General relativity and the other gravity field equation and Solution

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ABSTRACT

In the general relativity theory, we find the other gravity field equation and the other solution. For example, the other solution treats in Schwarzschild solution, Reissner-Nodstrom solution. Hence, the uniqueness of GR solution is denied by numberless solutions.

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1. Introduction

In the general relativity theory, our article's aim is that we find the other gravity field equation and the other solution.

First, the gravity potential $g_{\mu\upsilon}$ is

$$ds^2 = g_{\mu\upsilon} dx^{\mu} dx^{\upsilon} \tag{1}$$

In gravity potential $g_{\mu\upsilon}$, we introduce tensor $f_{\mu\upsilon}$ and scalar K.

$$f_{\mu\upsilon} = K g_{\mu\upsilon} \quad , \quad \frac{\partial K}{\partial x^{\lambda}} = 0$$

$$ds'^2 = f_{\mu\upsilon} dx^{\mu} dx^{\upsilon} = K g_{\mu\upsilon} \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial x^{\upsilon}}{\partial x'^{\beta}} dx'^{\alpha} dx'^{\beta}$$

$$= K g'_{\alpha\beta} dx'^{\alpha} dx'^{\beta} = f'_{\alpha\beta} dx'^{\alpha} dx'^{\beta}$$

$$g'_{\alpha\beta} = g_{\mu\upsilon} \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial x^{\upsilon}}{\partial x'^{\beta}} \quad , \quad f'_{\alpha\beta} = f_{\mu\upsilon} \frac{\partial x^{\mu}}{\partial x'^{\alpha}} \frac{\partial x^{\upsilon}}{\partial x'^{\beta}} \tag{2}$$

Inverse gravity potential $g^{\mu\upsilon}$,

$$f^{\mu\upsilon} f_{\mu\upsilon} = \delta_{\mu}^{\ \upsilon} = (\frac{1}{K} g^{\mu\upsilon})(K g_{\mu\upsilon}), \ f^{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} \tag{3}$$

In Christoffel symbol $\Gamma^{\rho}{}_{\mu\upsilon}$,

$$\Gamma'^{\rho}{}_{\mu\upsilon} = \frac{1}{2} f^{\rho\lambda} (\frac{\partial f_{\mu\lambda}}{\partial x^{\upsilon}} + \frac{\partial f_{\upsilon\lambda}}{\partial x^{\mu}} - \frac{\partial f_{\mu\upsilon}}{\partial x^{\lambda}})$$

$$= \frac{1}{2} (\frac{1}{K} g^{\rho\lambda})(K \frac{\partial g_{\mu\lambda}}{\partial x^{\upsilon}} + K \frac{\partial g_{\upsilon\lambda}}{\partial x^{\mu}} - K \frac{\partial g_{\mu\upsilon}}{\partial x^{\lambda}}) = \Gamma^{\rho}{}_{\mu\upsilon} \tag{4}$$

Therefore, in the curvature tensor $R^{\rho}{}_{\mu\upsilon\lambda}$,

$$R'^{\rho}{}_{\mu\upsilon\lambda} = \frac{\partial \Gamma'^{\rho}{}_{\mu\upsilon}}{\partial x^{\lambda}} - \frac{\partial \Gamma'^{\rho}{}_{\mu\lambda}}{\partial x^{\upsilon}} + \Gamma'^{\sigma}{}_{\mu\upsilon} \Gamma'^{\rho}{}_{\sigma\lambda} - \Gamma'^{\sigma}{}_{\mu\lambda} \Gamma'^{\rho}{}_{\sigma\upsilon}$$

$$= \frac{\partial \Gamma^{\rho}{}_{\mu\upsilon}}{\partial x^{\lambda}} - \frac{\partial \Gamma^{\rho}{}_{\mu\lambda}}{\partial x^{\upsilon}} + \Gamma^{\sigma}{}_{\mu\upsilon} \Gamma^{\rho}{}_{\sigma\lambda} - \Gamma^{\sigma}{}_{\mu\lambda} \Gamma^{\rho}{}_{\sigma\upsilon} = R^{\rho}{}_{\mu\upsilon\lambda} \tag{5}$$

In Ricci tensor $R_{\mu\upsilon}$,

$$R'_{\mu\upsilon} = R'^{\rho}{}_{\mu\rho\upsilon} = R^{\rho}{}_{\mu\rho\upsilon} = R_{\mu\upsilon} \tag{6}$$

In curvature scalar R

$$R' = f^{\mu\upsilon} R'_{\mu\upsilon} = \frac{1}{K} g^{\mu\upsilon} R_{\mu\upsilon} = \frac{1}{K} R \qquad (7)$$

Hence, in the gravity field equation of Einstein,

$$R'_{\mu\upsilon} - \frac{1}{2} f_{\mu\upsilon} R' = R_{\mu\upsilon} - \frac{1}{2} K g_{\mu\upsilon} (\frac{1}{K} R)$$

$$= R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = -\frac{8\pi G}{c^4} T_{\mu\upsilon} \qquad (8)$$

In Newtonian approximation, Energy-momentum tensor $T'_{\mu\upsilon}$ is

$$\nabla^2 f_{00} = \nabla^2 K g_{00} \approx -\frac{8\pi G}{c^4} K T_{00} = -\frac{8\pi G}{c^4} T'_{00} \qquad (9)$$

$$\rho c^2 = T_{00}, \quad K \rho c^2 = T'_{00}$$

$$T'_{\mu\upsilon} = K T_{\mu\upsilon} \qquad (10)$$

Einstein's gravity field equation is

$$R'_{\mu\upsilon} - \frac{1}{2} f_{\mu\upsilon} R' = R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = -\frac{8\pi G}{c^4} T_{\mu\upsilon} = -\frac{8\pi G}{c^4} \frac{T'_{\mu\upsilon}}{K} \qquad (11)$$

Therefore, tensor $f_{\mu\upsilon}$ satisfy the gravity field equation of Einstein.

$$f^{\mu\upsilon}[R'_{\mu\upsilon} - \frac{1}{2} f_{\mu\upsilon} R'] = \frac{1}{K} g^{\mu\upsilon}[R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R] = -\frac{8\pi G}{c^4} \frac{1}{K} g^{\mu\upsilon} T_{\mu\upsilon} = -\frac{8\pi G}{c^4} \frac{1}{K} T^{\lambda}{}_{\lambda}$$

$$= -\frac{8\pi G}{c^4} f^{\mu\upsilon} \frac{T'_{\mu\upsilon}}{K} = -\frac{8\pi G}{c^4} \frac{1}{K} T'^{\lambda}{}_{\lambda}$$

$$\rightarrow \; -R' = -\frac{1}{K} R = -\frac{8\pi G}{c^4} \frac{1}{K} T^{\lambda}{}_{\lambda} = -\frac{8\pi G}{c^4} \frac{1}{K} T'^{\lambda}{}_{\lambda} \;,$$

$$T'^{\lambda}{}_{\lambda} = T^{\lambda}{}_{\lambda}$$

$$R'_{\mu\upsilon} = R_{\mu\upsilon} = -\frac{8\pi G}{c^4} (T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) = -\frac{8\pi G}{c^4} (\frac{T'_{\mu\upsilon}}{K} - \frac{1}{2} \frac{f_{\mu\upsilon}}{K} T'^{\lambda}{}_{\lambda}) \qquad (12)$$

2. The other solution in Schwarzschild solution, Reissner-Nodstrom solution

$$f_{\mu\upsilon} = K g_{\mu\upsilon} \;, \quad \frac{\partial K}{\partial x^{\lambda}} = 0$$

$$ds^2 = g_{\mu\upsilon} dx^{\mu} dx^{\upsilon}, \; ds'^2 = f_{\mu\upsilon} dx^{\mu} dx^{\upsilon} \qquad (13)$$

For example, K is

$$K = 1 + n_2 \exp(-n_1 \frac{hc}{GM^2}) \text{ or } K = [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})] \cdot [1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})]$$

$n_1,\ m_1 > 0,\ n_1,\ n_2, m_1, m_2$ is number (14)

Schwarzschild solution(vaccum solution) is

$$ds^2 = -c^2(1 - \frac{2GM}{rc^2})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad (15)$$

The new solution is

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = K g_{\mu\upsilon} dx^\mu dx^\upsilon = K \cdot ds^2$$

$$= [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})]\,[-c^2(1 - \frac{2GM}{rc^2})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2]$$

$$K = 1 + n_2 \exp(-n_1 \frac{hc}{GM^2}) \qquad (16)$$

Reissner-Nodstrom solution is

$$ds^2 = g_{\mu\upsilon} dx^\mu dx^\upsilon$$

$$= -c^2(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \qquad (17)$$

The new solution is

$$ds'^2 = f_{\mu\upsilon} dx^\mu dx^\upsilon = K g_{\mu\upsilon} dx^\mu dx^\upsilon = K \cdot ds^2$$

$$= [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})] \cdot [1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})] \cdot [-c^2(1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4})dt^2 + \frac{dr^2}{1 - \frac{2GM}{rc^2} + \frac{kGQ^2}{r^2c^4}}$$

$$+ r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad , \quad K = [1 + n_2 \exp(-n_1 \frac{hc}{GM^2})] \cdot [1 + m_2 \exp(-m_1 \frac{hc}{kQ^2})]$$

(18)

3. Conclusion

We find the other solution in the General relativity theory. Hence, the uniqueness of GR solution is denied by numberless solutions.

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