# Generation of Chaos in the Microwave X-Band

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## Abstract:
The present work pertains to the exploration of signal based chaos generation in the Microwave X-Band frequency range. Specifically, two reflex klystrons are used as sources to two ports of the magic tee, while a third port is terminated. By taking the output from the fourth port, the envelope waveforms and phase portraits are seen to exhibit reasonable degree of richness and complexity. The presence of chaos is quantitatively ascertained using Lyapunov Exponents and Kolmogorov Entropies. The results outlined in the present work offer a far simpler alternative to generate easy-to-tune signal dependent chaos, suitable for telecommunication applications.

**Keywords**: Chaos Generation, Microwave, Signal based Chaos, Reflex Klystron, Magic Tee

## 1. Introduction
With characteristic signatures of determinism and an extremely sensitive dependence on initial conditions, chaos theory has emerged as one of the standing highlights in twentieth century science, with applications in both physical and life sciences [1-4]. Owing to the ‘practical randomness’ and unpredictability of chaos, it has also been used in secure communications, with chaos typically generated using op-amp based Chua circuits, which essentially pertains to system-dependent chaos [1].

In the present work, signal based chaos generation, where the control parameters are signal parameters such as amplitude and frequencies are explored in the X-Band. Specifically, two klystron based sources are connected to two of the four ports of a magic-tee [5]. The output is taken from the third port while a matched termination is attached to the fourth. The presence of chaos in the envelope of the generated signal is verified using standard parameters such as Kolmogorov Entropy and Lyapunov Exponent. In summary, the results outlined in the present work offer a far simpler alternative to generate easy-to-tune signal dependent chaos, suitable for telecommunication applications.

## 2. Design and Results
In the present work, reflex klystrons in the AM mode are used as X-Band (8GHz-12GHz) microwave signal sources. A reflex klystron is essentially a single cavity klystron based oscillation device, where microwave oscillations are produced by virtue of converting the velocity modulation of electrons to current modulation in the repeller space with a typical X-band (8-12GHz) signal of around 3W power [5].

The other key component in the present design is the magic-tee, which is a four port passive microwave component composed of a hybrid of the E-plane and H-plane tees. Thus, two signals applied to ports 1 and 2 of the magic tee result in a ‘sum’ signal generated in port 3, and a ‘difference’ signal generated in port 4.

The basic premise in the proposed design that, by terminating one of the output ports, and by applying two microwave sources with mismatched amplitudes and frequencies to the two input ports, it is possible to use the difference and sum functions of the magic tee to amplify the mismatches, which act as ‘initial conditions’, giving rise to chaotic output signal in due course. The setup is shown in Fig. 1.

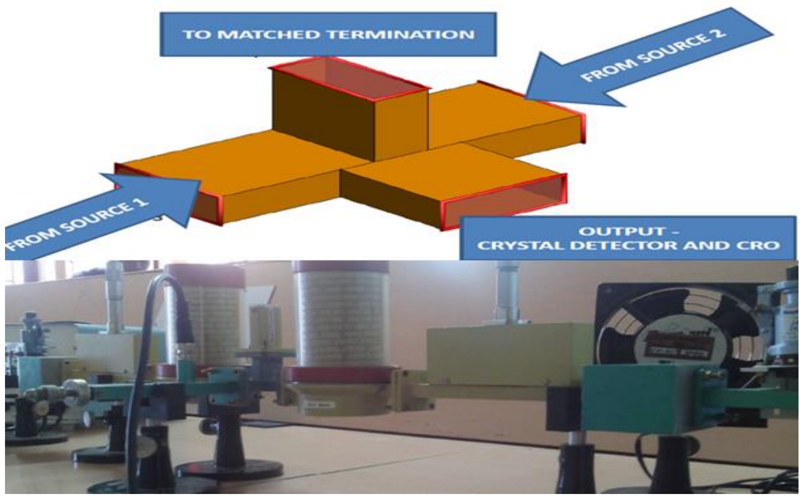


**Figure 1 The Experimental Setup**

The output is fed to a crystal detector, coupled with an oscilloscope, which then detects the envelope of the modulated signal. The experiment is performed for two different frequency combinations (f1=8.5GHz; f2=9.3GHz and f1=8.5GHz; f2=10.7GHz) of the two sources, and the observed waveforms are shown in Fig. 2, along with the corresponding phase portraits in Fig. 3.

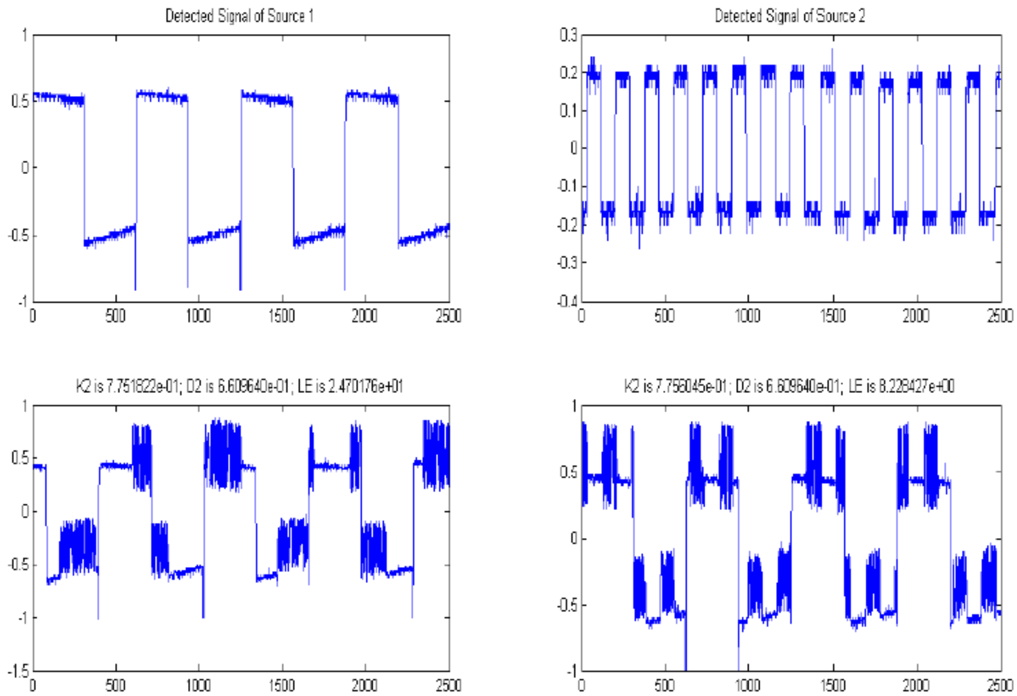


**Figure 2 Modulated Signal envelope waveforms for the two sources (top) and obtained outputs for two combinations of frequencies (bottom)**

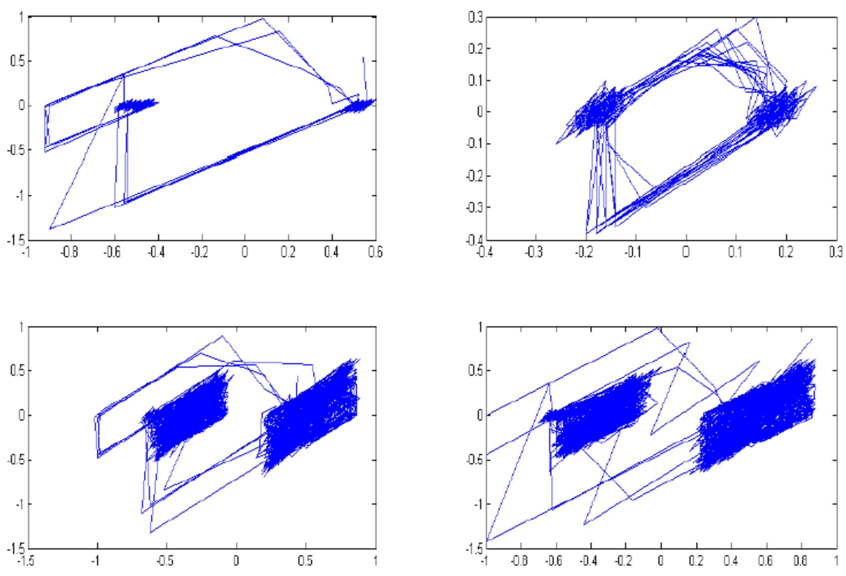

**Figure 2 Phase Portraits for the two sources (top) and obtained outputs for two combinations of frequencies (bottom)**

It is seen from both the waveforms and phase portraits that the two output signals exhibit reasonable amount of complexity and richness, characteristic of the underlying nonlinear processes.

Finally, to ascertain the presence of chaos in the signals, two techniques are used:

1. The most suitable measure to ascertain the presence of and to characterize chaos in the obtained output is the Largest Lyapunov Exponent (LLE), a measure of a system's sensitive dependence on initial conditions [6,7]. In the present work, Rosenstein's algorithm is used to compute the Lyapunov Exponents $\lambda_i$ from the voltage waveform, where the sensitive dependence is characterized by the divergence samples $d_j(i)$ between nearest trajectories represented by i given as follows, $C_j$ being a normalization constant [6]:

$$d_j(i) = C_j e^{\lambda_1 (i\Delta t)}; d(t) = Ce^{\lambda_1 t} \qquad (1)$$

2. The second parameter used to characterize chaos is the Kolmogorov Entropy, which is essentially a statistical measure of the uncertainty of the signal. By assigning each of the two quantifiable states (0 and 1) of the output amplitude as an event *i*, the Kolmogorov Entropy K *o*btained depends on their probabilities $p_i$ [7].

It is seen that the LLE values of the two output cases are obtained as 24.78 and 8.23, the presence of positive values in both cases positively asserting the presence of chaos. The corresponding K values are obtained as 7.751 and 7.756 nats/symbol respectively. Also, it can be concluded that the chaoticity and hence sensitivity is much higher in the first case (f1=8.5GHz; f2=9.3GHz), as seen from the higher LLE value. Finally, it is seen that the generated chaos is signal-based, as seen from the variation of K, LLE and phase portraits caused by change in input frequency.

## 3. Conclusion

In the present work, signal based chaos generation is explored in the Microwave X-Band. Specifically, two reflex klystrons used as signal sources are fed to two ports of a magic tee. By matching one of the output ports and observing the output in the other, waveforms and phase portraits indicate complexity and richness. The presence of chaos is quantitatively ascertained using Lyapunov exponents and Kolmogorov entropies. The results outlined in the present work offer a far simpler alternative to generate easy-to-tune signal dependent chaos, suitable for telecommunication applications.

## References


[1] Strogatz, S. H. (2014). Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. Westview press.

[2] Thompson, J. M. T., and Stewart, H. B. (2002). Nonlinear dynamics and chaos. John Wiley and Sons.

[3] Gilmore, R., and Lefranc, M. (2012). The topology of chaos: Alice in stretch and squeezeland. John Wiley and Sons.

[4] Hilborn, R. C. (2004). Sea gulls, butterflies, and grasshoppers: A brief history of the butterfly effect in nonlinear dynamics. American Journal of Physics, 72(4), 425-427.

[5] Kennedy. G, and Davis. R, Electronic Communication Systems. Tata McGraw Hill, 2003.

[6] James, R. G., Burke, K., and Crutchfield, J. P. (2014). Chaos forgets and remembers: Measuring information creation, destruction, and storage. Physics Letters A, 378(30), 2124-2127.

[7] Rosenstein, M. T., Collins, J. J., and De Luca, C. J. (1993). A practical method for calculating largest Lyapunov exponents from small data sets. Physica D: Nonlinear Phenomena, 65(1), 117-134.