ can be calculated as follows: For parameter e_1 and $\Lambda = t$:

$$\rho_t^{(1)}(\tilde{f},\tilde{g})(e_1) = \frac{(-0.088)(-0.200)+(-0.013)(-0.225)+(0.038)(0.175)+(0.062)(0.250)}{\sqrt{((-0.088)^2+(-0.013)^2+(0.038)^2+(0.062)^2)((-0.200)^2+(-0.225)^2+(0.175)^2+(0.250)^2)}} = 0.865$$

and for parameter e_2, e_3 and $\Lambda = t$:

$$\rho_t^{(1)}(\tilde{f},\tilde{g})(e_2) = 0.880 \ and \ \rho_t^{(1)}(\tilde{f},\tilde{g})(e_3) = -0.443.$$

Then,

$$\frac{1}{|E|}\sum_{e_k\in E}\rho_t^{(1)}(\tilde{f},\tilde{g})(e_k) = \frac{1}{3}(0.865+0.880+(-0.443)) = 0.434.$$

Similarly,

$$\frac{1}{|E|}\sum_{e_k\in E}\rho_i^{(1)}(\tilde{f},\tilde{g})(e_k) = \frac{1}{3}((-0.782)+(-0.664)+(-0.818)) = -0.755$$

$$\frac{1}{|E|}\sum_{e_k\in E}\rho_f^{(1)}(\tilde{f},\tilde{g})(e_k) = \frac{1}{3}((-0.567)+(-0.056)+(-0.510)) = -0.378$$

and so

$$\rho_{SVNRS_1}(\tilde{f},\tilde{g}) = \frac{1}{3}\sum_{\forall\Lambda\in\{t,i,f\}}\frac{1}{|E|}\sum_{e_k\in E}\rho_\Lambda^{(1)}(\tilde{f},\tilde{g})(e_k) = \frac{1}{3}(0.434+(-0.755)+(-0.378)) = -0.233.$$

This value shows that SVNRS-sets $\tilde{f}$ and $\tilde{g}$ have a bad negatively correlated.

In some practical applications, the parameters $e_k \in E\,(k\in I_k)$ may have different weights in the studied universe. Let $w_{\tilde{f}} = (w_f(e_1), w_f(e_2), ..., w_f(e_n))^T$ and $w_{\tilde{g}} = (w_g(e_1), w_g(e_2), ..., w_g(e_n))^T$ be the weight vectors of parameters in $SVNRS$-sets $\tilde{f}$ and $\tilde{g}$, respectively. Here, $w_f(e_k) \geq 0, w_g(e_k) \geq 0$ and $\sum_{e_k\in E} w_f(e_k) = 1$, $\sum_{e_k\in E} w_g(e_k) = 1$ for all $e_k \in E\,(k\in I_m)$. Then the correlation coefficient formula can be extended as follows:

$$\rho^w_{SVNRS_1}(\tilde{f},\tilde{g}) = \frac{1}{3}\sum_{\forall\Lambda\in\{t,i,f\}}\frac{1}{|E|}\sum_{e_k\in E} w(e_k)_{(\tilde{f},\tilde{g})}\rho_\Lambda^{(1)}(\tilde{f},\tilde{g})(e_k) \quad (4)$$

here

$$w_{(\tilde{f},\tilde{g})}(e_k) = 1 - \frac{|w_f(e_k) - w_g(e_k)|}{max\{w_f(e_k), w_g(e_k)\}}. \quad (5)$$

Note that if $w_f(e_k) = w_g(e_k)$ for all $e_k \in E$, then Eq.(4) reduce to Eq. (3).

Example of weighted correlation coefficient $\rho^{(1)}$ will be given in application section.

Theorem 3 *Properties listed in Theorem 4 valid for weighted correlation coefficient of two $SVNRS-sets$ $\tilde{f}$ and $\tilde{g}$.*

Proof The proof can be made similar way to proof of Theorem 4.

Now second type of correlation coefficient of $SVNRS$-sets will be given.

Definition 15 Let $\tilde{f}, \tilde{g} \in SVNRS_X^E$. Then, correlation coefficient of $SVNRS-$sets $\tilde{f}$ and $\tilde{g}$ is defined as follows:

$$\rho_{SVNRS_2}(\tilde{f}, \tilde{g}) = \frac{1}{3} \sum_{\forall \Lambda \in \{t,i,f\}} \frac{1}{|E|} \sum_{e_k \in E} \rho_\Lambda^{(2)}(\tilde{f}, \tilde{g})(e_k) \tag{6}$$

here

$$\rho_\Lambda^{(2)}(\tilde{f}, \tilde{g})(e_k) = \frac{C_\Lambda(\tilde{f}, \tilde{g})(e_k)}{max\big\{[C_\Lambda(\tilde{f}, \tilde{f})(e_k)], [C_\Lambda(\tilde{g}, \tilde{g})(e_k)]\big\}} \tag{7}$$

such that $\Lambda \in \{t = truth,\ i = indeterminacy,\ f = falsity\}$.

Theorem 4 *Let $\tilde{f}, \tilde{g} \in SVNRS_X^E$. Then, correlation coefficient $\rho_{SVNRS_2}(\tilde{f}, \tilde{g})$ satisfies the following properties:*

1. $\rho_{SVNRS_2}(\tilde{f}, \tilde{g}) = \rho_{SVNRS_2}(\tilde{g}, \tilde{f})$
2. *If* $\tilde{f} = \tilde{g}$ *then* $\rho_{SVNRS_2}(\tilde{f}, \tilde{g}) = 1$
3. $|\rho_{SVNRS_2}(\tilde{f}, \tilde{g})| \leq 1$

Proof 1. The proof is trivial.
2. The proof is clear.
3. Let us adopt the following notations;

$$\sum_{r=1}^{p} \Lambda^r_{f(e_k)}(x_j) = \tilde{x}_j,$$
$$\frac{1}{n} \sum_{s=1}^{n} \sum_{r=1}^{p} \Lambda^r_{f(e_k)}(x_s) = \overline{f}$$
$$\sum_{r=1}^{p} \Lambda^r_{g(e_k)}(x_j) = \tilde{y}_j,$$
$$\frac{1}{n} \sum_{s=1}^{n} \sum_{r=1}^{p} \Lambda^r_{g(e_k)}(x_s) = \overline{g},$$

$$(\rho^{(2)}(\tilde{f}, \tilde{g}))^2 = \frac{\Big(\sum_{j=1}^{n} \frac{1}{p^2}\Big(\tilde{x}_j - \overline{f}\Big)\Big(\tilde{y}_j - \overline{g}\Big)\Big)^2}{\Big(max\Big\{\big(\sum_{j=1}^{n} \frac{1}{p^2}\big(\tilde{x}_j - \overline{f}\big)^2\big), \big(\sum_{j=1}^{n} \frac{1}{p^2}\big(\tilde{y}_j - \overline{g}\big)^2\big)\Big\}\Big)^2}$$

$$\leq \frac{\Big(\sum_{j=1}^{n}\frac{1}{p^2}(\tilde{x}_j-\overline{f})^2\Big)\Big(\sum_{j=1}^{n}\frac{1}{p^2}(\tilde{y}_j-\overline{g})^2\Big)}{\Big(max\Big\{\big(\sum_{j=1}^{n}\frac{1}{p^2}\big(\tilde{x}_j-\overline{f}\big)^2\big),\big(\sum_{j=1}^{n}\frac{1}{p^2}\big(\tilde{y}_j-\overline{g}\big)^2\big)\Big\}\Big)^2}$$
$$\leq \frac{\Big(\sqrt{\sum_{j=1}^{n}\frac{1}{p^2}(\tilde{x}_j-\overline{f})^2}\Big)\Big(\sqrt{\sum_{j=1}^{n}\frac{1}{p^2}(\tilde{y}_j-\overline{g})^2}\Big)}{max\Big\{\big(\sum_{j=1}^{n}\frac{1}{p^2}\big(\tilde{x}_j-\overline{f}\big)^2\big),\big(\sum_{j=1}^{n}\frac{1}{p^2}\big(\tilde{y}_j-\overline{g}\big)^2\big)\Big\}}$$

Lets take $\sum_{j=1}^{n}\frac{1}{p^2}(\tilde{x}_j-\overline{f})^2=a$ and $\sum_{j=1}^{n}\frac{1}{p^2}(\tilde{y}_j-\overline{g})^2=b$. If $a\geq b$, then $\frac{\sqrt{a}\sqrt{b}}{a}=\sqrt{\frac{b}{a}}\leq 1$. If $a\leq b$, then $\frac{\sqrt{a}\sqrt{b}}{b}=\sqrt{\frac{a}{b}}\leq 1$. Thus, $|(\rho^{(2)}(\tilde{f},\tilde{g}))|\leq 1$ and $|\rho_{SVNRS_2}(\tilde{f},\tilde{g})|\leq 1$.

Example 4 Consider SVNRS-sets $\tilde{f}$ and $\tilde{g}$ given in Example 2. Then, correlation coefficient between SVNSR-sets $\tilde{f}$ and $\tilde{g}$ can be computed as follows: For parameter e_1 and $\Lambda=t$;

$$\rho_t^{(2)}(\tilde{f},\tilde{g})(e_1) = \frac{(-0.088)(-0.200)+(-0.013)(-0.225)+(0.038)(0.175)+(0.062)(0.250)}{max\Big\{((-0.088)^2+(-0.013)^2+(0.038)^2+(0.062)^2),((-0.200)^2+(-0.225)^2+(0.175)^2+(0.250)^2)\Big\}}$$
$$= 0.231,$$

and for parameter e_2, e_3 and $\Lambda=t$;

$$\rho_t^{(2)}(\tilde{f},\tilde{g})(e_2)=0.422 \ and \ \rho_t^{(2)}(\tilde{f},\tilde{g})(e_3)=-0.310.$$

Then,

$$\frac{1}{|E|}\sum_{e_k\in E}\rho_t^{(2)}(\tilde{f},\tilde{g})(e_k)=\frac{1}{3}(0.231+0.422+(-0.310))=0.114.$$

Similarly,

$$\frac{1}{|E|}\sum_{e_k\in E}\rho_i^{(2)}(\tilde{f},\tilde{g})(e_k)=\frac{1}{3}((-0.405)+(-0.532)+(-0.716))=-0.551,$$
$$\frac{1}{|E|}\sum_{e_k\in E}\rho_f^{(2)}(\tilde{f},\tilde{g})(e_k)=\frac{1}{3}((-0.378)+(-0.026)+(-0.202))=-0.202$$

and so

$$\rho_{SVNRS_2}(\tilde{f},\tilde{g})=\frac{1}{3}\sum_{\forall\Lambda\in\{t,i,f\}}\frac{1}{|E|}\sum_{e_k\in E}\rho_\Lambda^{(2)}(\tilde{f},\tilde{g})(e_k)$$
$$=\frac{1}{3}(0.114+(-0.551)+(-0.202))=-0.213.$$

This value shows that SVNRS-sets $\tilde{f}$ and $\tilde{g}$ have a bad negatively correlated.

If parameters in $\tilde{f}$ and $\tilde{g}$ have weights, then weighted correlation coefficient between $\tilde{f}$ and $\tilde{g}$ can be written as follows:

$$\rho^{w}_{SVNRS_2}(\tilde{f},\tilde{g}) = \frac{1}{3}\sum_{\forall \Lambda\in\{t,i,f\}}\frac{1}{|E|}\sum_{e_k\in E} w(e_k)_{(\tilde{f},\tilde{g})}\rho^{(2)}_{\Lambda}(\tilde{f},\tilde{g})(e_k). \quad (8)$$

5 Clustering algorithm for SVNRS-sets

In this section, an algorithm to make clustering under single valued neutrosophic refined soft environment based on intuitionistic fuzzy clustering algorithm in [48], and correlation coefficient formulas proposed for SVNRS-sets are developed.

Definition 16 Let $\tilde{f}_j$ $(j \in I_n)$ be n $SVNRS-$sets, then $R = (\varepsilon_{ij})_{n\times n}$ be a correlation matrix, where $\varepsilon_{ij} = \rho_{SVNRS}(\tilde{f}_i,\tilde{f}_j)$ is the correlation coefficient of two $SVNRS-$sets $\tilde{f}_i$ and $\tilde{f}_j$, which satisfies the following conditions:

1. $-1 \le \varepsilon_{ij} \le 1$ for all $i, j \in I_n$;
2. $\varepsilon_{ii} = 1, i \in I_n$;
3. $\varepsilon_{ij} = \varepsilon_{ji}$ for all $i, j \in I_n$.

Note that here item (1) is more general than item (1) of Definitions 3 and 10 in [48] and [43], respectively.

Now some definitions and theorems will be present in [48].

Definition 17 [48] Let $R = (\varepsilon_{ij})_{n\times n}$ be a correlation matrix, if $R^2 = R\circ R = (\hat{\varepsilon}_{ij})_{n\times n}$, then R^2 is called a composition matrix of R, where
$\hat{\varepsilon}_{ij} = max_k\big\{min\{\varepsilon_{ik},\varepsilon_{kj}\}\big\}$ for all $i, j \in I_n$.

Theorem 5 *[48] Let $R = (\varepsilon_{ij})_{n\times n}$ be a correlation matrix. Then the composition matrix $R^2 = R\circ R = (\hat{\varepsilon}_{ij})_{n\times n}$, is also a correlation matrix.*

Theorem 6 *[48] Let $R = (\varepsilon_{ij})_{n\times n}$ be a correlation matrix, then for any nonnegative integer k, the composition matrix $R^{2^{k+1}}$ derived from $R^{2^{k+1}} = R^{2^k}\circ R^{2^k}$ is also a correlation matrix.*

Definition 18 [48] Let $R = (\varepsilon_{ij})_{n\times n}$ be a correlation matrix, if $R^2 \subseteq R$, i.e.

$$max_k\big\{min\{\varepsilon_{ik},\varepsilon_{kj}\}\big\} \le \varepsilon_{ij} \text{ for all } i, j \in I_n.$$

then R is called an equivalent correlation matrix.

Theorem 7 *[48] Let $R = (\varepsilon_{ij})_{n\times n}$ denote a correlation matrix, then after having a finite times of compositions:*

$$R \to R^2 \to R^4 \to ... \to R^{2^k} \to ...,$$

there exists a positive integer k such that $R^{2^k} = R^{2^{(k+1)}}$, and R^{2^k} is also an equivalence correlation matrix.

Definition 19 [48] Let $R = (\varepsilon_{ij})_{n\times n}$ be a correlation matrix, then we call $R_\gamma = ({}_\gamma\varepsilon_{ij})_{n\times n}$, where

$$_\gamma\varepsilon_{ij} = \begin{cases} 0, \, if \, \varepsilon_{ij} < \gamma \\ 1, \, if \, \varepsilon_{ij} \geq \gamma \end{cases} \; i, j \in I_n \tag{9}$$

and γ is the confidence level with $\gamma \in [0, 1]$.

Here, since $-1 \leq \varepsilon_{ij} \leq 1,$ for all $i, j \in I_n$; in Definition 19 confidence level γ can be taken as $-1 \leq \gamma \leq 1$.

Algorithm-SVNRS-sets

Let $E = \{e_1, e_2, ..., e_m\}$ be a parameter set, $X = \{x_1, x_2, ..., x_k\}$ be an initial universe and $\{\tilde{f}_1, \tilde{f}_2, ..., \tilde{f}_n\} \subseteq SVNRS_X^E$. Let $w_{f_1}, w_{f_2}, ..., w_{f_n}$ be the weight vectors of the each $SVNRS-$set, respectively. Here $w_{f_1} = (w_{f_1}(e_1), w_{f_1}(e_2), ..., w_{f_1}(e_m))$ and $\sum_{i=1}^{m} w_{f_j}(e_i) = 1$ and $w_{f_j}(e_i) \geq 0$, $j \in I_m$.

Step 1: Find correlation coefficient related to the parameters of $SVNRS-$sets $\tilde{f}_i$ and $\tilde{f}_j$, for all $i, j \in I_n$ using Eq. 5.

$$w(e_k) = \begin{pmatrix} w_{(\tilde{f}_1,\tilde{f}_1)}(e_k) & w_{(\tilde{f}_1,\tilde{f}_2)}(e_k) & \dots & w_{(\tilde{f}_1,\tilde{f}_n)}(e_k) \\ w_{(\tilde{f}_2,\tilde{f}_1)}(e_k) & w_{(\tilde{f}_2,\tilde{f}_2)}(e_k) & \dots & w_{(\tilde{f}_2,\tilde{f}_n)}(e_k) \\ \vdots & \vdots & \ddots & \vdots \\ w_{(\tilde{f}_n,\tilde{f}_1)}(e_k) & w_{(\tilde{f}_n,\tilde{f}_2)}(e_k) & \dots & w_{(\tilde{f}_n,\tilde{f}_n)}(e_k) \end{pmatrix}$$

Step 2: Construct correlation matrix $R = (\varepsilon_{ij})_{n\times n}$ using Eq. 4, where $\varepsilon_{ij} = \rho^w_{SVNRS_1}(\tilde{f}_i, \tilde{f}_j)$.

Step 3: Check whether correlation matrix R satisfies $R^2 \subseteq R$, where $R^2 = R \circ R = (\hat{\varepsilon}_{ij})_{n\times n}, \hat{\varepsilon}_{ij} = max_k\{min\{\varepsilon_{ik}, \varepsilon_{kj}\}\}$ for all $i, j \in I_n$. If R does not satisfy condition $R^2 \subseteq R$, then the equivalent correlation matrix R^{2^k} will be formed :

$$R \to R^2 \to R^4 \to ... \to R^{2^k}, ..., \text{ until } R^{2^k} = R^{2^{(k+1)}}.$$

Step 4: Construct a $\gamma-$cutting matrix $R_\gamma = ({}_\gamma\varepsilon_{ij})_{n\times n}$ as in Definition 19 to classify the $SVNRS-$sets. Let R_{i_γ} and R_{j_γ} be ith and jth column(or row) matrices of R_γ, respectively. If $R_{i_\gamma} = R_{j_\gamma}$, then $SVNRS-$sets $\tilde{f}_i$ and $\tilde{f}_j$ are same characteristic. Therefore, all of $SVNRS-$sets $\tilde{f}_j$ can be classified by using this principle, for all $j \in I_n$.

6 Applied example

In this section, an application of clustering algorithm defined in section 5 is given.

Example 5 Assume that an investment company want to make classification for its investment experts. Therefore, human resource experts of company investigate the evaluations of investment experts about some firms according to previously obtained parameters. Under parameter set $E = \{e_1 = risk\ analy-sis, e_2 = growth\ analysis\}$, evaluations of investment experts $\tilde{f}_1, \tilde{f}_2, ..., \tilde{f}_6$ on firms x_1, x_2, x_3, x_4 and weight of parameters for each investment expert are given in Table 1-6 as tabular representation SVNRS-sets:

	$e_1, 0.6$	$e_2, 0.4$
$\tilde{f}_1$	$\langle x_1, (0.5, 0.1), (0.4, 0.2), (0.6, 0.5)\rangle$	$\langle x_1, (0.4, 0.4), (0.7, 0.4), (0.2, 0.1)\rangle$
	$\langle x_2, (0.7, 0.3), (0.6, 0.2), (0.8, 0.1)\rangle$	$\langle x_2, (0.5, 0.3), (0.6, 0.4), (0.9, 0.7)\rangle$
	$\langle x_3, (0.9, 0.2), (0.4, 0.3), (0.6, 0.2)\rangle$	$\langle x_3, (0.6, 0.2), (0.4, 0.4), (0.5, 0.1)\rangle$
	$\langle x_4, (0.2, 0.1), (0.5, 0.3), (0.7, 0.6)\rangle$	$\langle x_4, (0.3, 0.1), (0.2, 0.1), (0.7, 0.2)\rangle$

Table 1

	$e_1, 0.4$	$e_2, 0.6$
$\tilde{f}_2$	$\langle x_1, (0.4, 0.3), (0.7, 0.1), (0.2, 0.2)\rangle$	$\langle x_1, (0.3, 0.1), (0.4, 0.2), (0.6, 0.1)\rangle$
	$\langle x_2, (0.6, 0.3), (0.5, 0.1), (0.7, 0.2)\rangle$	$\langle x_2, (0.5, 0.3), (0.4, 0.1), (0.7, 0.2)\rangle$
	$\langle x_3, (0.4, 0.1), (0.7, 0.2), (0.6, 0.4)\rangle$	$\langle x_3, (0.1, 0.1), (0.2, 0.1), (0.4, 0.1)\rangle$
	$\langle x_4, (0.5, 0.4), (0.7, 0.3), (0.4, 0.2)\rangle$	$\langle x_4, (0.7, 0.3), (0.6, 0.5), (0.9, 0.7)\rangle$

Table 2

	$e_1, 0.8$	$e_2, 0.2$
$\tilde{f}_3$	$\langle x_1, (0.3, 0.1), (0.4, 0.4), (0.7, 0.6)\rangle$	$\langle x_1, (0.7, 0.5), (0.4, 0.3), (0.8, 0.2)\rangle$
	$\langle x_2, (0.5, 0.3), (0.7, 0.7), (0.5, 0.4)\rangle$	$\langle x_2, (0.8, 0.4), (0.6, 0.5), (0.3, 0.1)\rangle$
	$\langle x_3, (0.1, 0.0), (0.6, 0.5), (0.4, 0.1)\rangle$	$\langle x_3, (0.9, 0.5), (0.7, 0.7), (0.8, 0.6)\rangle$
	$\langle x_4, (0.9, 0.8), (0.5, 0.5), (0.4, 0.4)\rangle$	$\langle x_4, (0.3, 0.3), (1.0, 0.8), (0.2, 0.1)\rangle$

Table 3

	$e_1, 0.5$	$e_2, 0.5$
$\tilde{f}_4$	$\langle x_1, (1.0, 0.3), (1.0, 0.5), (0.5, 0.1)\rangle$	$\langle x_1, (0.4, 0.4), (0.5, 0.3), (0.7, 0.4)\rangle$
	$\langle x_2, (0.3, 0.1), (0.7, 0.4), (0.2, 0.1)\rangle$	$\langle x_2, (0.8, 0.5), (0.3, 0.1), (0.8, 0.2)\rangle$
	$\langle x_3, (0.4, 0.2), (0.6, 0.5), (0.9, 0.9)\rangle$	$\langle x_3, (0.6, 0.1), (0.2, 0.1), (0.5, 0.4)\rangle$
	$\langle x_4, (0.5, 0.3), (0.7, 0.6), (0.2, 0.1)\rangle$	$\langle x_4, (0.9, 0.2), (0.8, 0.6), (0.7, 0.5)\rangle$

Table 4

	$e_1, 0.3$	$e_2, 0.7$
$\tilde{f}_5$	$\langle x_1, (0.3, 0.1), (0.4, 0.2), (0.7, 0.5)\rangle$	$\langle x_1, (0.6, 0.4), (0.8, 0.5), (0.9, 0.2)\rangle$
	$\langle x_2, (0.8, 0.3), (0.7, 0.1), (0.7, 0.7)\rangle$	$\langle x_2, (0.8, 0.7), (0.9, 0.5), (0.1, 0.1)\rangle$
	$\langle x_3, (0.6, 0.1), (0.5, 0.4), (0.8, 0.0)\rangle$	$\langle x_3, (0.2, 0.2), (0.3, 0.3), (0.4, 0.4)\rangle$
	$\langle x_4, (0.9, 0.9), (0.8, 0.8), (0.7, 0.5)\rangle$	$\langle x_4, (0.6, 0.1), (0.7, 0.6), (0.8, 0.7)\rangle$

Table 5

	$e_1, 0.7$	$e_2, 0.3$
$\tilde{f}_6$	$\langle x_1, (0.4, 0.2), (0.5, 0.4), (0.3, 0.1)\rangle$	$\langle x_1, (0.5, 0.5), (0.9, 0.6), (0.3, 0.2)\rangle$
	$\langle x_2, (0.6, 0.3), (0.6, 0.6), (0.5, 0.5)\rangle$	$\langle x_2, (0.9, 0.5), (0.5, 0.2), (0.2, 0.2)\rangle$
	$\langle x_3, (0.8, 0.3), (0.5, 0.4), (0.4, 0.3)\rangle$	$\langle x_3, (0.8, 0.7), (0.5, 0.3), (0.5, 0.3)\rangle$
	$\langle x_4, (0.7, 0.4), (0.9, 0.8), (0.8, 0.7)\rangle$	$\langle x_4, (0.3, 0.2), (0.9, 0.7), (0.4, 0.3)\rangle$

Table 6

Step 1: Using Eq. 5, correlation coefficient between parameters of $SVNRS-$sets $\tilde{f}_i$ and $\tilde{f}_j$ $(i, j \in I_6)$ are obtained as follows:

$$w(e_1) = \begin{pmatrix} 1,000 & 0,667 & 0,750 & 0,883 & 0,500 & 0,857 \\ 0,667 & 1,000 & 0,500 & 0,750 & 0,750 & 0,571 \\ 0,750 & 0,500 & 1,000 & 0,625 & 0,375 & 0,875 \\ 0,834 & 0,750 & 0,625 & 1,000 & 0,600 & 0,714 \\ 0,500 & 0,750 & 0,375 & 0,600 & 1,000 & 0,429 \\ 0,857 & 0,571 & 0,875 & 0,714 & 0,429 & 1,000 \end{pmatrix}$$

$$w(e_2) = \begin{pmatrix} 1,000 & 0,667 & 0,500 & 0,800 & 0,571 & 0,750 \\ 0,667 & 1,000 & 0,333 & 0,833 & 0,857 & 0,500 \\ 0,500 & 0,333 & 1,000 & 0,400 & 0,286 & 0,667 \\ 0,800 & 0,833 & 0,400 & 1,000 & 0,714 & 0,600 \\ 0,571 & 0,857 & 0,286 & 0,714 & 1,000 & 0,429 \\ 0,750 & 0,500 & 0,667 & 0,600 & 0,429 & 1,000 \end{pmatrix}.$$

Step 2: Correlation coefficient of the $SVNRS-$sets $\tilde{f}_j$ $(j \in I_6)$ by using Eq. (4) and correlation coefficient of parameters for each $(\tilde{f}_i, \tilde{f}_j)$ $(i, j \in I_6)$ given in Step 1 are obtained as follows:

$$R = \begin{pmatrix} 1,000 & -0,279 & -0,007 & -0,408 & 0,033 & 0,195 \\ -0,279 & 1,000 & -0,116 & 0,450 & 0,127 & 0,060 \\ -0,007 & -0,116 & 1,000 & -0,163 & 0,064 & 0,178 \\ -0,408 & 0,450 & -0,163 & 1,000 & 0,109 & -0,081 \\ 0,033 & 0,127 & 0,064 & 0,109 & 1,000 & 0,182 \\ 0,195 & 0,060 & 0,178 & -0,081 & 0,182 & 1,000 \end{pmatrix}.$$

Step 3: R^2 can be obtained as follow

$$R^2 = R \circ R = \begin{pmatrix} 1,000 & 0,060 & 0,178 & 0,033 & 0,182 & 0,195 \\ 0,060 & 1,000 & 0,064 & 0,450 & 0,127 & 0,127 \\ 0,178 & 0,064 & 1,000 & 0,064 & 0,178 & 0,178 \\ 0,033 & 0,450 & 0,064 & 1,000 & 0,127 & 0,109 \\ 0,182 & 0,127 & 0,064 & 0,127 & 1,000 & 0,182 \\ 0,195 & 0,127 & 0,178 & 0,109 & 0,182 & 1,000 \end{pmatrix}.$$

Here, note that $R^2 \nsubseteq R$. The correlation matrix R is not an equivalent matrix. Therefore, we further calculate:

$$R^4 = R^2 \circ R^2 = \begin{pmatrix} 1,000 & 0,127 & 0,178 & 0,127 & 0,182 & 0,195 \\ 0,127 & 1,000 & 0,127 & 0,450 & 0,127 & 0,127 \\ 0,178 & 0,127 & 1,000 & 0,127 & 0,178 & 0,178 \\ 0,127 & 0,450 & 0,109 & 1,000 & 0,127 & 0,127 \\ 0,182 & 0,127 & 0,178 & 0,127 & 1,000 & 0,182 \\ 0,195 & 0,127 & 0,178 & 0,127 & 0,182 & 1,000 \end{pmatrix},$$

$$R^8 = R^4 \circ R^4 = \begin{pmatrix} 1,000 & 0,127 & 0,178 & 0,127 & 0,182 & 0,195 \\ 0,127 & 1,000 & 0,127 & 0,450 & 0,127 & 0,127 \\ 0,178 & 0,127 & 1,000 & 0,127 & 0,178 & 0,178 \\ 0,127 & 0,450 & 0,127 & 1,000 & 0,127 & 0,127 \\ 0,182 & 0,127 & 0,178 & 0,127 & 1,000 & 0,182 \\ 0,195 & 0,127 & 0,178 & 0,127 & 0,182 & 1,000 \end{pmatrix},$$

and

$$R^{16} = R^8 \circ R^8 = \begin{pmatrix} 1,000 & 0,127 & 0,178 & 0,127 & 0,182 & 0,195 \\ 0,127 & 1,000 & 0,127 & 0,450 & 0,127 & 0,127 \\ 0,178 & 0,127 & 1,000 & 0,127 & 0,178 & 0,178 \\ 0,127 & 0,450 & 0,127 & 1,000 & 0,127 & 0,127 \\ 0,182 & 0,127 & 0,178 & 0,127 & 1,000 & 0,182 \\ 0,195 & 0,127 & 0,178 & 0,127 & 0,182 & 1,000 \end{pmatrix} = R^8.$$

Thus, R^8 is an equivalent correlation matrix.

Step 4: Using Eq.(9) to form a $\gamma-$cutting matrix $R_\gamma = ({}_\gamma\varepsilon_{ij})_{n\times n}$ based on which, all possible classifications of the experts $\tilde{f}_j\,(j \in I_6)$ obtained as follow:

(1) If $0 < \gamma \leq 0.127$, then $\tilde{f}_i\,(i \in I_6)$ are of the same characteristic(or same type):

$$\{\tilde{f}_1, \tilde{f}_2, \tilde{f}_3, \tilde{f}_4, \tilde{f}_5, \tilde{f}_6\}$$

(2) If $0.127 < \gamma \leq 0.178$, then $\tilde{f}_i\,(i \in I_6)$ are classified in two characteristic:

$$\{\tilde{f}_1, \tilde{f}_3, \tilde{f}_5, \tilde{f}_6\}, \{\tilde{f}_2, \tilde{f}_4\}.$$

(3) If $0.178 < \gamma \leq 0.182$, then $\tilde{f}_i\,(i \in I_6)$ are classified in three characteristic:

$$\{\tilde{f}_1, \tilde{f}_5, \tilde{f}_6\}, \{\tilde{f}_2, \tilde{f}_4\}, \{\tilde{f}_3\}.$$

(4) If $0.182 < \gamma \leq 0.195$, then $\tilde{f}_i\,(i \in I_6)$ are classified in four characteristic:

$$\{\tilde{f}_1, \tilde{f}_6\}, \{\tilde{f}_2, \tilde{f}_4\}, \{\tilde{f}_3\}, \{\tilde{f}_5\}.$$

(5) If $0.195 < \gamma \leq 0.450$, then $\tilde{f}_i\,(i \in I_6)$ are classified in five characteristic:

$$\{\tilde{f}_1\}, \{\tilde{f}_2, f_4\}, \{\tilde{f}_3\}, \{\tilde{f}_5\}, \{\tilde{f}_6\}.$$

(6) If $0.450 < \gamma \leq 1.00$, then $\tilde{f}_i\,(i \in I_6)$ are classified in six characteristic:

$$\{\tilde{f}_1\}, \{\tilde{f}_2\}, \{\tilde{f}_3\}, \{\tilde{f}_4\}, \{\tilde{f}_5\}, \{\tilde{f}_6\}.$$

7 Conclusion

In this paper, the concept of single valued neutrosophic refined soft set and its set theoretical operations such as union, intersection and complement are defined and some of their basic properties are proved. Then, two formulas to compute correlation coefficient between two single valued neutrosophic refined soft sets are developed. Furthermore, the developed method is applied to clustering analysis based on clustering algorithm proposed by Xu et al. [48]. However, I hope that the main thrust of proposed formula will be in the field of equipment evaluation, data mining and investment decision making.

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