# Note on a Clifford Algebra Based Grand Unification Program of Gravity and the Standard Model

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**Abstract**

Further evidence is provided why a $Cl(5, C)$ gauge field theory in four dimensions furnishes the *simplest* Grand Unification model of Gravity and the Standard Model. In essence we have four copies of $Cl(4, R)$, one copy per each axis-direction in our observed $D = 4$-dim spacetime.

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A $Cl(5, C)$ unified gauge field theory formulation of conformal Gravity and $U(4) \times U(4) \times U(4)$ Yang-Mills in $4D$, and its implications for the Pati-Salam group $SU(4) \times SU(2)_L \times SU(2)_R$, Trinification GUT models of 3 fermion generations (based on the group $SU(3)_C \times SU(3)_L \times SU(3)_R$) and the Standard Model group $SU(3) \times SU(2) \times U(1)$ was analyzed in detail in [1], [2]. In this short note we provide further evidence why this unification model of gravity and the Standard Model in four dimensions, that is based on the complexified Clifford algebra $Cl(5, C)$, has precisely the right number of degrees of freedom [1] to accommodate the vielbein field $e^a_\mu$ of gravity, 12 gauge bosons and 48 fermions of the Standard Model. Clifford algebras are very natural to use because spinors are just the left/right ideal elements of the Clifford algebra [3].

Let us count the number of components of all the relevant fields in $D = 4$. There is the vielbein frame field $e^a_\mu$ of gravity, the internal index $a$ ranges from $1, 2, 3, 4$. The spacetime index $\mu$ ranges from $1, 2, 3, 4$, therefore there are a total of $4 \times 4 = 16$ entries in $e^a_\mu$. We do not include the spin connection $\omega^{ab}_\mu$ because in Riemanian geometry the spin connection is *not* an independent field; i.e it is a function of the vielbein.

[1] Not to be confused with the actual *physical* degrees of freedom

There are 8 gluons in $SU(3)$, and $3+1$ Electro-Weak bosons in $SU(2) \times U(1)$ giving a total of 12 gauge vector bosons, so the number of components in $D=4$ is $4 \times 12 = 48$.

There are 3 generations of fermions (observed so far) comprised of 16 chiral fermions in each family, with 8 fermions and their 8 anti-particles, where the neutrino is taken to be massive (consistent with the experimental evidence) . For example, the electron $e^-$, the electron neutrino $\nu_e$, the up (red, blue, green) quark, the down (red, blue, green) quark yield 8 fermions. Their anti particles furnish another 8 fermions adding up to 16 fermions for the electron family. Similar conclusions hold for the muon and tau family. Hence, we have then $3 \times 16 = 48$ fermions. The number of (real) components of a Weyl fermion in $D = 4$ is $four$, so the number of real components associated with the fermions of the Standard Model is $3 \times 16 \times 4 = 192$. The fact that 192 is also equal to $3 \times 8 \times 8$ might bear some connection to Octonions and the Triality property of the vector and spinorial representations of $SO(8)$ [6], [5], [2].

Adding all these numbers for bosons and fermions yields $16+48+192 = 256$ for the net number of degrees of freedom associated with the vielbein of gravity, 12 gauge bosons and 48 fermions of the Standard Model in four dimensions. Interestingly, this number of 256 components matches *precisely* the number of components of :

• **1** : A Clifford-valued gauge field $\mathbf{A}_\mu = A^C_\mu \Gamma_C$ in $D = 4$ and based on the complexified Clifford algebra $Cl(5, C)$ which is $2 \times 2^5 = 64$ dimensional. The generators $\Gamma_C$ are comprised of the unit element $\mathbf{1}$ of the Clifford algebra, the vector generator $\Gamma_i$, bivector $\Gamma_i \wedge \Gamma_j$, trivector $\Gamma_i \wedge \Gamma_j \wedge \Gamma_k, \ldots$ leading to $2^5$ generators. The complexified Clifford algebra provides an additional factor of two. To sum up, in $D = 4$ one has then $4 \times 64 = 256$ components encoded in the Clifford-valued gauge field $\mathbf{A}_\mu = A^C_\mu \Gamma_C$ after one accounts for the four degrees of freedom associated with the spacetime vector index of $\mathbf{A}_\mu$. This fact corroborates that our model [1] based on the group $Cl(5, C) \sim Cl(4, R) \times Cl(4, R) \times Cl(4, R) \times Cl(4, R)$ is on the right track. In essence we have four copies of $Cl(4, R)$, one copy per each axis-direction in our observed $D = 4$-dim spacetime,

• **2** A Clifford-valued scalar field $\mathbf{\Phi} = \Phi^C \Gamma_C$ based on the Clifford algebra $Cl(8, R)$ which is $2^8 = 256$ dimensional. A $Cl(8)$ algebraic approach to unification in $D = 8$ has been advanced by Smith [5]. Its relation to $E_8$ has also been studied by Smith [5] and analyzed and reviewed in detail by us in [2].

In [1], [2] we discussed how the Higgs scalars, like a complex $SU(2)$ doublet, can be incorporated within a Clifford gauge theory formulated in Clifford Spaces ($C$-spaces). For this reason we shall not repeat it here. This numerical coincidence (or design) found in this short note deserves further investigation. There might be a plausible connection to the Noncommutative Geometric approach to grand unification with gravity via the Spectral Action principle involving the Dirac operator [4].

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