An estimation of power-consumption and thrust-generation at a medium-sized thruster of Helios-type

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Abstract: The following paper is not a complete self-sustained model of the electromagnetic system of a plasma-thruster. It is just an estimation of inputted energy and outputted thrust. The points of the ionization system where transfer of electrical energy mainly occurs are highlighted by a mathematical treatment to estimate the power consumption. The formation of the charge-density as basis for the electric field that generates the repulsion-force by extraction and acceleration of the ionized fuel-gas was analyzed in order to estimate the thrust generation. Nevertheless, this approach could be used to be a basis of a self-sustained model. Since it was planned to obtain an estimation of the needed electrical input energy, the pure electric thruster of the two Helios concepts [1] was chosen as model-type.

Definition of symbols of the mathematical modeling (if given with pre-defined values):

l_1 – length of pre-stage (:= 0,05m)
l_2 – length of main-stage (:= 0,3m)
l_3 – length of extraction system (:= 0,1m)
d_1 – diameter of pre-stage (:= 0,03m)
d_2 – diameter of main-stage (:= 0,1m)
N_1 – number of windings secondary Tesla-coil (:= 100)
N_2 – number of windings primary Tesla-coil (:= 5)
N_3 – number of windings magnetic coil (:= 20)
d_{N1} – winding-diameter secondary coil (:= 0,003m = 3mm)
d_{N3} – winding-diameter magnetic coil (:= 0,005m = 5mm)
f – frequency of the induction coil (:= 500000Hz)
R_{coil} – resistivity of the secondary coil
$v_{e,1}$ – velocity of a free electron in pre-stage
$v_{e,2}$ – velocity of a free electron in main-stage
$s_{e,1}$ – free-path of a free electron in pre-stage
$s_{e,2}$ – free-path of a free electron in main-stage
a_e – acceleration of a free electron by electric field
δ – penetration-depth electromagnetic field into plasma
r_1 – distance pre-stage electrode to point of minimal action
$U_{ion,Xe}$ – ionization energy of Xenon
$U_{kin,e}$ – kinetic energy of a free electron
$n_{Xe,1}$ – gas-density in pre-stage
$n_{Xe,2}$ – gas-density in main-stage
p – gas-pressure in pre-stage (:= 1Pa)
T – temperature (:= 300K)

Introduction

General benefits

The high-thrust sector of electric propulsion tries to combine two goals. First a continuous thrust with the possibility of switching on and off on demand. This makes the mission planning easier for the main reason of more flexibility of reaction on occurring disturbances.
The second goal is a thrust as high as possible in order to reduce the time of travel respectively to raise the payload.

The first goal, continuous thrust, is a fundamental and crucial feature of electric propulsion in general and makes this kind of propulsion the most promising basis.
The Helios-concepts are designed to achieve the second goal on this basis. These concepts aim for ionization of a fuel-gas under high-pressure conditions to basically produce a sufficient number of thrust-generating particles for reaching high thrust-levels. To fulfill this achievement as efficient as possible an ionization in two stages is used. The first stage has the one and only task to improve the initial conditions and provide the possibility that the main-stage is able to ionize the fuel-gas without huge effort of energy.

The task of the main-stage is not only the production of charged particles but also to couple in electric energy into these particles. The targeted particles for this process are the electrons. Since the ions are orders of magnitude heavier than the electrons in case of a high-frequent change of electric field direction the electrons are almost complete carrier of current and the transfer of extern electric energy in kinetic particle energy is most efficient.

Also with focus on efficiency the transfer of the kinetic energy from the accelerated electrons onto the ions, the actual thrust-generating particles, occurs. This happens not via collision, what is not efficient because of the mass difference, but via effect of an electric field. This field arises from a resulting charge-distribution as consequence of the different flow-rates of electrons and ions. The strength of this electric field depends on the difference of velocity between electrons and ions; so extern electro-dynamic energy is most efficiently transferred into thrust-generating kinetic energy of ions.

The extraction system combines the benefits of gridded ion-source and plasma-source. Like at ion-sources the extracted particles are accelerated by electro-static fields, an optional origin of high particle velocities. But the field source is not a static grid but dynamic charge-densities, on this way ions and electrons are extracted together what prevents the formation of current-limiting space charge zones.

The whole technology concept Helios follows two guidelines: to generate a high thrust-level and to do this by a transfer as efficient as possible of external electric energy into thrust.

Thruster model

For the following estimation a medium-sized thruster model is assumed. The thruster-nozzle, and therewith the main-stage of ionization, has a diameter of 10cm. The length of the main-stage is 30cm. Length and diameter of the pre-stage are 5cm and 3cm respectively. The Tesla transformer consists of a primary coil with 5 windings and a secondary coil with 100 windings and winding-diameter of 3mm. Both coils are made of copper. The fuel-gas is Xenon with a pressure of 1Pa in the pre-stage. The magnetic coil of the extraction system has 20 windings and a length of 10cm.

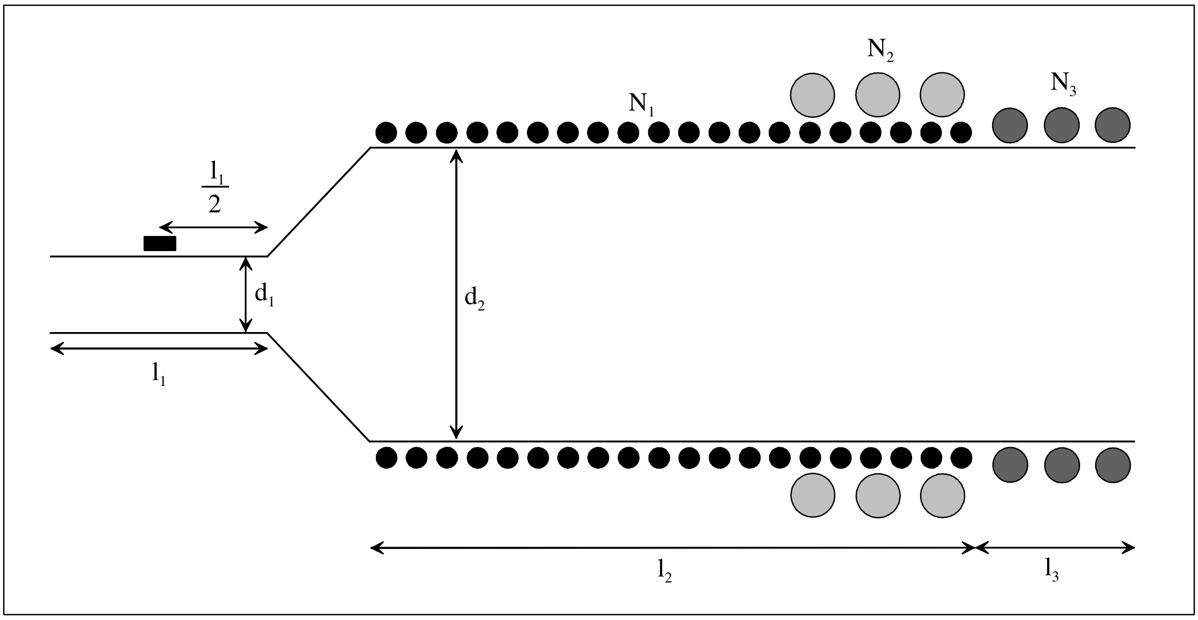


Figure1: Scheme of a Helios-1 thruster with included parameter definition.

<u>"Helios-1" power-consumption</u>

Tesla transformer

The underlying basis technology of the ionization-system of a Helios-1 propulsion system is a weak air-coupled transformer; a so-called Tesla-transformer. At this system there are two points of interaction with the plasma: the end-capacity and the coil of the secondary oscillation circuit of the transformer. At these points occurs the coupling in of electric energy with the aim of ionization and acceleration of particles. It follows a mathematical treatment of the mechanisms at these two points to work out an estimation of the energy transfer.

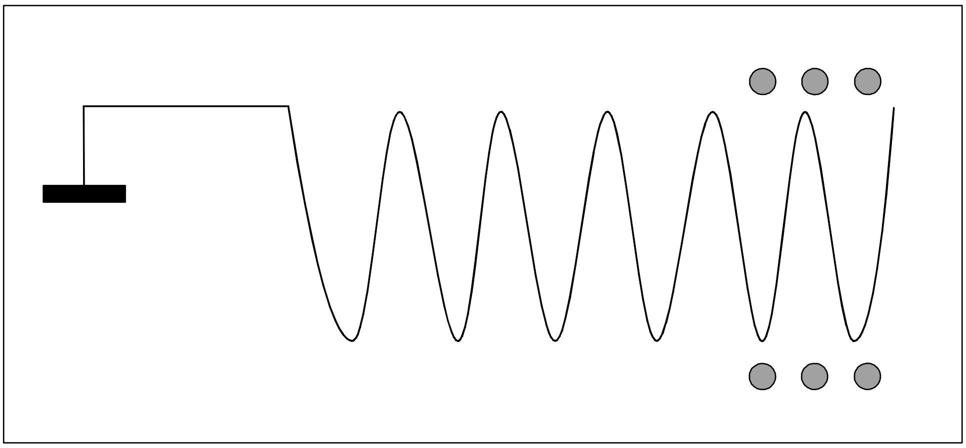


Figure2: Scheme of the Tesla-transformer as ionization system. With secondary coil and capacity and the primary coil.

Pre-stage

For a more convenient treatment as radial-symmetric problem the electrode of the pre-stage is assumed as point-shaped in the following (see fig.3).

The ionization mechanism is the electron collision; at the event of collision the kinetic energy of a free electron is transferred into inner energy of the electron-shell of neutral fuel-gas atom. This means at the distance r_1 the electric field of the electrode has to accelerate the electron on a kinetic energy-level equals the ionization-energy of Xenon.

$$U_{ion,Xe} = U_{kin,e} = \frac{1}{2} m_e v_{e,1}^2 \tag{1}$$

The primordially existing electrons generated by natural radioactivity or cosmic rays are assumed in rest before accelerated by electric field.

$$v_{e,1} = a_e t \text{ ; with } t = \sqrt{2\frac{s_{e,1}}{a_e}} \tag{2}$$

$$v_{e,1} = \sqrt{2 a_e s_{e,1}} \tag{3}$$

The free-path $s_{e,1}$ a free electron is able to cover while acceleration is limited by the density of particles $n_{Xe,1}$.

$$s_{e,1} = \sqrt[3]{\frac{1}{n_{Xe,1}}} \text{ ; assumed homogenous distribution of particles} \tag{4}$$

Via the ideal gas law the density of particles in a gas is related to the pressure of this gas.

$$p = \bar{n}RT, n = \bar{n}N_A \tag{5}$$

$$(5) \rightarrow (4): \; s_{e,1} = \sqrt[3]{\frac{RT}{pN_A}} \tag{6}$$

Within this distance the electric field has to accelerate the free electron on a level of kinetic energy equals the ionization energy of Xenon. The free electron has the charge e and the electrode the charge Q.

$$m_e a_{e,1} = \frac{1}{4\pi\varepsilon_0}\frac{eQ}{r^2} \tag{7}$$

The electrode as field source is made of metal. Hence, all charge of the electrode consists of a positive or negative amount of electrons with single charge e.

$$Q = Ne \tag{8}$$

$$(8) \rightarrow (7): \; m_e a_{e,1} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}N \tag{9}$$

$$a_{e,1} = \frac{1}{4\pi\varepsilon_0 m_e}\frac{e^2}{r^2}N \tag{10}$$

This means for achieving the ionization energy level:

$$(1) \rightarrow U_{ion,Xe} = \frac{1}{2} m_e v_{e,1}^2 \tag{11}$$

$$(3) \rightarrow (11): U_{ion,Xe} = m_e a_{e,1} s_{e,1} \tag{12}$$

$$(4) \rightarrow (12): U_{ion,Xe} = m_e a_{e,1} \sqrt[3]{\frac{RT}{pN_A}} \tag{13}$$

$$(10) \rightarrow (13): U_{ion,Xe} = \sqrt[3]{\frac{RT}{pN_A}} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} N \tag{14}$$

An efficient ionization is guaranteed in a case when the electric field strength is sufficient to ionize at a point with distance r_1 from the electrode inside the discharge area as far as possible away from the electrode (see fig.3).

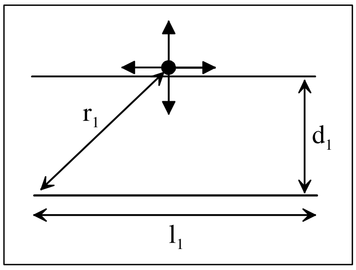


Figure3: The electrode of the pre-stage as source of a radial-symmetric electric field with l_1 and d_1 as length respectively diameter of the pre-stage.

$$\text{Fig.2} \rightarrow \; r_1^2 = d_1^2 + \frac{l_1^2}{4} \tag{15}$$

$$(15) \rightarrow (14): U_{ion,Xe} = \sqrt[3]{\frac{RT}{pN_A}} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{d_1^2 + \frac{l_1^2}{4}} N \tag{16}$$

$$N = 4\pi\varepsilon_0 U_{ion,Xe} \sqrt[3]{\frac{pN_A}{RT}} \frac{d_1^2 + \frac{l_1^2}{4}}{e^2} \tag{17}$$

Main-stage

With the transfer from pre-stage to main-stage the diameter of the ionization chamber increases from d_1 to d_2 (see fig1.). The stream of fuel-gas expands and the free path of a free electron to become accelerated on ionization energy increases.

A number N_{gas} of the fuel-gas stream fills a volume-segment dV_1 in the pre-stage and a volume-segment dV_2 after passing into the main-stage.

$$dV_1 = \frac{\pi d_1^2}{4} dz \tag{18}$$

$$dV_2 = \frac{\pi d_2^2}{4} dz \tag{19}$$

$$n_{Xe,1,2} = \frac{N_{gas}}{dV_{1,2}} \tag{20}$$

$n_{Xe,1} dV_1 = n_{Xe,2} dV_2$; with number of particles N_{gas}=const. (21)

(18), (19) → (21): $n_{Xe,2} = n_{Xe,1} \frac{d_1^2}{d_2^2}$ (22)

(22) → (4), (5) → (22): $s_{e,2} = \sqrt[3]{\frac{RT}{pN_A} \frac{d_2^2}{d_1^2}}$ (23)

$s_{e,2}$ is the distance a free electron is able to cover in the main-stage to gain a kinetic energy level by electric field acceleration as high as necessary to ionize.

Crucial for the treatment of interaction between electromagnetic field and plasma is the depth δ an extern field is able to penetrate the plasma. The applied electromagnetic field is generated by oscillating currents inside a coil surrounding the plasma region. Hence the generated field is circular closed and the direction of field-action changes periodically. Since the period of field change is high-frequent and the inertia of ions is much higher then the one of electrons; almost only the electrons and hardly the ions are able to follow the field-action. In this radio-frequent scenario the plasma features a metal-like behavior: electrons are carrier of current and are influenced by electric field. A formalism related to metal can be used to treat penetration depth.
The electrons excited to move by external field are electric field-sources themselves. They weak the external field [2]. Since these sources of field-weakening are homogenously distributed the decrease of extern field is exponential.

$$E = E_0 e^{-\frac{d}{\delta}} \tag{24}$$

For d=δ the external field is increased on 1/e of the initial value. By common convention this distance defines the penetration depth.
The penetration depth is inversely proportional to the frequency f of the external electromagnetic field and the conductivity σ of plasma.

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \mu_r \sigma}} \tag{25}$$

Normally the plasma is diamagnetic and it can be determined μ_r=1. The conductivity σ depends on the density n_e and the free time of flight τ of the electrons; since the electrons are the only carrier of current.

$$\sigma = \frac{n_e e^2 \tau}{m_e} \tag{26}$$

By neglecting the initial electrons generated in the pre-stage and with assumed dominance of generation of single-charged ions the ionization in the main-stage is a symmetric process that produces a nearly symmetric density of charged particles.

$$n_i \approx n_e \tag{27}$$

Within the region of the penetration depth of external electromagnetic field the optimum case of complete ionization of the gas-stream is assumed.

$$n_{gas} = n_i \approx n_e \tag{28}$$

For the treatment of electrons as carriers of current and not as ionizers by impact a collision-less case is assumed. Hence the free time of flight τ of an electron is identically with the half of the time-period D of the external oscillating field.

$$\tau = \frac{D}{2} = \frac{1}{2f} \tag{29}$$

Regarding the penetration depth of the field and the cylindrical symmetry of the field-generating electrode a zone inside the plasma is formed where the electric field-action causes ionization. This zone has the shape of a hollow-cylinder with wall-thickness δ [2].

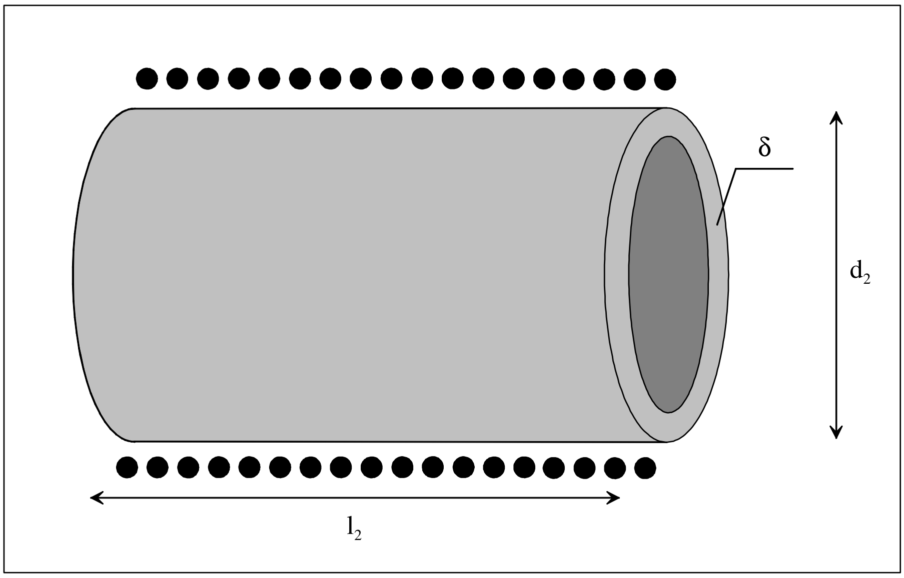


Figure4: Scheme of interaction between induction coil and plasma.

Since plasma features a metal-like behavior in the radio-frequent range the interaction between induction coil (secondary Tesla-coil) and plasma can be seen as mutual inductance between an outside-coil with N_1 windings and an inside-coil with only one winding (see fig.4).
Within the induction coil (outside) flows a time-varying current I and generates in the plasma an induction-voltage U_{ind}.

$$U_{ind} = \frac{\pi \mu_0 N_1 d_2^2}{4 l_2} \frac{\Delta I}{\Delta t} \tag{30}$$

The voltage U_{ind} is induced inside the hollow plasma-cylinder and acts there as electric field with average strength E_2 on the charge-carriers.

$$E_2 = \frac{U_{ind}}{\pi(d_2 - \delta)} \tag{31}$$

$$(30) \rightarrow (31):\ E_2 = \frac{\mu_0 N_1 d_2^2}{4 l_2 (d_2 - \delta)} \frac{\Delta I}{\Delta t} \tag{32}$$

A free electron in the main-stage is accelerated with $a_{e,2}$ by the electric field E_2 and has to gain a kinetic energy-level equals the ionization energy of Xenon within the distance of $s_{e,2}$.

$$m_e a_{e,2} = eE_2 \tag{33}$$

$$(32) \rightarrow (33):\ a_{e,2} = \frac{e}{m_e} \frac{\mu_0 N_1 d_2^2}{4 l_2 \pi (d_2 - \delta)} \frac{\Delta I}{\Delta t} \tag{34}$$

$$U_{ion,Xe} = \frac{1}{2} m_e v_{e,2}^2 = m_e a_{e,2} s_{e,2} \tag{35}$$

$$(34) \rightarrow (35),\ (23) \rightarrow (35):\ U_{ion,Xe} = \frac{e\mu_0 N_1 d_2^2}{4 l_2 \pi (d_2 - \delta)} \sqrt[3]{\frac{RT}{pN_A} \frac{d_2^2}{d_1^2}} \frac{\Delta I}{\Delta t} \tag{36}$$

The time-depending variation of the current inside the coil is with this:

$$\frac{\Delta I}{\Delta t} = U_{ion,Xe} \sqrt[3]{\frac{pN_A}{RT} \frac{d_1^2}{d_2^2}} \frac{4 l_2 \pi (d_2 - \delta)}{e\mu_0 N_1 d_2^2} \tag{37}$$

Power estimation

To estimate the power consumption the calculation of the current flowing in the induction current is essential. The therefore necessary boundary conditions are established with the previous work above.

First: The current I flowing in the coil has to transport N electrons to the electrode of the pre-stage within a half of the period D of its electrical oscillation. These electrons inside the electrode are the source of the electric field that ionizes in the pre-stage. Since this coil current varies with time from 0 to I (assumed linearity) the value of I/2 can be taken as averaged transport current.

$$eN = \frac{I}{2} \Delta t \tag{38}$$

$$(17) \rightarrow (38):\ I\Delta t = 8\pi\varepsilon_0 U_{ion,Xe} \sqrt[3]{\frac{pN_A}{RT}} \frac{d_1^2 + \frac{l_1^2}{4}}{e} := A \tag{39}$$

Second: The variation of coil current while a half of the time-period D has to induce a voltage inside the main-stage that is sufficient to ionize the fuel-gas there.

$$(37) \rightarrow \frac{\Delta I}{\Delta t} = U_{ion,Xe} \sqrt[3]{\frac{pN_A}{RT} \frac{d_1^2}{d_2^2} \frac{4 l_2 \pi (d_2 - \delta)}{e \mu_0 N_1 d_2^2}} := B \tag{40}$$

When the initial current is 0 it can be taken ΔI=I.
Utilization of boundary conditions:

$$I \Delta t = A \Rightarrow \Delta t = \frac{A}{I} \tag{41}$$

$$\frac{I}{\Delta t} = B \Rightarrow \Delta t = \frac{I}{B} \tag{42}$$

$$\Rightarrow \frac{A}{I} = \frac{I}{B} \Rightarrow I^2 = AB \Rightarrow I = \sqrt{AB} \tag{43}$$

$$I = \sqrt{32} \frac{\pi}{e} \sqrt{\frac{\varepsilon_0}{\mu_0}} U_{ion,Xe} \sqrt[3]{\frac{pN_A}{RT} \frac{d_1}{d_2}} \sqrt{\frac{l_2 \left(d_1^2 + \frac{l_1^2}{4} \right) (d_2 - \delta)}{N_1 d_2^2}} \tag{44}$$

For the generation of a current I inside a coil made of copper with N_1 windings, a diameter of d_2 and a winding-diameter of d_{N1} an electric power of P_{coil} is necessary.

$$R_{coil} = \rho_{Cu} \frac{l_2}{A_{winding}} = 4 \rho_{Cu} N_1 \frac{d_2}{d_{N1}^2} \tag{45}$$

$$P_{coil} = R_{coil} I^2 \tag{46}$$

$$(45) \rightarrow (46),\ (44) \rightarrow (46):\ P_{coil} = 128 \rho_{Cu} \frac{\varepsilon_0}{\mu_0} \frac{\pi^2}{e^2} U_{ion,Xe}^2 \left(\frac{pN_A}{RT} \frac{d_1}{d_2} \right)^{\frac{2}{3}} \frac{l_2 \left(d_1^2 - \frac{l_1^2}{4} \right) (d_2 - \delta)}{d_{N1}^2 d_2} \tag{47}$$

A crucial and dominant process at a high-frequent discharge is the elastic collision between electrons and ions. Because of the mass difference thereby the electrons are reflected almost without any loss of kinetic energy. With every change of direction of the applied high-frequent field the electron gains an additional amount of kinetic energy. In the actual case an electron that gains the tenth part of ionization energy in the first period of applied field will receive the full kinetic energy for ionization of Xenon within 10µs. To apply the full ionization energy within the first period is not efficient.

$$U_{ion,Xe} \rightarrow \frac{1}{10} U_{ion,Xe}:\ P_{coil} = \frac{128}{100} \rho_{Cu} \frac{\varepsilon_0}{\mu_0} \frac{\pi^2}{e^2} U_{ion,Xe}^2 \left(\frac{pN_A}{RT} \frac{d_1}{d_2} \right)^{\frac{2}{3}} \frac{l_2 \left(d_1^2 - \frac{l_1^2}{4} \right) (d_2 - \delta)}{d_{N1}^2 d_2} \tag{47a}$$

Extraction system

The task of the extraction system is to align the movement of the thrust-generating particles in order to establish a directed thrust outward the plasma engine. The static magnetic field produced by the magnetic coil of the extraction system should be strong enough to force even the heavier ions on orbits with diameters smaller than the thruster diameter d_2.

$$evB = 2m\frac{v^2}{d} \quad ; d \le d_2 \tag{48}$$

The thrust-generating force on the ions acts outward the thruster. The vector of this velocity component is parallel with the magnetic field-lines of the extraction coil and remains uninfluenced. Furthermore the ions do not follow the electromagnetic oscillating induction field. The only component of velocity of the ions that is able to interact with the magnetic field is the thermal velocity v_{th}.

$$v_{th} = \sqrt{\frac{3k_B T}{m_{Xe}}} \tag{49}$$

$$m_{Xe} = 131{,}29u = 2{,}18 \cdot 10^{-25}\,kg \tag{50}$$

$$(48) \rightarrow (49):\ B = \sqrt{\frac{3k_B m_{Xe} T}{e^2 d_2^2}} \tag{51}$$

The magnetic field of the extraction system is produced by a cylindrical coil with N_3 windings, a length l_3, a diameter d_2 and a direct current I_{mag} inside the coil.

$$I_{mag} = \frac{\sqrt{l_3^2 + d_2^2}\,B}{\mu_0 N_3} \tag{52}$$

An electrical power P_{mag} is necessary to generate the current I_{mag} inside the coil.

$$R_{mag} = 4\rho_{Cu} N_3 \frac{d_2}{d_{N3}^2} \tag{53}$$

$$P_{mag} = R_{mag} I_{mag}^2 \tag{54}$$

$$(52), (53) \rightarrow (54):\ P_{mag} = 4\frac{\rho_{Cu}}{\mu_0^2}\frac{B^2}{N_3}\frac{d_2\left(l_3^2 + d_2^2\right)}{d_{N3}^2} \tag{55}$$

Thrust generation

The causal mechanism of generation of thrust is the difference in velocity of ions and electrons. This creates a depletion of electrons within the plasma and thereby an increasing resulting charge-density of positive charges.

The force-interaction between this positive charged plasma and the extracted ions directly generates the repulsion: The ion is accelerated outward the thruster; hence the whole thruster with the same force into opposite direction.

The force results from the electric field produced by the plasma E_{total} and the number of extracted ions N_i.

$$F = N_i e E_{total} \tag{56}$$

In the moment of force-interaction the ion is within or very close to the field-generating charge-density. Hence the distance is very small compared to surface of charge-density. With this the formalism of a field E caused by an infinitely extended plate with charge-density σ can be used.

$$E = \frac{\sigma}{2\varepsilon_0} \tag{57}$$

The resulting charge-density σ_{pl} is the difference of the charge-densities of electrons σ_e and ions σ_i.

$$\sigma_{pl} = \sigma_i - \sigma_e \tag{58}$$

The particular charge-densities are determined by the balance between the rate of creation $\dot{\sigma}_{cre}$ and the rate of outflow $\dot{\sigma}_{out}$ of the particular particle species.

$$\sigma_{i/e} = \left(\dot{\sigma}_{cre,i/e} - \dot{\sigma}_{out,i/e}\right)\Delta t \tag{59}$$

A symmetric and in the region of electric field penetration complete ionization was assumed: The particle creation of ions and electrons is equal.

$$\dot{\sigma}_{cre,i} = \dot{\sigma}_{cre,e} \tag{60}$$

The rate of outflow is calculated from the density of particles and the velocity of the particular particle species.

$$\dot{\sigma} = env \tag{61}$$

$$n_i = n_e = n_{Xe} \tag{62}$$

The total velocity of the electrons is composed by the thermal velocity $v_{th,e}$ and the velocity v_{ion} the electrons received in the ionization-system by induction. The velocity of ions is determined by the thermal velocity.

$$\dot{\sigma}_{out,e} = en_{Xe}\left(v_{th,e} + v_{ion}\right) \tag{63}$$

$$\dot{\sigma}_{out,i} = en_{Xe} v_{th,i} \tag{64}$$

$$v_{th,i} = \sqrt{\frac{3k_B T}{m_{Xe}}}, v_{th,e} = \sqrt{\frac{3k_B T}{m_e}}, v_{ion} = \sqrt{\frac{2U_{ion,Xe}}{m_e}} \tag{65}$$

(59) → (58): $\sigma_{pl} = (\dot{\sigma}_{cre,i} - \dot{\sigma}_{out,i} - \dot{\sigma}_{cre,e} + \dot{\sigma}_{out,e})\Delta t$ (66)

(60) → $\sigma_{pl} = \left(\dot{\sigma}_{out,e} - \dot{\sigma}_{out,i}\right)\Delta t$ (67)

(63), (64) → (67): $\sigma_{pl} = en_{Xe}\left(v_{th,e} + v_{ion} - v_{th,i}\right)\Delta t$ (68)

(65) → (68): $\sigma_{pl} = en_{Xe}\left(\sqrt{\frac{3k_B T}{m_e}} + \sqrt{\frac{2U_{ion,Xe}}{m_e}} - \sqrt{\frac{3k_B T}{m_{Xe}}}\right)\Delta t$ (69)

(69) → (57): $E_{pl} = \frac{en_{Xe}}{2\varepsilon_0}\left(\sqrt{\frac{3k_BT}{m_e}} + \sqrt{\frac{2U_{ion,Xe}}{m_e}} - \sqrt{\frac{3k_BT}{m_{Xe}}}\right)\Delta t$ (70)

E_{pl} is the equations in case of a strict separation of field-source and influenced charged particles. But this scenario is not realistic since the out-flowing charged particles are the cause for the electric field it selves; on the one hand side. And on the other hand the generated field influences the flow behavior of the particles what again changes the strength of the generated field; and so on.. This scenario forms a not-linear problem; an effect that changes its own cause.

This includes two different mechanisms that have to be taken into account. First, the electric field of the plasma decelerates the out-flowing electrons. Second, the thrust-generation itself; the extracted ions deplete the resulting charge-density that produces the electric field. Without a non-linear treatment it can be assumed that these mechanisms deliver two additional factors E and F influencing the velocities of charged particles producing to electric field. With a view on the boundary conditions of these factors a mathematical expression of the factors can be found.

The out-flowing negative charges leave behind a distribution of positive charges. If negative and positive charges would have the same weight, and since the positive and negative have the same charge, the impulse transferred by the generated field would be shared symmetrically what means a factor of deceleration of out-flowing particles of E=1/2. And if the mass of the positive charges would be infinite, the out-flowing negative charges completely lose their kinetic energy; what means E=0.

$$m_i = m_e \Rightarrow E = \frac{1}{2}\ ,\ m_i \to \infty \Rightarrow E = 0\ :\ E = \frac{m_e}{m_e + m_i} \qquad (70a)$$

The left-behind positive charges under the influence of the generated field try to follow the negative charges. If the mass of positive and negative charges is identical as well as the charge-value the transferred impulse would be separated symmetrically. What means a reduction with the factor F=1/2 of the velocity of the resulting out-flowing charges. If the mass of the positive charges is infinite, there occurs no out-flowing of positive charges and the field-generating charge density remains uninfluenced; this means F=1.

$$m_i = m_e \Rightarrow F = \frac{1}{2}\ ,\ m_i \to \infty \Rightarrow F = 1\ :\ F = \frac{m_i}{m_e + m_i} \qquad (70b)$$

The new expression for the electric field with inclusion of the two mechanisms is $E^*_{pl.}$

$$E^*_{pl} = \frac{en_{Xe}}{2\varepsilon_0} EF\left(\sqrt{\frac{3k_BT}{m_e}} + \sqrt{\frac{2U_{ion,Xe}}{m_e}} - \sqrt{\frac{3k_BT}{m_{Xe}}}\right)\Delta t \qquad (70c)$$

(70a), (70b)→(70c): $E^*_{pl} = \frac{en_{Xe}}{2\varepsilon_0}\frac{m_e}{m_e + m_i}\frac{m_i}{m_e + m_i}\left(\sqrt{\frac{3k_BT}{m_e}} + \sqrt{\frac{2U_{ion,Xe}}{m_e}} - \sqrt{\frac{3k_BT}{m_{Xe}}}\right)\Delta t$ (70d)

Thus for the thrust F of the plasma engine means this:

$$F = N_i e E_{pl}^* \tag{71}$$

The penetration depth δ of the induction field forms a hollow cylindrical-shaped region where the fuel-gas is completely ionized. Out of the head-surface A_δ of this region the thrust-generating ions are extracted. V_2 and A_2 are volume respectively cross-section area of the main-stage.

$$N_i = \frac{n_{Xe} V_2}{A_2} A_\delta = n_{Xe} l_2 \frac{\pi}{4}\left(d_2^2 - (d_2 - 2\delta)^2\right) \tag{72}$$

(70d), (72) → (71):

$$F = \frac{\pi e^2}{8\varepsilon_0} n_{Xe}^2 l_2 \left(d_2^2 - (d_2 - 2\delta)^2\right) \frac{m_e}{m_e + m_i} \frac{m_i}{m_e + m_i} \left(\sqrt{\frac{3k_B T}{m_e}} + \sqrt{\frac{2U_{ion,Xe}}{10 m_e}} - \sqrt{\frac{3k_B T}{m_{Xe}}} \right) \Delta t \tag{73}$$

Results

With the pre-defined parameters the mathematical model above delivers the following values:

Ionization system

Penetration depth: (25) → $\boxed{\delta = 9{,}097 \cdot 10^{-4} m}$

Power consumption: (47a) → $\boxed{P_{ion} = 3{,}5133 \cdot 10^4 W}$

Extraction system

Magnetic field: (51) → $\boxed{B = 6{,}4962 \cdot 10^{-3} T}$

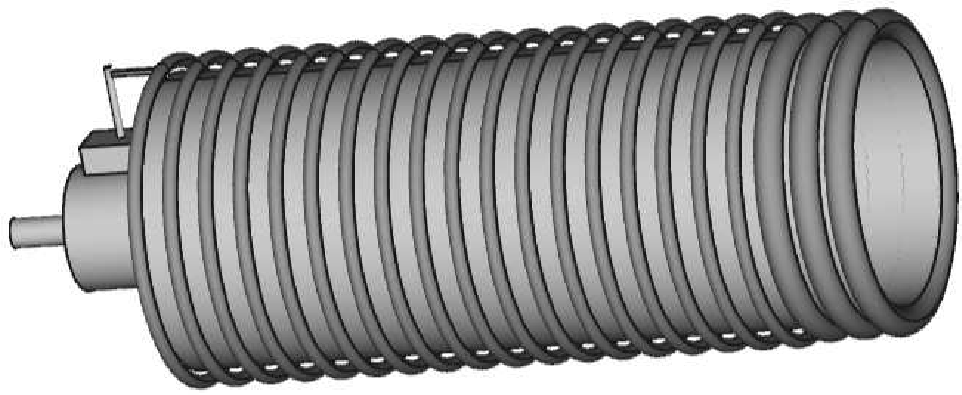
Power consumption: (55) → $\boxed{P_{mag} = 36{,}44 W}$

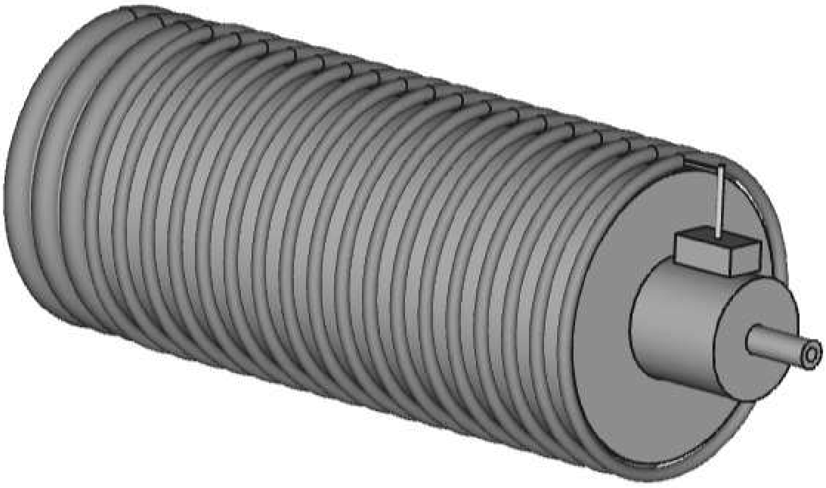
Thrust

The underlying mechanism for thrust generation is the formation of charge-distributions by flowing of charges with different rates. This is a process that evolutes in time. To keep errors as small as possible the smallest time-frame for evolution of the charge density was assumed; the half period of the induction field. $\Delta t = 1 * 10^{-6}$s

Thrust-force: (73) → $\boxed{F = 187{,}07 N}$

Three-dimensional impression of Helios-1





[1] D. Kirmse: “Two concepts of high-thrust plasma-propulsion engines for use as main-thruster units for interplanetary exploration” 2014

[2] D. Kirmse: “On the interaction of coil and plasma at a Radio-Frequency Ion-Thruster (RIT)” 2014