Generic form of the Poulet numbers having a prime factor of the form 30n+23

Marius Coman
email: mariuscoman13@gmail.com

Abstract. In this paper I observe that many Poulet numbers P having a prime factor q of the form 30*n + 23, where n positive integer, can be written as P = m*(q^2 – q) + q^2, where m positive integer, and I conjecture that any Poulet number P having 23 as a prime factor can be written as P = 506*m + 529, where m positive integer.

Observation:

Many Poulet numbers P having a prime factor q of the form 30*n + 23, where n positive integer, can be written as P = m*(q^2 – q) + q^2, where m positive integer.

Examples:

: 8321 = 53*157 and 8321 = 2*(53^2 – 53) + 53^2, so [q, m] = [53, 2];
: 85489 = 53*1613 and 85489 = 30*(53^2 – 53) + 53^2, so [q, m] = [53, 30];
: 88561 = 11*83*97 and 88561 = 12*(83^2 – 83) + 83^2, so [q, m] = [83, 12];
: 91001 = 17*53*101 and 91001 = 32*(53^2 – 53) + 53^2, so [q, m] = [53, 32];
: 208465 = 5*173*241 and 208465 = 6*(173^2 – 173) + 173^2, so [q, m] = [173, 6];
: 215265 = 3*5*113*127 and 215265 = 16*(113^2 – 113) + 113^2, so [q, m] = [113, 16];
: 275887 = 263*1049 and 275887 = 3*(263^2 – 263) + 263^2, so [q, m] = [263, 3];
: 278545 = 5*17*29*113 and 278545 = 21*(113^2 – 113) + 113^2, so [q, m] = [113, 21];
: 422659 = 3*53*2687 and 422659 = 154*(53^2 – 53) + 53^2, so [q, m] = [53, 154].

Conjecture:

Any Poulet number P having 23 as a prime factor can be written as P = 506*m + 529, where m positive integer.

Verifying the conjecture:
(For the first seven such Poulet numbers)

: 2047 = 23*89 and 2047 = 3*506 + 529, so m = 3;
: 6601 = 7*23*41 and 6601 = 12*506 + 529, so m = 12;
: 15709 = 23*683 and 15709 = 30*506 + 529, so m = 30;
: 30889 = 17*23*79 and 30889 = 60*506 + 529, so m = 60;
: 137149 = 23*67*89 and 30889 = 270*506 + 529, so m = 270;
: 272251 = 7*19*23*89 and 272251 = 537*506 + 529, so m = 537;
: 340561 = 13*17*23*67 and 340561 = 672*506 + 529, so m = 672.

Note the following 13 Poulet numbers having a prime factor of the form 23*n + 30 (from the first 29 such Poulet numbers) which can't be written in the way showed above: 13747 = 59*233, 3277 = 29*113, 31417 = 89*353, 60787 = 89*683, 65077 = 59*1103, 72885 = 3*5*43*113, 88357 = 149*593, 130561 = 137*953, 194221 = 167*1163, 196021 = 7*41*683, 253241 = 157*1613, 256999 = 233*1103, 280601 = 277*1013. In all of these cases, the prime factor of the form 23*n + 30 is the biggest prime factor (a particular case is the number 256999 having both of the factors of the form 23*n + 30).