Non-Zero OEN Ternary Number System

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Abstract:
THIS PAPER DISCUSSES THE ORIGINS, ARITHMETIC OPERATIONS AND CONVERSION OPERATIONS INVOLVING A NON-ZERO TERNARY NUMBER SYSTEM, COMPRISING OF THE NUMBERS 1, 8 AND 9. THIS NUMBER SYSTEM, CHRISTENED THE "OEN NUMBER SYSTEM", PRESENTS ITSELF AS A STRONG CONTENDER FOR CODING AND ENCRYPTING APPLICATIONS DUE TO THE ABSENCE OF ZERO, WHICH MAKES IT IMPOSSIBLE TO USE CONVENTIONAL CODING/ ENCRYPTING ALGORITHMS, THUS PROVIDING A HIGHER DEGREE OF SECURITY.

Introduction:
A 'number system' is a set of numbers, (in the broadest sense of the word), together with one or more operations, such as addition or multiplication. There are innumerous number systems in existence, with a lot more combinations possible. To completely define a number system, the essential components are: the degree/radix, the constituent "numbers", procedures for various mathematical systems such as addition/subtraction, multiplication/division, and conversion to and from other number systems. There are a lot of popular number systems like binary, hexadecimal, octal, and so on. Usually the number systems can be grouped based on the radix, such as unary, binary, ternary, quaternary and so on.

Origins of the number system:
This idea of a number system struck incidentally on August 2003, during a few ramblings, which are narrated as follows:
$17^3 = 4913$, and the digital root (digit sum) of 4913 is 4+9+1+3 = 17. Hence proceeding with other cubes and squares, a table was formed, connecting various numbers and the digit sums of their squares and cubes.

Number	Square	Cube	Square digit sum	Cube digit sum
1	1	1	1	1
2	4	8	4	8
3	9	27	9	9
4	16	64	7	1
5	25	125	7	8
6	36	216	9	9
7	49	343	4	1
8	64	512	1	8
9	81	729	9	9
10	100	1000	1	1

As is evident from the table, the digit sums of the squares are irregular, whereas those of the cubes form a regular repeating pattern of 1,8,9,1,8,9… So, a number system involving the numbers 1,8 and 9 was conceived, such that the units digits of all these numbers in ascending order would form the 1,8,9,1,8,9.. Sequence. Since the number system involved 1,8 and 9, this was called the OEN Number system, O,E and N standing for One, Eight and Nine, respectively. A few samples of the OEN Number system, with their decimal equivalents are as follows:

Decimal	OEN
1	1
2	8
3	9
4	11
5	18

6	19
7	81
8	88
9	89
10	91
11	98
12	99
13	111
14	118
15	119

The Proposed Number System:

The characteristics of the proposed number system, the OEN Number system are as follows:

1. The radix is 3.
2. The constituents are 1,8 and 9.

Mathematical Operations on the OEN Number System:

The method of performing addition/ subtraction/ multiplication/ division is the same as that for any number system. A "look up table" consisting of basic operations for 1 digit numbers are given below:

ADDITION	1+1=8	SUBTRACTION	8-1=1
	1+8=9		9-8=1
	1+9=11		11-9=1
	8+8=11		11-8=8
	8+9=18		18-9=8
	9+9=19		19-9=9
MULTIPLICATION	1*1=1	DIVISION	1/1=1
	8*8=11		11/8=8
	9*9=89		89/9=9
	1*8=8		8/8=1
	1*9=9		9/9=1
	8*9=19		19/9=8

Using the lookup table given above, the arithmetic operations can be performed. But, the OEN Number system is a NON-ZERO number system. Hence certain operations such as borrowing follow a different procedure. These are given in the following section.

SPECIAL RULES FOR ARITHMETIC OPERATIONS:

BORROWING:

While borrowing from a number, the next significant digit should be reduced by 1, and '9' is added to the digit that requires borrowing. The reason is that in decimal system usually '10' is added to the number to be borrowed. Here, as there is no '10' possible in OEN, 9 is used.

ZERO DIFFERENCE:

If on subtracting 2 digits, the result is '0', borrowing should be used as '0' is not possible in OEN number system. Hence, subtracting any number by itself is not allowed in this non-zero system.

ZERO REMAINDER:

If in the middle of a division process, on dividing one number by a divisor, '0' is obtained as the remainder, the division should be employed with a lower divisor and the non zero remainder used.

These rules are substantiated with illustrations in the following sections.

CONVERSION RULES:

DECIMAL TO OEN:

Divide the given number by 3. Find the OEN value of the quotient and remainder and let these values be ‘a’ and ‘b’ respectively. Now, ‘a’ must be concatenated with ‘b’ to form the OEN number ‘ab’. If the remainder happens to be ‘0’, reduce 1 from a and let this value be ‘a-1’. Now, concatenate with ‘9’ to form the OEN value, ‘a-1 9’.

OEN TO DECIMAL:

Split the OEN number ‘ab’ into 2 parts such that the units place becomes a new remainder ‘b’ and the rest of the number becomes ‘a’. Find the decimal equivalent of ‘a’ and ‘b’. Now let the decimal equivalent of ‘ab’ be ‘d’. d = a*3 + b.

ILLUSTRATIONS:

CONVERSIONS:

1. $(67)_{10}$ = 8111OEN. 67/3 = 22 and 1 remainder. Value of 22 is 811 (can be found by recursive division). Therefore, value of 67 is ‘811 1’, that is 8111OEN.
2. $(99)_{10}$ = 9189OEN. 99/3 = 33 and 0 remainder. But, according to above mentioned conversion rule, 32 should be considered as quotient. OEN value of 32 = 918. On concatenating with 9, we obtain 9189.
3. 8988OEN = $(89)_{10}$. Splitting 8988 into ‘898’ and ‘8’. Decimal values are 29 and 2 respectively. Now according to the rule, d = 29*3+2 = 89.

ARITHMETIC OPERATIONS:

1. 891+119 = 1181. (9+1 = 11 – 1 carry, 9+1+1 = 18, 1 carry 8+1+1 = 11).
2. 899-111 = 188. (9-1 =8 and 8-1 =1).
3. 891–119 = 111. (1-9 is not possible. Use **borrowing** rule. Therefore, the 9 in tens place becomes 8 and 1 of units place is 1+9=11. 11-9=1 and 8-1=1).
4. 81889-9889 = 9889. (9-9=0 not possible. Using **zero difference** rule, borrow 1 from 8 and hence 8 becomes 1, and 9 becomes 9+9=19. Now 19 – 9 = 9. Using laws of subtraction and borrowing rule, we proceed with other digits and obtain 9889.
5. 9999*88 = 988119. (This is done using the lookup table and carry operations).
6. 9999/11 = 899. (11*9=99 for the first digit of quotient. But remainder will be 0. According to **zero remainder** rule, we need to use 11***8**=88. Remainder now being 11.Next 11***9**=99. 119-99=9. 11***9**=99. Hence the quotient is 899).

ADVANTAGES AND APPLICATIONS:

1. Since OEN Number system is ternary (radix 3), it is easy to implement.
2. Usually algorithms can be built to encode and decode any number system, and a single algorithm can be used for all umber systems with the same radix. But, OEN is a non zero number system, where ‘0’ has to be replaced with an appropriate place value. Hence the conventional encode/decode algorithms cannot be used for OEN Number systems. This provides a higher level of privacy/security for OEN Number systems.
3. Even if other non-zero ternary number systems exist, the algorithms used for those cannot be used for OEN Number system as the place value of ‘0’ will change.
4. Due to these properties, OEN Number systems can be used for Secret and Secure Communications such as those used in the defense services.
5. OEN Number system can also be used for encryption/decryption, password services etc..

CONCLUSION:

Thus a non-zero ternary system, the “OEN” number system was proposed and the operations and conversion procedures were listed out. This number system stands as a strong contender in various coding and encrypting applications.