Quantum Computation using Chaotic Circuits

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ABSTRACT:

One of the most successful theories of modern physics, quantum mechanics has undergone rigorous testing and validation, one of the primary motives being fanciful and informative applications of quantum information theory such as teleportation and superdense coding. Over the years, many explanations have been given for the mechanism and interpretation of the various quirks and mysteries of quantum mechanics, the latest entrant being a suggestion of nonlinearity and chaos underlying quantum mechanics. The present work purports to a formulation of quantum bit as a SU(2) Lie Group using a Chua's chaos generator circuit as the basis. PSPICE simulations of the same are performed. Various basic quantum gates such as the Pauli X, Y, Z, Hadamard and CNOT gates are implemented using this formulation. Finally, the chaos-qubit formulation is validated using a real time application – quantum teleportation. The ability to successfully demonstrate the teleportation of a single qubit numerically suggests that the chaotic interpretation of quantum mechanics has some validity. Furthermore, it ushers in the era of low cost, high capacity, high security information systems using nonlinear electrical circuits.

KEYWORDS:

Chaos Theory, Quantum Computation, Chua Circuits, Quantum Gates, Quantum Teleportation, PSPICE.

INTRODUCTION:

At the heart of nearly all of science today is Quantum Physics. Quantum mechanics is a theory so counter-intuitive, yet repeatedly tested and validated for its accuracy and precision [1]. Of the many applications of this branch of science, quantum information and computation stands out [2]. This is essentially due to the fact that quantum information promises fanciful yet incredibly useful applications such as quantum teleportation and superdense coding, which could potentially usher in the era of higher data security, capacity and ultrahigh processing speeds [3-6].

A theory as counterintuitive as quantum mechanics is bound to have its controversies. Most of the controversies surrounding this theory stem from the various interpretations given by Einstein, Bohr, Bohm and the like regarding its various 'quirks' such as wavefunction collapse, quantum entanglement, symmetry breaking and so on [7-9].

One relatively new interpretation of quantum mechanics is from a nonlinear perspective [10-13]. Specifically McHarris in [10] has argued about the possibility of nonlinearity and chaos underlying quantum mechanics. While a full fledged quantitative formulation awaits, the points raised by McHarris et al. seem to be intuitively valid. A brief summary of the key points, comparing chaotic and quantum behaviors raised are as follows:

1. Nonlinear dynamics in its chaotic realm bridges the gap between the statistical nature of quantum mechanics and a more deterministic, more fundamental perspective without having to introduce hidden variables, adding completeness to a theory which was in Einstein's words 'correct yet incomplete' [14, 15].
2. A one to one correspondence between cause and effect arising due to the determinism of chaos satisfies Einstein's perspective of Quantum Mechanics, whereas the extreme sensitivity nature of chaos drastically reducing the certainty of prediction and supplying a practical statistical interpretation satisfies Bohr's perspective of Quantum Mechanics, thus potentially resolving one of the most intense debates on Quantum Mechanics.
3. The classical derivation of Bell's inequality and subsequent violation of the same in the case of entanglement corresponds to an inaccuracy in the formulation [16-18]. Specifically, the subtle assumptions of non-correlation statistics have greatly suppressed the correlations that could occur in real time and raise the level of the inequality [10].
4. The existence of attractors and basins in the dynamics of chaotic systems suggests a tendency of such systems to quantize themselves, without external influence [10, 19].
5. A possibility of a nonlinear basis, motivated by instability, to explain spontaneous symmetry breaking and parity nonconservation [20, 21].
6. The possible equivalence of quantum diffraction and intermittent periods of order in chaos [10].

Chaos theory, as the flagship of Nonlinear Science, has found extensive applications in recent years in fields as diverse as biology, astrophysics, psychology and finance [22-26]. Specifically, in electronics, the Chua circuits are capable of creating extremely intricate chaotic patterns [27-29]. The circuits and the resulting patterns have been extensively used for various pattern formation studies and for secure communications based applications [30, 31].

The present article takes advantage of the extensive knowledge accumulated in the domain of Chua circuits, and proposes a quantum information system where the chaotic signal is postulated as the 'qubit'. The implementation of various quantum gates and quantum teleportation using this formulation are then discussed.

The implementation of a chaos based quantum system yields a number of advantages – firstly, quantum information applications such as quantum teleportation and superdense coding hitherto possible only in expensive superconducting systems or high magnetic field environments can hence be performed with much lesser cost [32-35]. Secondly, it provides an experimental platform to verify most of the postulates of quantum mechanics including McHarris' chaos-quantum equivalency [10]. Thirdly, a lot of unified theories in particle physics such as the one proposed by Lloyd use quantum bits as the basis [36, 37]. A chaos enabled implementation of such 'computational universe' models could potentially answer a lot of unanswered questions in science such as dark matter, black hole singularities and so on [36].

METHODOLOGY:

GENERATION OF CHAOS

The first step in creating a chaos-quantum information equivalency is to develop a chaotic system. A Chua circuit as given in [38] acts as the basis system for generation of chaos. The circuit model is given in Fig.1.

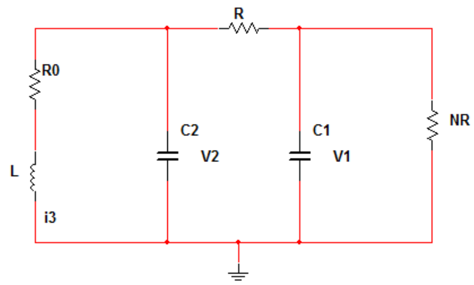

Figure 1 Generalized Chua Circuit

This circuit is a solution to the system of three coupled ordinary differential equations given as follows, with NR being the nonlinear resistance (Chua Diode) [38].

$$\frac{dV1}{dt} = \frac{1}{C1}[(V2 - V1)G - f(V1)],$$

$$\frac{dV2}{dt} = \frac{1}{C2}[(V1 - V2)G + i3],$$

$$\frac{di3}{dt} = -\frac{1}{L}(V2 + R0i3),$$

where $G = \frac{1}{R}$, $f(V) = bV + \frac{1}{2}(a - b)(|V + E| - |V - E|)$ and $E = 1.075V$.

The Chua system is implemented in PSPICE using operational amplifier IC TL082 as mentioned in [38]. It is observed that for the following RLC component values, a pattern similar to one reported as PC85 in [38] is obtained:

$$C1 = -7.686pF; C2 = 9.796nF; G = 1mS; L = -0.735mH; R0 = 2.18\Omega;$$

$$a = 0.169mS; b = -0.477mS$$

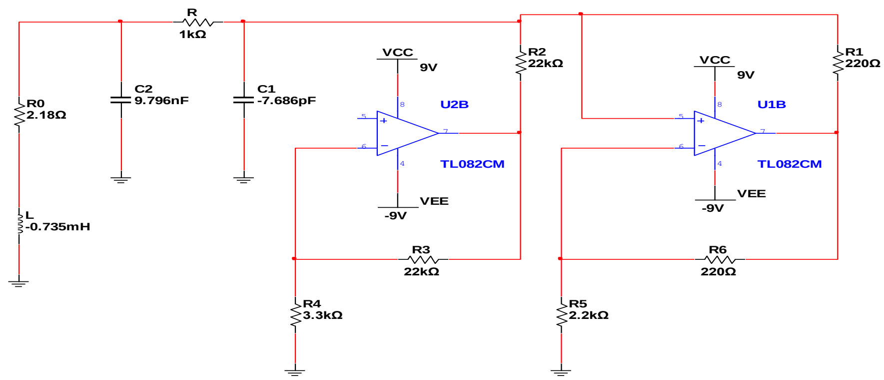

Figure 2 PSPICE Implementation of Chua Circuit using Op Amps

In the present work, we take one of the three outputs (V1, V2 and i3) as the parameter of interest. Specifically, we choose V1, which is plotted in Fig. 3.

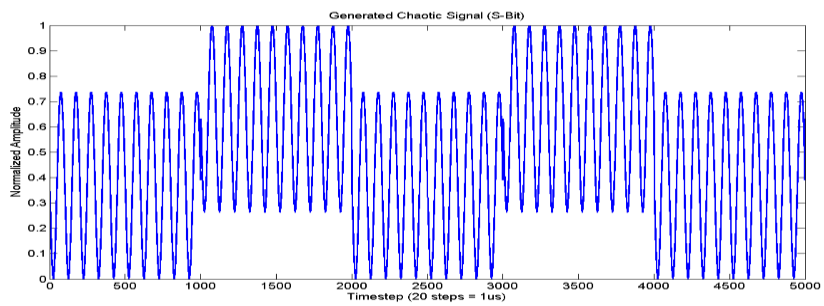

Figure 3 Generated Chaotic Signal V1

The corresponding signal phase portrait is shown in Fig. 4.

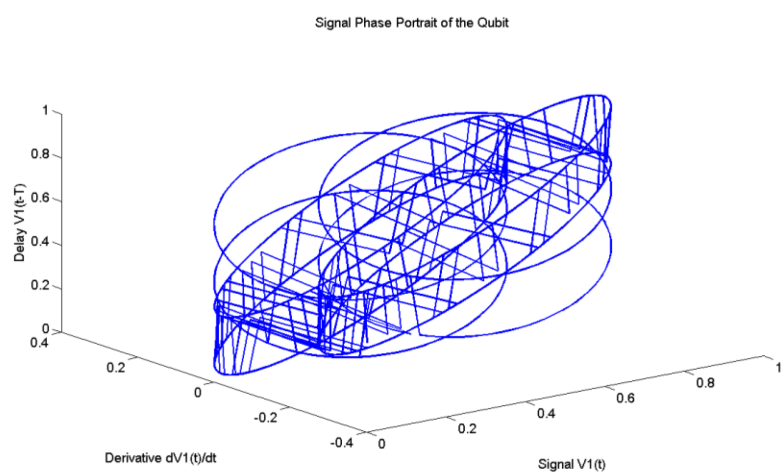

Figure 4 Signal Phase Portrait of Chua Circuit Output V1.

The largest Lyapunov exponent (LE) of the chaotic signal is obtained as 1.295 using Rosenstein's algorithm [39, 40], confirming its chaotic nature.

QUBIT FORMULATION

The next step is to formulate the chaotic signal V1 as a qubit. In order to perform this, we first note the mapping between the Bloch sphere, a standard representation of a qubit, and the SU(2) Lie Group [41, 42]. The Bloch sphere representation of a qubit is given in Fig. 5.

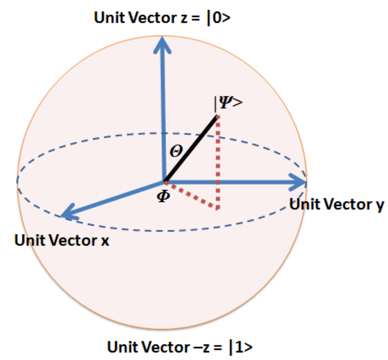

Figure 5 Bloch Sphere Representation of a Qubit

The quantum mechanical operators corresponding to the coordinates of the Bloch Sphere are described as the three 2x2 Pauli spin matrices defined as follows [43]:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \; and \; \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

It is noted that these matrices are precisely the generators of the SU(2) Lie group, and this forms the bottomline of the chaos-qubit formulation in the present work [44].

The 1D chaotic signal V1 is taken as the qubit. The Z operator is given by the flip of the dependant variable, amplitude, whereas the X operator is given by the flip of the independent variable, time.

Level logic is assumed, logical ‘0’ is Ground (0V), and logical ‘1’ is the Supply Voltage Vdd (V). Thus if Vdd is 1V, then we have |0> = 0V, and |1> = 1V. Any voltage level in between 0V and 1V can thus be viewed as a superposition of the two states. For example, 0.7V corresponds to 0.7|1>+0.3|0>.

Thus, the ‘superposed’ state is merely any state lying in between ‘pure’ 1 (0|0>+1|1>) (1V) and ‘pure’ 1 ‘pure’ 0 (1|0>+0|1>) (0V).

One of the significant hallmarks of a quantum bit is the uncertainty concept, which enables the bits to take the superposed states. In other words, during measurement, the superposed state collapses into one of the two ‘pure’ states and until that point in time, the result is uncertain.

This uncertainty can be viewed as the resultant of the complexity of the system. In the present work, the complexity, and hence, the apparent illusion of ‘uncertainty’ is achieved using chaos.

QUANTUM GATES USING CHAOS:

Using the qubit formulation described above, various quantum gates are implemented as follows. It is noteworthy that by fixing the Z and X operations as flipping without and with offset respectively, most of the quantum logic operations can be obtained by a combination of flips and offsets.

For all gate implementations in the present work, the value of |1> is set as 1V and that of |0> is set at 0V. All other states can be found by a simple superposition of these states.

1. The Pauli X gate, denoted by the σ_x matrix exchanges |0> and |1> states. This is implemented mathematically as X = 1-V1 and the waveforms are shown in Fig. 6. This is a 'phase flip' operation.

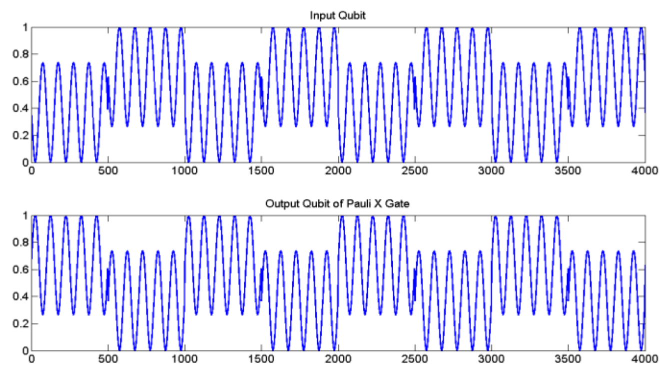

Figure 6 Input and Output of Pauli X Gate

2. The Pauli Z Gate denoted by σ_z transforms |1> to -|1>, leaving |0> unchanged. This is a 'sign flip' operation, represented mathematically as Z=-V1. The waveforms are as shown in Fig. 7.

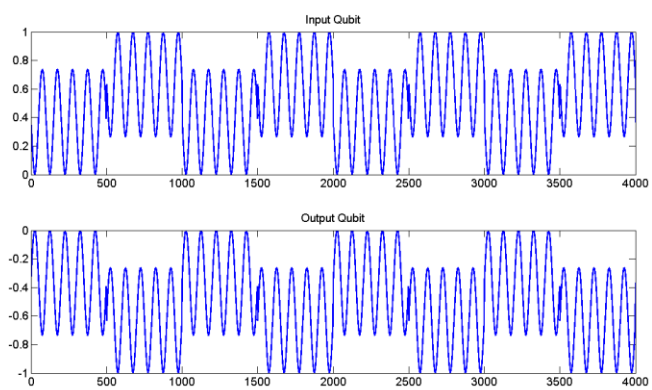

Figure 7 Input and Output of Pauli Z Gate

3. The Pauli Y Gate denoted by the σ_y matrix is a hybrid of the sign and phase flip operations and is given by the cyclic relation $\sigma_z \sigma_x = i\sigma_y$. The waveforms are given in Fig. 8.

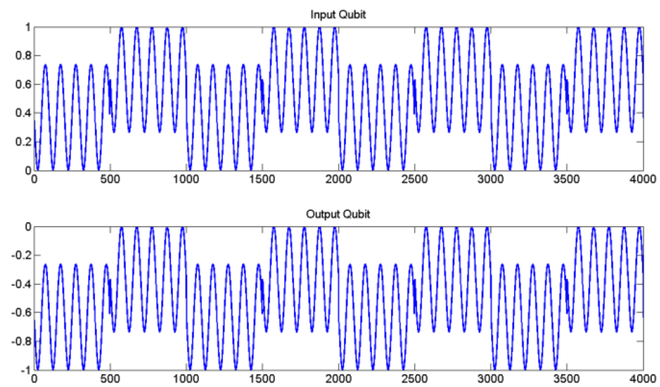

Figure 8 Input and Output of Pauli Y Gate

The Hadamard Gate denoted by the matrix $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ performs the quantum interference operation, transforming pure states to superposed states and vice versa. Mathematically, H = ((1-V1)-0.5)*1.414 and can be physically realized using a combination of X/Z gates and clampers. The waveforms are as shown in Fig. 9.

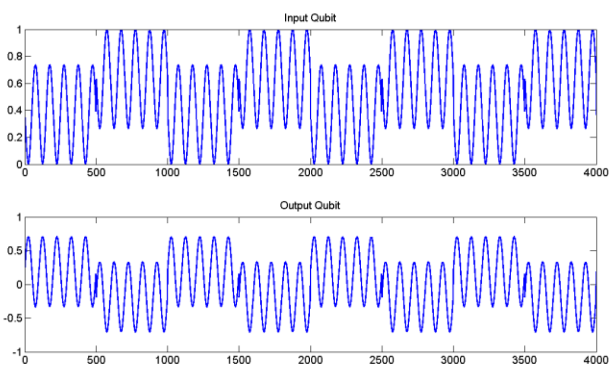

Figure 9 Input and Output of Hadamard Gate

The Controlled NOT Gate or CNOT Gate is the quantum equivalent of an XOR gate and is represented as CNOT(V1a,V1b) = (V1a,V1a-V1b) for 2 qubits V1a and V1b. The matrix and waveforms are as shown in Fig. 10.

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

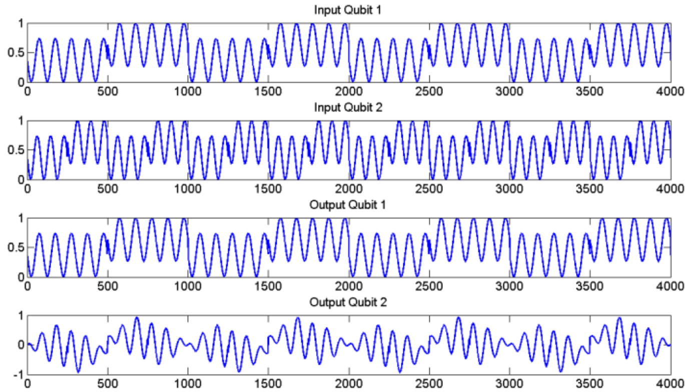

Figure 10 Input and Output of CNOT Gate

One of the most fascinating applications of the Quantum Computing is in creating entangled states. This occurs when the phase relationship between two Qubits forces the observer to describe one particle in terms of another. In other words, any measurement made on one particle instantaneously affects the other. This is what Einstein famously described as 'spooky action at a distance' [9, 45-47].

The most celebrated way of generating entanglement is by creating the Bell states. The Bell states for a two qubit system are given as follows:

|B00> = 0.707(|00>+|11>), |B01> = 0.707(|01>+|10>), |B10> = 0.707(|00>-|11>) and |B11> = 0.707(|01>-|10>)

The generation of the Bell states can be explained using a combination of the Hadamard and the CNOT gates. The schematic is shown in Fig. 11.

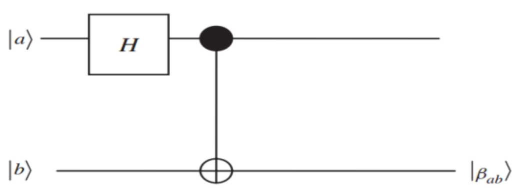

Figure 11 Schematic of Quantum Entanglement

By using the Hadamard gate, the first input is converted into the superposition form. This in turn, acts as a control for the CNOT gate, thus effectively 'imposing' its information onto the second bit. Thus the Bab (a,b=0,1) state is an entangled state, containing the information of a and b.

The inputs and outputs of the Bell State Generation are shown in Fig. 12.

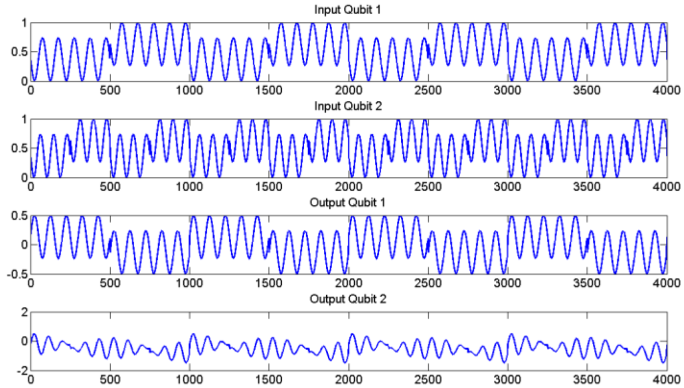

Figure 12 Input and Output of Bell State Generation

QUANTUM TELEPORTATION:

In order to validate the chaos-qubit formulation and the quantum gate formulations mentioned above, a 1-qubit quantum teleportation is implemented. Quantum teleportation is defined as transmitting a quantum state from one place to another without that state traversing the space in between [2-6].

One bit quantum teleportation can be done using appropriate combinations of entanglement, CNOT, Hadamard, measurement and the Pauli X and Z gates.

The schematic of a one bit Quantum Teleportation circuit using a Pauli X gate is shown in Fig. 13.

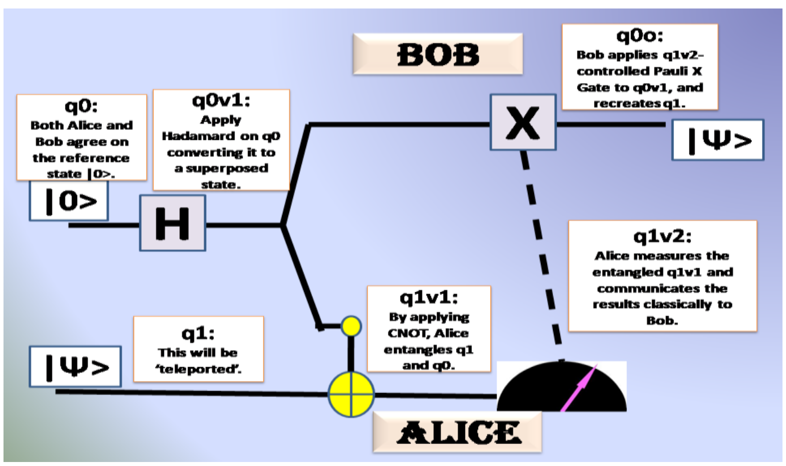

Figure 13 Schematic of 1 qubit quantum teleportation

The input q1 and teleported waveform q0o are shown in Fig. 14.

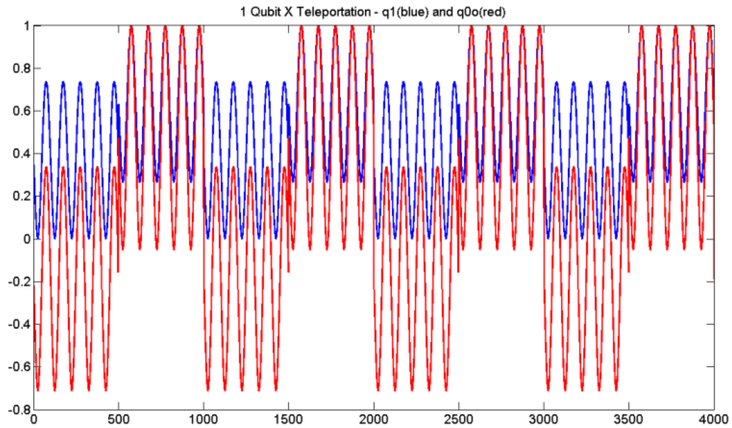

Figure 14 Input and Output os Quantum Teleportation - Input q1 in blue and output q0o in red

As can be seen, the 'teleported' qubit q0o bears a fairly close resemblance to the original qubit q1, affirming the validity of the proposed chaos-qubit formulation.

CONCLUSION

After a short review of the possibilities of nonlinearity and chaos underlying quantum mechanics, the present work proposes a chaos-qubit formulation where a chaotic waveform is regarded as the qubit and the sign and phase flip operations as the corresponding SU(2) generators. Following this, various quantum gates are demonstrated using this formulation including applications such as quantum interference and quantum entanglement. Finally, to verify the formulation, a 1 qubit X gate quantum teleportation is implemented and the closeness of the original and teleported qubits indeed validate the formulation.

Though a detailed investigation into the intricacies and quantitative formulation of a chaos induced quantum mechanics await, the present work took a small step in applying the chaos-quantum equivalency in the domain of quantum information processing. The success of quantum information using chaos opens the doors for a wide variety of applications, most notable of which are cost effective quantum computation and computational universe simulations.

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